PLENARY SESSION

Tuesday, September 14, 2021 | Zoom Session from 04:00 to 07:00 UTC Block 1

NON-PLENARY SESSIONS

Monday, September 13, 2021 | Zoom Session from 07:30 to 11:30 UTC
Tuesday, September 14, 2021 | Zoom Session from 07:30 to 11:30 UTC
Wednesday, September 15, 2021 | Zoom Session from 07:30 to 11:30 UTC

PLENARY SESSION | SYMMETRY, STRUCTURE, AND INFORMATION
Tuesday, September 14 | 04:00 - 07:00 UTC Block 1

INVITED LECTURE | 04:00 - 05:00 UTC

THE DEVELOPMENT AND ROLE OF SYMMETRY IN ANCIENT SCRIPTS
Peter Z. REVESZ, Department of Computer Science and Engineering, University of Nebraska-Lincoln (United States of America)

ENANTIOMORPHIC TALKS ON SYMMETRY | 05:00 - 07:00 UTC

SYMMETRY AND INFORMATION: AN ODD COUPLE (?)
Dénes NAGY, International Society for the Interdisciplinary Study of Symmetry (Hungary and Australia)

ANTINOMIES OF SYMMETRY AND INFORMATION
Marcin J. SCHROEDER, IEHE, Tohoku University, Sendai, Japan

PANEL DISCUSSION
Moderators: Dénes Nagy and Marcin J. Schroeder
Confirmed Panelists: Ted Goranson, Peter Revesz, Vera Viana, Takashi Yoshino

INVITED LECTURE
Tuesday, September 14 | 04:00 - 05:00 UTC

THE DEVELOPMENT AND ROLE OF SYMMETRY IN ANCIENT SCRIPTS

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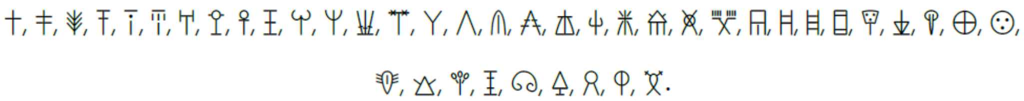
Many ancient scripts have an unexpectedly high number of signs that contain a type of symmetry where the left and right sides of the signs are mirrored along a central vertical line. For example, within our English alphabet, which is a late derivative of the Phoenician alphabet, the following letters have this type of symmetry: A, H, I, M, N, O, T, U, V, W, X and Y. That is, a total of 12/26 = 46.2% of the letters of the English alphabet has this type of symmetry. Similarly, the ancient Minoan Linear A script, which existed from about 1700 to 1450 BC, contains the following mirrored signs:



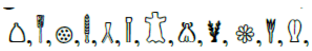
These 42 signs are about half of the most frequent signs in the Linear A script, which are estimated to be about 90 signs. In this paper we try to answer the question of "Why is there such a high percent of mirrored signs in ancient scripts?"

We believe that the unexpectedly high number of symmetric signs is due to a development of writing that is called *boustrophedonic*, or literally as 'the ox goes' meaning that at the end of a line the next line continues right below the end and goes in the opposite direction. Hence a left-to-right line is followed by a right-to-left line, which is again followed by a left-to-right line and so on. This is reminiscent of how oxen plow a plot of land. The main problem with boustrophedonic writing is that when we look at a particular line, we do not automatically know which way it should be read, unlike in modern English texts, where every line is read from left-to-right. As a modern example, suppose we would like to write 'GOD' in a row that is to be read from right-to-left. This looks like an easy task that can be done by simply writing: 'DOG'. The problem is that the reader may not recognize that the row needs to be read from right-to-left, hence 'God' becomes 'dog' for the reader. Ancient scribes compensated for this problem by vertically mirroring any nonsymmetric letter so that the orientation of the words would indicate the direction. Using this concept, instead of 'DOG', they would have written:

'ꓷO⅁'

While boustrophedonic writing with mirroring of asymmetric signs is an attractive solution, and it occurs also in the Mycenaean Linear B script and the Indus Valley Script, it causes the problem of having to know and correctly write the mirrored versions of the asymmetric signs. Many people make mistakes when writing mirrored letters and can read mirrored letters much slower than ordinary texts. We believe that these factors combined with the observation that only a few frequently occurring asymmetric signs are enough to indicate the writing direction led to the development of symmetric forms for most signs.

We test this hypothesis by considering earlier scripts where there are no examples of boustrophedonic writings. For example, the Cretan Hieroglyphic script, a predecessor of the Linear A script, contains significantly fewer symmetric signs, and only the following signs of the Phaistos Disk, which may belong to an even earlier layer of scripts, are symmetric signs:

 and .

That is, only 13/45 = 28.9% of the Phaistos Disk signs are symmetric. Hence, on the island of Crete, the percent of scripts signs with symmetry nearly doubled within a few centuries, showing the importance of symmetry in writing.

ENANTIOMORPHIC TALKS ON SYMMETRY
Tuesday, September 14 | 05:00 - 05:30 UTC

SYMMETRY AND INFORMATION: AN ODD COUPLE (?)
Dénes NAGY (snagydenes@gmail.com)
International Society for the Interdisciplinary Study of Symmetry (Hungary and Australia)

Symmetry (from the Greek *syn* + *metron*, "common measure"), *structure* (from the Latin *structūra*, "fitting together, building"), and *information* (from the Latin *īnfōrmātiō*, "formation, conception, education") are scholarly terms that have ancient roots but gained new meanings in modern science. It is also common that all of these played important roles in interdisciplinary connections, linking even science and art.

1. We argue that *symmetria* - *asymmetria* could have played a relevant role at the birth of mathematics as an abstract science with a deductive methodology (we present a partly new hypothesis which unites those ones by Szabó and by Kolmogorov), then we discuss the related, but different modern meaning-family of symmetry.
2. The structural approach gained special importance in geometric crystallography from the mid-19th century (14 Bravais-lattices), which, using symmetry considerations, led to a major breakthrough in the 1890s by presenting the complete list all of the possible ideal crystal structures, that is the 230 space symmetry groups (Fedorov, Schoenflies, and Barlow). In the early 20th century, the focus on structures in linguistics (Saussure) also inspired the later developments in social sciences, the intensive study of relations and structuralism as a methodology (Lévi-Strauss, Piaget, and others). From the mid-1930s, a group of French mathematicians used a similar path in order to present "new math" (Bourbaki group).
3. Theory of communication led to the study of information in mathematical-statistical context and finally a method for measuring information (Hartley, Shannon, Wiener), which became useful for the emerging computer science in the mid-20th century. Then information theory was also used for the study of aesthetical questions (Moles, Bense).

Looking back, we may see interesting changes:

- The original Greek concept of *symmetria* was related to measurement, but the usual modern understanding of symmetry implies rather a yes/no question: an object or a process is either symmetric or not. We argue that it is important to go back to the roots and to consider symmetry-measures. In fact, the concept of dissymmetry (as the lack of some possible elements of symmetry) gained special importance in structural chemistry (Pasteur), theoretical physics (P. Curie), and crystallography (Shubnikov and Koptsik), pointed out to such a direction.
- The concept *information* was originally not related to measurement, but the modern mathematical approach introduced measures in bits, the number of yes/no questions (Hartley, Shannon). On the other hand, the meaning of information was lost in the case of the mathematical-statistical approach. There were important works related to the meaning of information (MacKay, Bar-Hillel and Carnap, Shreider). It would be important to unite these two and modify them according to the

new needs. Quantum computing needs a new information theory related not to bit, but to qubit; here we may need symmetry considerations (cf., Bloch sphere representation).

In some sense, the “odd couple” of symmetry (as an ordering principle) and information (knowledge based on measurements) came together for solving the Maxwell-demon problem. In this thought experiment, which seemingly violates the law of entropy, the demon as the doorman between the two chambers of a closed container filled with gas, introduces new order by separating the high-speed and the low-speed gas molecules by opening the door always in due time. This method would solve our heating and cooling problem in everyday life. The demon, however, should use information, specifically measuring the speed of molecules for the purpose of his work (Szilard). Thus, it would be “a” delated very expensive heating and cooling. Another example where symmetry and information work together: The vertices of some regular and semi-regular polyhedra inscribed into a sphere present the centers of circles in the case of densest packing of a given number of equal circles on this sphere (Tammes problem), which is important for spherical coding. The term *information asymmetry* is well-established in economic science. The fact that it may create an imbalance of power in transactions, and, in the worst case, market failure led to various studies and, actually, the Nobel-prize of three economists (Akerlof, Spence, and Stiglitz)

We suspect that some generalized symmetry and information concepts, which are required by the recent developments in science and art, may help each other.

ENANTIOMORPHIC TALKS ON SYMMETRY
Tuesday, September 14 | 05:30 - 06:00 UTC

ANTINOMIES OF SYMMETRY AND INFORMATION

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This is a proposal of the resolution of several apparent antinomies within the studies of information, symmetry, and of the mutual relationship between symmetry and information. A selection of examples of such antinomies is followed by a nut-shell overview of their solution.

The earliest example of the opposition in the views on information can be found in the critical reaction to the claim of Shannon's foundational work denying importance of the semantic aspects of communication. This denial exposed his work to the objection that it is not about information. The issue was never completely resolved, although it faded with increased popularity of naive claims that the problem disappears when we demand in the definition that whatever information is, it has to be true.

The relationship between the measure of information given by Shannon in the form of entropy and the measure called negentropy introduced by Schrödinger as a magnitude which although being non-negative has its value opposite to the non-negative entropy is antinomial. This curious pairing, although apparently sufficiently harmless not to attract much attention, is a tip of the iceberg of much deeper internal opposition in the view of information. Shannon's view of information is tied to the recipient of a message i.e. it is observer's view. Schrödinger's negentropy is a numerical characteristic of the acquired freedom in forming organized structure within the system.

An example representing antinomies of symmetry has a form of an opposition of two oppositions. One of them is between the artificial, intentional character of symmetry associated with human aesthetical preference, and the natural character of asymmetry associated with spontaneous, unconstraint generation of forms. The other, reversed opposition is provided by the biological evolution in which the steps in the transition to higher form of life are marked by diverse forms of breaking symmetry leading from the highly symmetric proto-organismic simple systems to the complex human organism with its asymmetric functional specialization.

Finally, there is an example of the opposition in views on the relationship between information and symmetry with its main axis between the claim that information has its foundation in asymmetry and the view that physics is essentially a study of symmetries, so if information is physical we should base its study on the analysis of its symmetries. The former position originates in the Curie Principle that symmetric causes cannot have asymmetric effects justifying the focus on asymmetry, as it can guide us to the actual causes of phenomena. The early expression of Bateson's metaphor of "information as a difference which makes difference" was in his explanation of the rules of biological asymmetry.

The elimination of these and other antinomies is based on the recognition of the two manifestations of information, selective and structural. The latter requires involvement of symmetry understood as invariance with respect to groups of transformations. The key point is that the apparent antinomies of information, symmetry, and of their relationship are consequences of the fallacious idea of asymmetry, which obscures the relations and transitions between diverse forms of symmetry.

ENANTIOMORPHIC TALKS ON SYMMETRY - PANEL DISCUSSION
Tuesday, September 14 | 06:00 - 07:00 UTC

MODERATORS: Dénes Nagy and Marcin J. Schroeder

INTRODUCTION TO THE PANEL DISCUSSION

The theme of this discussion and the conference is Symmetry, Structure, and Information. Each of these three ideas escapes a commonly accepted definition. On the other hand, if you ask a passerby whether he or she understands the words symmetry, structure, information most likely the answer would be "sure". In a very unlikely case, the answer could be "not at all, but I really would like to understand symmetry" showing that the person knows a lot. Then invite him or her to attend the Congress on Symmetry in Porto (https://symmetrycongress.arq.up.pt/) next July.

We can expect a question about the objectives of discussing the triad of symmetry, structure, and information. After all, if we add one more idea of complexity then we have a collection of the most elusive and at the same time most important notions of the philosophical and scientific inquiries. Isn't it better to focus on each of them separately and only after we have clear results of such inquiries to attempt synthesis? This is the main question addressed to the panelists and the audience.

This question can be reformulated or complemented by the question about the importance, or its lack, of the mutual relationships between the ideas in the leitmotif of the conference. This includes importance for philosophical, theoretical, or practical reasons.

Finally, we can consider the question about what is missing from the picture which is painted by the title with the three ideas only. What ideas, notions, concepts we should include, or even we should give priority in our inquiries of symmetry?

NON-PLENARY SESSIONS | MONDAY, SEPTEMBER 13 2021
Zoom session from 7:30 to 11:30 UTC

September 13 | 07:30 to 08:00 UTC

TILINGS FOR CONJOINED ORIGAMI CRANES USING LESS SYMMETRIC QUADRILATERALS
Takashi YOSHINO

September 13 | 08:00 to 08:30 UTC

DEMONSTRATION OF THE CONEPASS TO CONSTRUCT THE GOLDEN ANGLE
Akio HIZUME

September 13 | 08:30 to 09:00 UTC

ARTISTIC INTUITION: HOW SYMMETRY, STRUCTURE AND INFORMATION CAN COLLIDE IN ABSTRACT PAINTING
Marina ZAGIDULLINA

September 13 | 09:00 to 09:30 UTC

FUTURE ETHNOMATHEMATICS FOR A 'NEW BLETCHLEY'
Ted GORANSON

September 13 | 09:30 to 10:00 UTC

FRACTAL-LIKE STRUCTURES IN INDIAN TEMPLES
Sreeya GHOSH, Paul SANDIP and Chanda BHABATOSH

September 13 | 10:00 to 10:30 UTC

INNER ANGLES OF TRIANGLES IN PARAMETER SPACES OF PROBABILITY DISTRIBUTIONS
Takashi YOSHINO

TILINGS FOR CONJOINED ORIGAMI CRANES USING LESS SYMMETRIC QUADRILATERALS

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We consider the variations of tilings for Renzuru (連鶴), conjoined origami cranes. The traditional Renzuru were summarized in the famous book titled *The Secret Heritage of How to Fold Thousands of Cranes* (秘伝千羽鶴折形) published in 1797 (Okamura [01]). All of the origami cranes in the book were folded from squares. On the other hand, recent researches revealed that the origami cranes could be folded non-square paper sheets (Justin [02] and Kawasaki [03]). Incorporating the results, we construct a list of variations of Renzuru consisted of congruent quadrilaterals.

We focused our consideration on "periodic Renzuru tilings," which is defined in this study as satisfying all the three following conditions:

1. the periodic tilings of congruent quadrilaterals or congruent polygons consisting of some quadrilaterals on an infinite plane,
2. origami cranes could be folded ideally from all the constituting quadrilaterals,
3. each crane was connected with the others at the bill, tail, and wingtips.

This study aims to obtain the complete list of the tilings satisfying the above three conditions. We focused on the case that one type of quadrilateral is used for tilings hereafter. In other words, we only considered the tilings with congruent quadrilaterals, which are foldable to origami cranes.
According to Justin [02] and Kawasaki [03], the necessary and sufficient conditions for a sheet shape of origami cranes are the quadrilateral having an inscribed circle. This condition is equivalent to "the sums of opposite edges of the quadrilateral are equal."
On the other hand, Grünbaum and Shephard [04] made a complete list of the tilings using one type of convex quadrilateral. According to the results, the variation of the tilings is up to fifty-six.
Our method for completing the list of the periodic Renzuru tilings consists of two steps: First, we select the "edge-to-edge" tilings among the fifty-six tilings. The edge-to-edge tilings were defined that their vertices must not appear on the edges of the adjacent quadrilaterals.

We found ten types of periodic Renzuru tilings consisting of one type of quadrilateral shape. In our presentation, we will show the results (Periodic Renzuru tilings with one quadrilateral shape), with all the tilings. Eight tilings consist of the vertices of the degree of four. On the other hand, the remaining two tilings include the vertices of degrees of three and six. They are new types of conjoined origami cranes having 3-fold and 6-fold symmetries. Their examples will be shown in the presentation.
We started to list the periodic Renzuru tilings using two different quadrilaterals consisted of a pentagon. At present, we found six tilings that satisfy the definition of the periodic Renzuru tilings. We will discuss this type of tilings if we finish listing them.

References

[01] Okamura, M. (2006) *The World of Conjoined Cranes*, Hon-no-Izumi Sha (in Japanese)
[02] Justin, J. (1994) Mathematical Remarks about Origami Bases, *Symmetry: Culture and Science*, Vol.5, No.2, 153-165
[03] Kawasaki, T. (1998) *Roses, Origami & Math*, pp.162, Morikita Shuppan (Japanese Edition).
[04] Grünbaum, B. and Shephard, G. C. (1987) *Tilings and Patterns*, W. H. Freeman and Company, 700 pp.

DEMONSTRATION OF THE CONEPASS TO CONSTRUCT THE GOLDEN ANGLE

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The golden ratio is the simplest fractal structure. Acoustically, it is the highest dissonance, and engineering-wise, it is the ratio with the best stirring efficiency. Plants are known to have thrived on this golden ratio for hundreds of millions of years [01]. By growing one leaf after another at the Golden Angle (about 137.5°), the plants serve the dual purpose of evenly basking in sunlight and being hard to topple.
For the last 35 years I have been working with the peaceful use of the Golden Ratio, learning mainly from plants and quasicrystals.
In particular, I am proud to have embodied the "idea" of the plant in the Origami Fibonacci Turbine using the Golden Angle, which I invented in 2018 [02].
Nevertheless, how do plants construct the Golden Angle the first place?
This has been an open question for many years.
Once this is known, the last missing link regarding plants will have been solved.
Modern scholars are also working on this problem, but they tend to fall into a premeditated simulation. No matter how much they look at the plant's growing point under a microscope with any amount of magnification, they are unlikely to get an answer. At the root of this difficulty is the fact that it is impossible to construct a golden angle with a ruler and a compass, which seems to be the crux of the problem. Therefore, I dare to change the way I formulate the problem. That is, "Let's invent any elegant tool to construct the Golden Angle precisely, beyond the ruler and the compass". In this way we can expand our freedom of thought considerably.
After much thought experimenting and trial and error, I came up with a solution in 2020, which I report here.
A cone could be created by making a single radial cut in the circle. Surprisingly, this operation contains a mechanism for producing the exact Fibonacci sequence. By this simple principle, the Golden Angle can be rigorously constructed. I have named it the "*Conepass*" [03, 04, 05]. The term is a combination of the words "Cone" and "Compass".
The Kepler Triangle, which constitutes the cross section of the pyramid, also plays an important role in the constructing process.
In the symposium, I will demonstrate the constructing of the Golden Angle with a *Conepass*. Audiences will be surprised by the simplicity of the process.

References

[01] Kazuo Azukawa Sunflower Seeds, *Sugaku Seminar* (1985 in Japanese)
[02] Akio Hizume, Origami Fibonacci Turbine, *11th Congress and Exhibition of SIS* (2019).
[03] Akio Hizume, Cone-Pass revised, preprint, *ResearchGate* (2020).
[04] Akio Hizume, Cone-Pass #2, preprint, *ResearchGate* (2020).
[05] Akio Hizume, Cone-Pass, how to construct the Golden Angle, *Bulletin of Musashino Art University no.51*, (2021 in Japanese)

ARTISTIC INTUITION: HOW SYMMETRY, STRUCTURE AND INFORMATION CAN COLLIDE IN ABSTRACT PAINTING

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This case-study is based on the media-aesthetic analysis of one work of art by French artist Guy de Montlaur (1918-1977; see: *The Destinies*…, 2018) through the lens of symmetry-structure-information logics (as Gregory Bateson understands it; see Bateson, 1971).

As Bateson suggests, symmetry is the basic rule of nature (a way to balance the material world), structure is the material realization of this general law (or "the morphology" of living entities), and information is a force that organizes the process of materialization of symmetry in structure. Gregory Bateson, who develops his father's (William Bateson) ideas (see Bateson, 1894), believes that not only information itself but also the *lack* of information becomes a stimulus to morphological changes as an answer to this lack.

As it was broadly proved by Piaget or Weyl, the boundary between biological, mathematical, philosophical approaches to structure-symmetry investigations is blurred. That is why the case study from the art-field can contribute to the general theory of symmetry in its specific relation with structure and information. This case is of particular interest to researchers of the current communication field because there is an overlap of three domains of communication: its materiality (media), its code (a key) and its content (a message). Also, by analysing the painting by Guy de Montlaur, we contribute to the transmediality theory as a part of a broader Information theory (how one type of art can be converted into another one).

The case-study is based on the following data:

1. The painting "Quatrain" refers to Arthur Rimbaud's "*Quatrain*", 1871 (1), where the poet develops his theory of colours, related to sounds ("*Voyelles*"). This theory ("A noir, E blanc, I rouge, U vert, O bleu", or, more broadly speaking, the correspondence between "a person's state of mind" and colours) has a long cultural tradition, where the struggle between "symmetry" and "asymmetry" is important. In the presentation, the asymmetrical approach by Newton is compared with Goethe‘s symmetric approach. Here, the code for "one's state of mind" through the colours can be seen (= to be deciphered at the level of the meaningfulness of colours).
2. Rimbaud's "*Quatrain*" is a "trap": actually, the poet symmetrically points only two colours (the 1st line is in symmetry with the 3d line, with the dominant E – whiteness, which joins together the star and the woman's body; the 2nd and the 4th lines contain the dominant A – blackness, constructing a parallel between "infinity" and "man" as a background of the star and the woman. This provocation has been reflected in the painting through the colours and shapes. (It is easy to note comparing the sketch of the painting and the painting itself, for example, transforming of "the ear" into "the heart".)
3. Coming back to Bateson's idea: "the trap" (or "false-colours") can be considered as *a lack* of information that stimulates the imagination and helps see "beyond" the shapes and colours. The symmetrical rule works: in the drawing (the sketch), the symmetrical centre is at the intersection of

"white" star-woman and "black" infinity-man. The symmetry of the whole structure of this painting is meaningful (in the drawing, it is clear: the "ear-heart" reflects a black figure along the ascending diagonal, and the "white star" reflects "bleeding breast" along the descending diagonal.

Conclusions:

(For the art-criticism field):

Abstract painting can be percepted and understood (perceptive-cognitive complex, see Redies, 2013); For example, indication of a source of the artist's inspiration (in our case – the poem by Rimbaud). may help us understand the concept behind it, and even more: we can easily track precise and concrete images disguised using some more additional information.

(For the theory of symmetry and the communication field):

Even knowing the source of inspiration, the public need to get a key to transmediality (how one code is transformed into another one). Symmetry can be such a key: its mechanism makes it clear to see how "the need for parallelism" works: many codes and images are grouped into a major binary opposition; in the case of Montlaur's painting it is black and white symbolic juxtaposition (The artist has used a lot of different colours though). Any piece of art can be considered as a phenomenon of communication, where three domains merge: materiality (as a medium), codes (keys) and content (a message, a sense), these domains can be interpreted as structure, symmetry, information.

References:

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Bateson, W. *Materials for the Study of Variation,* Mac Millan and Co., 1894.

Piaget, J. *Structuralism*. New York. Harper & Row, 1971.

Weyl, H. *Symmetry,* Princeton. Princeton University Press, 1952.

Redies, C. Combining universal beauty and cultural context in a unifying model of visual aesthetic experience, *Frontiers in Human Neuroscience,* Issue 9, 2015, article 218, pp. 1-20, doi 10.3389/fnhum.2015.00218.

The Destinies of Abstract Expressionism: For the Centenary of Guy de Montlaur's Birth (1918-1977): collective works. Moscow: RSUH, 2018.

List of paintings:

Guy de Montlaur. *Quatrain***.** Huile sur toile, 60 cm x 92 cm. Paris, mai 1975 (https://montlaur.net/retrospective/2021-06-06-quatrain.html)

Notes:

Arthur Rimbaud, *Quatrain* (1871):

L'étoile a pleuré rose au coeur de tes oreilles,
L'infini roulé blanc de ta nuque à tes reins
La mer a perlé rousse à tes mammes vermeilles
Et l'homme saigné noir à ton flanc souverain.

FUTURE ETHNOMATHEMATICS FOR A 'NEW BLETCHLEY'

Ted GORANSON (ted.goranson@anu.edu.au)
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As human engineers, we are more capable in understanding and working with the world; our scientific insights have grown enormously. One can argue about where we are on that path, at the beginning or middle, but some areas clearly reveal relative ignorance; we don't understand many causal mechanisms in the human body; outstanding mysteries in physics are profound.

Tools for understanding have a food chain, and at the beginning of that food chain is the ability to perform abstract reasoning and develop supporting tools and methods. This paper speculates on what future tools for abstraction will look like, with an emphasis on useful power. We suppose spatial reasoning and deep abstraction support each other.

The metaphor of the paper is *ethnomathematics*, being the study of sophisticated abstractions in societies that lacked publishable notations. We presume that publishable mathematical notation and conforming syntax constrain intuitive abstraction, so it may be time to rethink what we could do without the 'constraints of the page'. We follow *symmetry* as a guide in anticipating what these might be.

The Success of Symmetry

A consensus view has the roles of intuition, mathematical structures, and science related in a specific way, expressed for example by Eugene Wigner in 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences', in how one can approach a mystery. The best way, he credibly claims, is to take mathematical abstractions that have been applied to a similar physical phenomenon and follow a dual path of expanding the abstractions and application to the problem in tandem. He notes but does not explore in any depth the role of intuition in these two linked paths. That is, the detective needs to have some natural feel for how mathematical structures evolve, and at the same time have some similar natural feel for how nature works, what some Japanese scientists' term '*Katachi*' (1).

Wigner and his colleagues were working with a set of abstractions with core symmetries that led to the standard model of physics. This is a complex set of overlapping symmetries at different levels that subsumes quantum physics and stands as a candidate for the most successful theory in science. While Wigner and colleagues were working with particle physics, this combined quantum/standard theory anchors most of the natural sciences (2). Foundational discoveries are routinely made by extrapolating from 'missing symmetries.'

An essential element in this process is that the mathematics has a sense; the world also presents a sense. Talented thinkers can merge these two in a common metaphoric system. We believe this has a visual component, is strongly influenced by narrative, and is constrained by neural biology that evolved for one purpose and is adapted for others. Using these insights, what is next 'symmetry'? For reasons described in the paper, we'll describe it as a *future ethnomathematics*.

Contributing Disciplines

A transdisciplinary project is called for, perhaps with the following contributing disciplines.

Ethnomathematics. As humans, we've had the capability for speech for at most 200,000 years, written language, possibly 6,000, and mathematical notation at most 300. Yet we know sophisticated communication regarding toolcraft goes back at least 2 million. We see complex abstract reasoning and representation in current preliterate societies unaffected by the biases of print. In particular, string figures and gesture narratives may indicate abstraction models based on reality, body and neural 'hardwiring'.
Form Morphology. Space has intrinsic structural properties that many disciplines have explored. Can we develop an abstraction strategy from form itself to anchor future 3D and Virtual Reality "notations"?
Hippocampus Complex Neurobiology. We've evolved a neural toolset for mapping and navigation that has been adapted for different purposes. Can we better understand these dynamics to inform our 'natural' strengths? Can these inform future bionic strategies? Is eidetic memory leverageable?
Type and Category Theory. We do have mathematics that are optimised for reasoning about abstraction. Can we employ these where we once promoted group and set theory?
Katachi. Are there insights from Eastern traditions that can advise this program?
Symmetry. The International Society for the Interdisciplinary Study of Symmetry probably has mechanisms and members that can help. When formed, the society subsumed ethnomathematics as it was. Very likely it can be employed in this program.
Narrative. In everyday life, the way we structure and communicate experience is through narrative structures. These have accessible dynamics and dependencies, including intricate introspective mechanisms that should at least inform the program.

A Resulting Institution
A good case can be made that Bletchley Park was the single research program with the most valuable results in history. Though the immediate focus was decryption in the context of war, nearly every advance in information science has a fundamental root in the associated work. A key, we suggest, is that the agenda was guided by pure mathematicians rather than engineers, as say, the Apollo Program. Another was the intense urgency of the sponsors. We suppose we can recreate that environment if the skills and goals are crisply defined and create a research program with similar benefits.

A Goal
We believe that the tools and methods available to information engineers have structural limits. Ontologies focus on entities/states, so transformations are second class citizens. Semiotics in current forms does not address this profound deficiency. Reasoning is set-theoretic, so reasoning over the open set must be by improvised methods. The use of modal and/or probabilistic colourings limits vision into non-ergodic effect spaces. Underlying relations have conventionally been group theoretic, which biases against situated influences. Our fundamental tools are advancing quickly; for example, homotopy type theory promises a new foundation for relevant mathematics. Category theory now provide robust proof methods and implementations using functional methods. Therefore, we believe this program, or something like it, could give us the next generation of pervasive metaphors that Wigner supposed, to expand 'effectiveness'.

1. The idea behind *katachi* is that the elegance of nature has both order and beauty. The more successful you are in internalising the combination of beauty in order and vice versa, the better scientist you will be. A Katachi Society nurtures this.
2. A secondary controversy concerns whether the universe is inherently geometric (in a larger sense), or whether the geometries are an effect from complex emergence of overlapping behaviour. Many senior figures align on both sides. We will prefer the former.

FRACTAL-LIKE STRUCTURES IN INDIAN TEMPLES

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A fractal is a symmetrically expanding pattern which exhibits self-similarity. They are recursive patterns across different scales such that any small portion resembles the whole. Fractals found in nature include, branches of trees, phyllotaxy, animal circulatory systems, snowflakes, clouds, lightning, geographic terrain, river systems and many more. An object possesses self-similarity if each of its parts is geometrically similar to the whole. The Hindu philosophy believes that every being is a miniature form of the Cosmos. This concept is portrayed through the fractal-like architectural features of the Indian temples. The architectures of the Indian temples have evolved from the cave temples to the palatial temple complexes through the passage of time. From the Gupta period i.e., the 3rd century CE onwards, complex aesthetic features of the Indian temples gradually developed [01,02]. Geometry played a pivotal role in the construction of the Indian temples. The architectural principles of these temples are described in the *Shilpa Shastras (texts dealing with arts and crafts)* and the *Vastu Shastras (texts dealing with building architecture).* The *garbhagriha* or the sanctum enshrining the deity follows a geometrical design. The spire rises symmetrically above the sanctum and is the most prominent and visible part of an Indian temple. Different regions in India have different names and styles of the temple spire.

- In *Nagara* Architecture of North India, the spire known as the *Shikhara,* is usually curved in shape.
- In *Dravidian* Architecture of South India, the spire known as the *Vimana,* is usually four-sided pyramidal.
- In Vesara Architecture of Central India, the spire has a synthesis of the Nagara and Dravidian styles.
- In the temple architecture of Bengal [03], the spire, known as a *Ratna,* usually rises conically. The number of these *ratnas* above the main sanctum can vary from one temple to another.

Though, the structural features of the spires are different for different architectural styles, self-similar patterns can be observed in all.

The main focus of this current work is to highlight the fractal-like patterns in the different styles of the Indian temple spires as perceived through their images. To achieve this, self-similar patterns would be recognised through detailed visual analysis and feature matching techniques of the images and hence similarities between images would be measured [04]. Furthermore, calculation of fractal dimensions of the spires using Box Counting algorithm would be attempted [05].

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INNER ANGLES OF TRIANGLES IN PARAMETER SPACES OF PROBABILITY DISTRIBUTIONS

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We consider triangles in parameter spaces of probability distributions by using the formalism of information geometry. Information geometry treats the parameter spaces of probability distributions by using the methods of differential geometry. One of the significant results in information geometry is a formula referred to as the "Generalized Pythagorean Theorem." Although the orthogonality is sometimes discussed using the theorem, the triangle shape is seldomly mentioned because the purpose of information geometry is to obtain the most optimized situation. Hereafter, we discuss inner angles of triangles in parameter spaces in order to consider their structures.

We present a short summary of information geometry in order to explain some concepts used in the following discussion. We restricted our consideration within the parameter spaces of the probability distributions belonging to the exponential family. The probability distribution function of the family is written as,

$$p\,(x, \boldsymbol{\theta}) = \exp\{\theta^i h_i(x) + k(x) - \psi(\boldsymbol{\theta})\},$$

where x is a random variable and $\boldsymbol{\theta} = (\theta^1, \theta^2, \cdots, \theta^n)$ is a set of parameters, $\boldsymbol{x} = (h_1(x), h_2(x), \cdots, h_n(x))$ represents a set of random variables generated from x, $k(x)$ is for a measure ($d\mu(\boldsymbol{x}) = \exp k(x)\, dx$) and $\psi(\boldsymbol{\theta})$ is for the normalization condition written as,

$$\psi\,(\boldsymbol{\theta}) = \log \int \exp\,(\boldsymbol{\theta} \cdot \boldsymbol{x})\, d\mu(\boldsymbol{x}).$$

The function $\psi(\boldsymbol{\theta})$ is convex and is known as the cumulant generating function. The dual affine parameter $\boldsymbol{\eta}$ is obtained from the convex function as,

$$\boldsymbol{\eta} = \nabla\psi\,(\boldsymbol{\theta}) = \log \int \boldsymbol{x}\; p\,(\boldsymbol{x}, \boldsymbol{\theta})\, d\mu(\boldsymbol{x}) = \mathbf{E}[\boldsymbol{x}].$$

the dual geodesic line which connects two points P and Q are represented respectively by using $\boldsymbol{\theta}_{\mathrm{P}}$, $\boldsymbol{\theta}_{\mathrm{Q}}$, $\boldsymbol{\eta}_{\mathrm{P}}$, and $\boldsymbol{\eta}_{\mathrm{Q}}$ as,

$$\boldsymbol{\theta}_{\mathrm{PQ}}(t) = (1-t)\boldsymbol{\theta}_{\mathrm{P}} + t\boldsymbol{\theta}_{\mathrm{Q}} \text{ and } \boldsymbol{\eta}_{\mathrm{PQ}}(t) = (1-t)\boldsymbol{\eta}_{\mathrm{P}} + t\boldsymbol{\eta}_{\mathrm{Q}}.$$

We focused on the triangles consisting of three points P, Q, and R in parameter spaces. The orthogonality of the two geodesic lines at the point Q was written as,

$$\dot{\boldsymbol{\theta}}_{\mathrm{PQ}} \cdot \dot{\boldsymbol{\eta}}_{\mathrm{QR}} = (\boldsymbol{\theta}_{\mathrm{Q}} - \boldsymbol{\theta}_{\mathrm{P}}) \cdot (\boldsymbol{\eta}_{\mathrm{R}} - \boldsymbol{\eta}_{\mathrm{Q}}) = 0$$

The generalized Pythagorean theorem was obtained by applying this relation to the Kullback- Leibler divergence. As the simple extension of this definition, we defined the inner angle at the point Q as,

$$\cos\angle\mathrm{Q} = \frac{(\boldsymbol{\theta}_\mathrm{Q} - \boldsymbol{\theta}_\mathrm{P}) \cdot (\boldsymbol{\eta}_\mathrm{R} - \boldsymbol{\eta}_\mathrm{Q})}{\|\boldsymbol{\theta}_\mathrm{Q} - \boldsymbol{\theta}_\mathrm{P}\| \|\boldsymbol{\eta}_\mathrm{R} - \boldsymbol{\eta}_\mathrm{Q}\|}. \tag{1}$$

The other inner angles could be defined the same way as above. The explicit form and numerical value could be ideally obtained by substituting the equations of $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ depending on the probability distribution.

In the cases of Gaussian distributions, the parameters $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ were represented as,

$$\boldsymbol{\theta} = \left(\frac{\mu}{\sigma^2}, -\frac{1}{2\sigma^2}\right) \tag{2}$$

and

$$\boldsymbol{\eta} = (\mu, \mu^2 + \sigma^2), \tag{3}$$

where μ and σ^2 were mean and variance of a Gaussian distribution, respectively. The formula of angle could be obtained by substituting Eqs. 2 and 3 to Eq. 1, however, it had a highly complicated form. Therefore, we omitted to demonstrate the complete form here.

All numerical calculations of the sum of inner angles of triangles resulted in $\pi/2$ (not π). The results were obtained by using random numbers. Therefore, we predicted that the sum of the inner angles of the triangles in Gaussian parameter space always became $\pi/2$ although we do not prove it analytically.

We also consider the discrete probability distributions because the distributions belong to the exponential family. Their probability distribution is written as $\boldsymbol{p} = (p_0, p_1, p_2, \cdots, p_n)$. And the parameters $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ are derived as,

$$\theta_i = \log\frac{p_i}{p_0} (i = 1,2, \cdots n),$$

and

$$\eta_i = p_i \ (i = 1,2, \cdots n).$$

It should be noted that the number of elements of both parameters was not n+1 but n because of the constraint of $\Sigma p_i = 1$.

We carried out the numerical calculations of the sum of the inner angles in the cases of $n = 1$, $\boldsymbol{p} = (1 - p_1, p_1)$, and $n = 2$, $\boldsymbol{p} = (1 - p_1 - p_2, p_1, p_2)$. Finally, we predicted that the sum of the inner angles became π in the case of $n = 1$, however, the values varied in the case of $n = 2$.

In conclusion, the sum of the inner angles depends on the parameter spaces, even in the exponential family. In some cases, we predict that the sum becomes constant. We should consider other specific cases (e.g., regular triangles) to understand the properties of the triangles in the parameter spaces. Furthermore, we should discuss the triangle centres (incentre, circumcentre, etc.) and other properties of triangles. In the forthcoming presentation, we will discuss such topics.

NON-PLENARY SESSIONS | TUESDAY, SEPTEMBER 14, 2021
Zoom session from 7:30 to 11:30 UTC

September 14 | 07:30 to 08:00 UTC

GROWN MATTER DESIGN, SYMMETRY, STRUCTURE, & INFORMATION
Juan Manuel Villa CARRERO, Ismael Humberto Garcia PÁEZ and Maria Camila ARAQUE

September 14 | 08:00 to 08:30 UTC

NUMERICAL TOOLS IN DESIGN PROCESSES OF VARIOUS TYPES OF SPATIAL STRUCTURES
Janusz RABIELAK

September 14 | 08:30 to 09:00 UTC

"CONVENIRENT ... EXACTISSIME": FOR EMPIRICAL RIGOR IN THE APPLICATION OF THE CONCEPT OF (BILATERAL) SYMMETRY
Michael SELZER

September 14 | 09:00 to 09:30 UTC

BREAKING SYMMETRY AND INFORMATION IN DESIGN
Patricia MUÑOZ

September 14 | 09:30 to 10:00 UTC

PATTERNS IN ORIENTAL CARPETS: SYMMETRY, STRUCTURE, AND KNOWLEDGE
Carol BIER

September 14 | 10:00 to 10:30 UTC

LOGICAL POLYGON AS A TOOL FOR PHILOSOPHICAL REASONING: ONE QUESTION ON INTROSPECTION
Oksana CHERKASHINA

GROWN MATTER DESIGN, SYMMETRY, STRUCTURE, & INFORMATION

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This article results from our interest in experimenting with matter and its forms in its natural state, this eagerness aims to understand and conduct the natural systems of formation for use especially in architecture, design, and obtaining relevant structures. In particular, this paper explains the experimentation with the aggregation of mineral concentrations in supersaturated aqueous environments in proximity with seed particles, using the information emitted by their aggregation and growth processes as a basis for studying and exploring the potential for form generation, and the scaling of these systems.

Our motivation with this research project was to establish a base of principles for the design and implementation of dynamic architectural systems, ensuring directionality, connectivity, and the ability to predict the properties of growth that allow assisting in the generation of forms, i.e., design the growth of these natural processes for functionalities beyond the orthodox practice in architecture.

The project was developed in three stages, the first focused on data capture and analysis in time and space, through the study of the growth of crystal structures in proximity with seed particles, which provided the necessary information for understanding the phenomenon of particle aggregation studied. In a second stage, with the help of a Rhinos-Grasshopper environment, the hypothesis about the studied aggregation phenomenon allowed its digital simulation, and therefore the prediction of its directionality and structure at a larger scale, which required a larger volume of data obtained through predictive analysis of the results of the first stage with an algorithm deployed within the Anaconda developer's platform. The third stage experimented with the directionality of the material structure and conditioned it to deltas of temperatures, and lighting and dark conditions, as well as physical perturbations in an analogously simulated aqueous medium.

The results indicate that third-party perturbations or perturbations direct the aggregations of crystalline particles and tend toward symmetric orders. We also validate those different levels of temperature and illumination affect their development, thereby interfering with particle aggregation, and thus modelling. These findings allow us to conclude that the direction of particle aggregations varies directly in proportion to the energy levels with which it interacts, e.g., light, temperature, or perturbations. These illations open a stage of validation at larger scales and complexities, e.g., they envision automatic repair and self-organization, as well as reveal possible artificial self-manufacturing or self-assembly.

NUMERICAL TOOLS IN DESIGN PROCESSES OF VARIOUS TYPES OF SPATIAL STRUCTURES

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The notion of spatial structure refers to a very large group of structural systems applied in architecture and in numerous fields of engineering. Within the group one can distinguish also numerous subgroups of families of such systems having different forms and having being designed also for different useful purposes. Spatial structures are mostly defined as the structural systems consisting of component parts uniformly arranged in their spaces and moreover the force transmission process between these parts is also done in the spatial way. They are the multifold statically indeterminate systems that is why the structural analyses and designs of them are sometimes very complex, sophisticated and time consuming. The space structure of big geometric dimensions is usually subjected to act of the huge load forces. If the structure is moving, flying in the air or sailing on the sea or lake then symmetry rules have to be kept during the design, assembly and exploitation stages of it. The very complex design processes of such objects are accelerated and make easier by application of suitable programming languages and computer software. Digital models defined in the numeric technology make all the design stages more efficient due to effective interchange of information between all participants involved in the investment project.

In the intended paper will be presented methods or procedures of the design or analyses of various types of the spatial structures worked out by the author. There will be spoked results of application of some methods useful in the design of the truss structural systems. Numerical models of selected types of the tension-strut structures will be analyzed and there will be presented their graphical visualizations. The truss and tension-strut structures are often devoted for the roof covers support systems having large clear spans. These types of structural systems, due to their big geometric dimensions and due to great values of the loading forces, have to be mostly of symmetric forms. It implies further that in the design procedures the symmetric ways and methods of analyses are also mostly used. Moreover mutual, direct, and permanent exchange of information, between all participants of the design or investment teams, has to be efficiently maintain. Specific type of the foundation structure belongs also to the group of spatial structures and can be applied as the safe base for foundation of the very heavily loaded objects situated on grounds of very small load carrying capacity or in the earthquake areas. Conceptual designs of new types of airplane structures will be shown as results of visualizations of their numerical models defined in the programing language Formian. Both recently spoken structures have the same characteristics and requirements like trusses regarding the design procedures including the necessity of the precise, unobstructed interchange of information between all groups of designers and manufacturers of them. It is obvious that basic principles of symmetry have to be obey in the design processes of these structural systems, what is very clearly visible on examples of defining of their numerical models.

"CONVENIRENT … EXACTISSIME": FOR EMPIRICAL RIGOR IN THE APPLICATION OF THE CONCEPT OF (BILATERAL) SYMMETRY

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The concept of symmetry introduced by L. B. Alberti in the fifteenth century pertains to forms whose two lateral halves mirror each other (*destra sinistris convenirent*) in even the most minute detail (*exactissime … in minutissimis*) [01].

"Symmetry" has acquired numerous additional meanings since the 15th century, but its most common use is probably still for the mirroring of the two lateral halves of a form - a usage that is affirmed in all standard dictionaries and that (as Weyl correctly states) has "a concrete, precise, meaning" [02]. And it is as a concrete, precise, term for bilateral symmetry that I use "symmetry" here.

Alberti's concept is prescriptive as well as descriptive. He declared that Nature's forms, our prototypes of beauty, are invariably symmetric; and that therefore, if we want the things, we make to be beautiful, we must take pains to make them symmetric. Alberti claimed that this principle was recognized by the ancient Greeks and Romans, leading them to ensure that their buildings, works of art, and other artifacts were always shaped symmetrically.

That claim has been amplified greatly since Alberti's day. It is (for example) a commonplace among art historians, anthropologists, and others, that symmetry is "one of the most ancient and fundamental characteristics … of art the world over" [03]. Natural scientists are among those who make similar claims. A Nobel-winning physicist, for example, exults in "the graceful symmetry" of Nature's forms, among them "a noble tree's branches … a snowflake…" [04]. It is clear that "symmetry", in these and a myriad of similar statements, is intended as "bilateral symmetry", concretely and precisely understood.

My purpose in this paper is to call attention to the fact that few of the forms that for five centuries have routinely been described as symmetric, (usually *nemine contradicente)*, are in fact symmetric. (This holds true, ironically, even of forms that Alberti and Weyl themselves declared are symmetric). It is, for example, at most only in the rarest instances that Nature's forms are symmetric [05]; or that the ancient Greeks made anything symmetric [06]; or that symmetric forms are found in works of primitive art [07]; nor, by the same token is the human body symmetric [08]. No graphic evidence of even one symmetric snowflake has ever been produced [09]; and symmetry was never "an abiding principle" of Byzantine art [10] … That only symmetric forms are beautiful is also untrue. Symmetric modern office buildings are not beautiful; asymmetric facades of Gothic cathedrals are not ugly [11]. Our understanding of symmetry must surely be vitiated by its derivation, all too often, from factually incorrect descriptions such as these.

These "symmetry fallacies" (as I call them) are among the more consequential and persistent in Western intellectual history. In this paper, I suggest that the misperception of forms as symmetric arises from a deep bias against asymmetry that is endemic in our culture; I speculate about the causes of that bias and describe some mechanisms that enable it. I enumerate the negative consequences of the symmetry fallacies, not least among them the lamentable state of modern architecture, and our profound misunderstanding of asymmetry. I conclude with a plea for empirical rigor in the application of the concept of symmetry.

[01] *De re aedificatoria,* ix.7; vi.3. I share much scholarly opinion (eg Gadol, *Leon Battista Alberti …,* pp.109-110), that Alberti's *convenire* is to be understood as "mirroring". With at most slight modifications, Alberti's view about symmetry in the Classical world continues to be put forward today. *Gardner's Art through the Ages,* p.102, for example, alleges that the Ancients had a "passion for symmetry"!
[02] Weyl, *Symmetry,* p.6.
[03] Boas, *Primitive Art* (Dover ed.) p.4. Boas is sometimes called "the father of American anthropology".
[04] Lederman, *Symmetry and the Beautiful Universe,* p.13.
[05] See examples in Selzer, *Symmetry Fallacies,* chapter 2.
[06] *Ibid,* chapter 3.
[07] *Ibid,* chapter 4.
[08] Comp. "Bilateral symmetry is the symmetry of the human body and for that reason of towering importance to mankind": Wittkower, *Idea and Image,* p.128.
[09] "Snowflakes have hexagonal symmetry", Nakaya, *Snow Crystals, Natural and Artificial,* p.38.
[10] *Oxford Dictionary of Byzantium,* article on "symmetry".
[11] "If one part always answers accurately to another part, it is sure to be a bad building; and the greater and more conspicuous the irregularities, the greater chances are that it is a good one" – *The Works of Ruskin,* vol. X, p.268.

BREAKING SYMMETRY AND INFORMATION IN DESIGN

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In visual configurations, symmetry provides order, structure, and confirms what can be expected following its rules. However, disruption of order is a tool to communicate through tension, introducing what is different.

This communicational resource is broadly used in professional practice. This presentation will discuss two design examples, in different areas: graphic and industrial design. In the first one, transgression is intense. In Rafal Olbinski´s book cover of *Nuclear illusion and reality*, there is a beautiful landscape, with a pond, where a swan swims, and you can see its reflection in the water. However, the infringement is that the living, swimming swan is shown in the reflection and, where the source of the image should be, you see its skeleton. The living being is just an immaterial, vanishing image, and what is left in the world are its bones, obviously related to the perils suggested in the title of the book. The simultaneous mention of opposite concepts intensifies the acknowledgement of each and of the concepts conveyed through them. It is a counterpoint, a dialogue between both parts.

But it is not enough to break the rules, the way in which this is done is also important. The regulation of difference is significant to adjust the intensity of what is communicated but there is another factor to consider. In order to achieve a greater amount of tension in communication there must be a balance between confirmation and destruction of what is expected because a deceptive appearance must be sustained. One of our greatest dreads, exploited in numerous films and books is the fear of monsters. It is nourished by the suspicion that our familiar background may become threatening and dangerous because the unconceivable permanently lurks in the depths of what is commonplace. To use the words of Raúl Dorra, “a monster can be defined in terms of the breach of a rule…it is the outcome of a tension between similarity and disparity”. Similarity provides a dangerous proximity, and disparity provides fear of the unknown, of what is different and should be left aside.

In industrial design, breaking isometric symmetry is more subtle. Operational issues, as well as fabrication factors, have promoted and assured the domination of isometric symmetry in everyday objects. Examples of homeometry are frequent, but it is difficult to find stronger divergences from regularity. We will discuss the Câliner Sofa, designed by Yael Goldenberg. Reflection of the motif is combined with a non-uniform extension. This sofa is meant for two users, and the change in scale represent the difference in their bodies and interactions.

Symmetry provides structure to our built environment, conveying balance and repose. However sometimes there is a need to express tension, or to mention two opposing concepts in one single object. Infringing order in some of its levels: in structure or in motifs, is a powerful design resource.

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Dorra, R. (2000) ¿Para qué los monstruos? *Monstruos*, Secretaría de Estado de Cultura de Jujuy & Universidad Nacional de Jujuy: Jujuy.
Zuckerman, S. (1982) *Nuclear Illusion and Reality*. Viking Press: US.

Note: An image of the book cover and of the sofa referred in the text can be found in this link: https://drive.google.com/drive/folders/1B7nukyzNSKU4OK7vFj3dCAsELf5u_diS?usp=sharing

PATTERNS IN ORIENTAL CARPETS: SYMMETRY, STRUCTURE, AND KNOWLEDGE

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A simple technology and natural materials combine to give carpets a complicated three-dimensional structure of interlaced warps (longitudinal elements) and wefts (transverse elements), with supplementary segments of discontinuous wefts (called knots), which wrap around successive warps to create a pile. Although three-dimensional in structure, the multiplicity of patterns carried by the pile are two-dimensional, covering the plane with a systematic organization [01] based on symmetry.

The earliest known carpet, excavated at Pazyryk in the 1940s in a barrow in Russia where it was preserved in permafrost, likely dates from the fourth century BCE and already bears features we have come to accept as typical of Oriental carpets known from the 15th century CE onwards [02]. A central field, bearing a periodic pattern in the plane, is surrounded by multiple borders with frieze patterns.

The process of manufacture - the technology that leads to the production of a carpet – is relatively very simple. It relies upon a loom that holds the set of warp yarns taut. By hand, the weft is manipulated, one yarn at a time, to interlace with the warps; as weaving progresses, cut sections of colored yarns are introduced. The pattern is emergent, as the weaver wraps each knot, row by row, in a unitary process that is at once systemic [03]. Like a seashell, the algorithmic patterns grow along the leading edge [04]. The patterns of a rug thus document the history of its manufacture.

When I first visited Turkey in the mid-1960s I met a young woman, a rug-weaver from a village in central Anatolia. What I didn't realize at the time was how much this woman and her work would affect my intellectual development as curator and scholar over the years. In each village home, a simple loom was set up in the living quarters; women and girls seemed to weave at every opportunity. Many of the older women were illiterate. Although they couldn't read or write, they were math literate in a manner I had never experienced. The understanding of number, shape, and the nature of space, which pertains to the process and patterns of rug-weaving, results in a math literacy that expresses embedded knowledge of spatial relationships that differ from the numerical quantifications of statistics or studies of rates of change based on the quantification of data. In sharp contrast to the use of linear perspective, focused on a vanishing point that results in the illusionary rendering of three-dimensions on a picture plane, the rich variety of rug patterns document in real time the spatial domain of two-dimensions [05].

What is also significant from a scientific point of view is that the alignment of older and younger working in tandem, parallel to one another, is now recognized as a model for overcoming dyslexia. The coherent systems of teaching and learning, planning, and producing, counting, and patterning, result in a depth of understanding that exists in traditional weaving cultures. Careful observation and thoughtful considerations of the processes of weaving in traditional cultures can yield a multitude of possibilities for advancing our understanding of mathematical thinking.

[01] The central field pattern represents a plane symmetry group, and the border patterns exhibit linear symmetries. See C. Bier, "Elements of Plane Symmetry in Oriental Carpets," *The Textile Museum Journal* 31 (1992), 53-70; see also several brochures for exhibitions organized by The Textile Museum (C. Bier, curator): "Visions of Infinity: Design and Pattern in Oriental Carpets" (1990); "What is an Oriental

Carpet?” (1993); “Symmetry and Pattern: The Art of Oriental Carpets,” (1996), available at https://works.bepress.com/carol_bier/ (accessed 28 July 2021).

[02] S. I. Rudenko, *Frozen Tombs of Siberia: The Pazyryk Burials of Iron-age Horsemen*, tr. M. W. Thompson. Berkeley: University of California Press, 1970.

[03] C. Bier, “Choices and Constraints: Pattern Formation in Oriental Carpets,” *Forma* 15 (2000), 127-132.

[04] H. Meinhardt, *The Algorithmic Beauty of Seashells*, 4th ed. Heidelberg: Springer, 2009.

[05] C. Bier, “From Grid to Projected Grid: Oriental Carpets and the Development of Linear Perspective,” *Proceedings of the Textile Society of America* (2010), https://digitalcommons.unl.edu/cgi/viewcontent.cgi?article=1009&context=tsaconf (accessed 28 July 2021). See also, C. Bier, “CarpetMath: Exploring Mathematical Aspects of Turkmen Carpets,” *Journal of Mathematics and the Arts* 4/1 (2010), 29-47.

LOGICAL POLYGON AS A TOOL FOR PHILOSOPHICAL REASONING: ONE QUESTION ON INTROSPECTION

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Logical Polygon is a graphical representation of logical relations among propositions about relations. A proposition is a statement that can be true or false and expresses a thought which affirms or denies a) a connection between an object and a property, b) relations among objects, or c) the fact of existence of an object. Logical Polygon deals mostly with cases b), although can be used for a) also.

Constructed by the author of this work, the Polygon helps to reason correctly and easily about whether two propositions can be true together, whether they can be false together, whether the truth (or falsity) of one of them leads to the truth (or falsity) of the other – that is, to reason about relations among propositions. The Polygon is an analogue of the well-known in logic Square of Opposition but is applicable to propositions with not just one-place, but *n*-place predicates (*n* is a natural number).

Logical Polygon consists of geometric figures of certain kind, principles of their construction and rules of application. Its geometric figures are complex, symmetric structures that are different for different *n*. They all have vertices standing for logical forms of propositions and lines showing the relations between propositions of such forms. As we have shown earlier (see: Cherkashina, 2019, also for more general information on the Polygon), symmetry plays a significant role in how the Polygon is constructed and works.

Symmetric relations are such that if they hold between *x* and *y*, they hold between *y* and *x*, too "*x* being a relative of *y*" is a symmetric relation, "*x* being taller than *y*" is not; most of the relations between propositions considered in the Polygon are symmetric in this sense. The symmetry of those relations facilitates some of the reasoning about them and shows itself in the relations' visual representations, making one line enough to express the relations between propositions a and b – and b and a.

One more interesting feature of the Polygon is the kind of symmetry that keeps figures the same when a figure is rotated 180 degrees with simultaneous substitution of lines showing contrariety for lines showing subcontrariety and vice versa and reversing the arrows for subalternation. There are also other kinds of symmetry in Polygon's figures (to be presented at the Congress, illustrated).

We may say that if the information contained in the logical form of one proposition is incompatible with the information contained in that of the other, two propositions can't be true together. What interests us about the logical form of the propositions in this work are their quality (affirmative/negative, depending on whether the proposition affirms or denies the relation) and quantity (depending on the proposition's expressing the thought about all objects of a specified kind, or about at least some of them - universal/particular...universal/particular, the number of elements here = *n*). (For an example, see below; for classification of propositions about relations see Ivlev, pp. 40-41.)

In the presented work we use the Polygon to give an answer to a question discussed in *philosophy of mind:* whether S. Shoemaker's claim on self-blindness and T. Williamson's claim on luminosity contradict each other and, if not, whether they can be true together. S. Shoemaker claims that self-blindness (the impossibility of noticing "mental facts to which normal people have introspective access") is impossible, and T. Williamson claims that the conditions with which we engage in our everyday life are non-luminous (roughly, luminous conditions are such that people in every case of having them are able to know that they have them).

Addressing the question without the Polygon met the difficulty of requiring more formal logic than it is comfortable for many of the philosophers not specializing in logic. Meanwhile, the Polygon can be used quite easily by specialists in different fields.

After analyzing what "impossibility" and what objects were meant in each case, we reformulate the claims as *comparable* propositions that (both in the same terms) affirm or deny the same relation between the same kinds of objects in the same order:

A. All mental states are for all creatures in (at least) some cases introspectively accessible. (Shoemaker) – form: universal-universal-particular (*UUP*), affirmative.
B. All mental states [in everyday life] are for all creatures in (at least) some cases not introspectively accessible. (Williamson) – form: *UUP*, negative.

"… being introspectively accessible for … in cases …" is a 3-place predicate. Looking at the respective figure of the Polygon, we see that *UUP* affirmative (A) and *UUP* negative (B) are not in a relation of contradiction.

Applying the Polygon to check whether there are other Aristotelian relations (contrariety, subcontrariety, subalternation) between them, we discover that the propositions A and B are logically independent from each other. That is, from the point of view of their logical form they can be true together or false together, or any one of them can be true while the other is false.

We should note that the Polygon is meant to solve the logical part of questions about relations among propositions. As we have seen in this example, before using the Polygon (the same as for any other logical tool) we need to make sure that the information has the form that allows to work with it. Here this condition means that the propositions must be comparable. For that, the philosophical interpretation of the initial claims was needed. If the interpretation was correct, the answer obtained with the Polygon is the answer to the initial questions.

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NON-PLENARY SESSIONS | WEDNESDAY, SEPTEMBER 15, 2021
Zoom session from 7:30 to 11:30 UTC

September 15 | 07:30 to 08:00 UTC

'THE OLYMPIC FLOTILLA': FLOATING-TRANSPORTABLE ARRAY OF 'MEGA EVENTS' FACILITIES FOR RENT

Michael BURT and Yehiel ROSENFELD

September 15 | 08:00 to 08:30 UTC

ROMAN COSMATESCO: SYMMETRY UNDERFOOT

Eugene A. KATZ

September 15 | 08:30 to 09:00 UTC

INTERACTION OF SYMMETRY, STRUCTURE, AND INFORMATION IN THE OXIPO-MODEL OF LEARNING

Ferenc MEZŐ and Katalin MEZŐ

5September 15 | 09:00 to 09:30 UTC

QUALITY OF INFORMATION AND SYMMETRY

Miro BRADA

September 15 | 09:30 to 10:00 UTC

PERIODIC LAW IN THE CONTEXT OF COGNITIVE DEVELOPMENT

Dmitry WEISE

September 15 | 10:00 to 10:30 UTC

A COMBINED RESEARCH PLATFORM OF STRUCTURAL MORPHOLOGY, DEALING WITH THE ORDERED 3D SPACE

Michael BURT

'THE OLYMPIC FLOTILLA': FLOATING-TRANSPORTABLE ARRAY OF 'MEGA EVENTS' FACILITIES FOR RENT

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The domineering characteristic of our times is the staggering increase in volume and frequency of human multi-modal communication and physical mobility. It is accompanied by an explosive number of mega-events, of cultural, economic, and political nature, with the Sport industry and mass entertainment and recreation through sport, from the Olympics down, as a prime leading phenomenon.

The key to assuming the privilege of hosting a mega-event rests in economic capacity of providing the required facilities and services.

The grim factual reality is that most of the world nations, with all their multitude of humanity, are unable to compete for the privilege of hosting the Olympic games, and in fact any other international mega-sport or other event, mostly on economic grounds.

Since most of humanity and its potential hosting mega-cities are sprawled along the coastal waters of the Ocean-sea system, it is right to promote the idea of generating an 'Olympic Flotilla', with all the facilities and related services, required for a mega-sport event (from the Olympics down) and any mega-events related to mass entertainment-recreation; *a flotilla that is capable of traversing the oceans, to be rented for the duration of the events***.**

The design intentions and developments must aim high: Sport facilities, capable of accommodating up to 150,000 spectators under one roof and providing hotel accommodation services for hundreds of thousands of very demanding visitors- participants.

The paper suggests a conceptual framework of dealing with the space structures and the array of floating vessels and their general design proposals: Wide-span multi-layered I.P.L space frame plate structures (developed and patented by Michael Burt in the past), and a multi-hull minimal drought solution for the ocean-going carrying vessels that could be maneuvered in relatively shallow waters, when served by the coastal infrastructures.

Such mega space structures, to be economically viable, have to strive for minimalistic material tonnage and highest possible geometric periodicity level, to ensure utmost technological industrialization processing, that might be provided only by highly symmetrical nature of the mobilized structural morphology.

ROMAN COSMATESCO: SYMMETRY UNDERFOOT

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Cosmatesque is a style of geometric decorative inlay stonework typical of the architecture of Medieval Italy, and especially of Rome and its surroundings. It was used most extensively for the decoration of church floors, but was also used to decorate church walls, pulpits, and bishop's thrones. The name derives from the Cosmati, the leading family workshop of craftsmen in Rome who created such geometrical marble decorations during XII-XIV centuries. Even the tomb of King Henry III in Westminster Abbey in London was decorated by the Cosmati.

Cosmati floors are made from variously shapes and sizes (circles, squares, rhombuses, triangles) of porphyry, white, yellow, green, and black marble. Moreover, they inlaid not only the floors, but also the walls, columns, portals, pulpits of temples. The stone used by the Cosmati artists were often salvaged material from the ruins of ancient Roman buildings, the large roundels being the carefully cut cross sections of Roman columns.

In my talk I will focus on the geometrical aspects of this highly symmetrical art. In particular, I will demonstrate that seventeen plane symmetry groups (with the possible exception of one group, p3) have been realized in these artistic ornaments. Furthermore, a few other very important scientific concepts can be studied with the examples of this medieval art. Together with *symmetry*, they include *antisymmetry*, *dissymmetry*, *symmetry paradox*, *fractals* (*the Sierpiński triangle*), *Mobius strip*, *rhombille tiling* (*the reversible cubes*), *knots* (*the Borromean ring*) and others.

Most of the illustrations in the lecture are author photographs made in Rome.

INTERACTION OF SYMMETRY, STRUCTURE, AND INFORMATION IN THE OXIPO-MODEL OF LEARNING

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According to the OxIPO-model, learning is a kind of information processing, that has got four main components (organizing, input, process, output):

Learning = Organization x (Input + Process + Output)

The concepts of 'symmetry', 'structure', and 'information' are closely related in this model.
Symmetry and Information in the OxIPO-model. In the OxIPO-model, we can differentiate three learning strategies by the symmetry of input and output information. These strategies are:

1. productive (creative) learning: input < output. This is an asymmetric relation between the input (e.g., quantitative, and qualitative properties of the content of a book) and the output (e.g., students' knowledge after reading and learning the book). In the case of productive learning, the students will have more and/or better-structured information at the end of the learning than the book contains.
2. reproductive learning (without understanding): input = output. This relation is a kind of symmetry between the information of input and output. E.g.: the students can learn words, but they cannot understand or apply their knowledge.
3. unproductive learning: input > output. This is an asymmetric relation between input and output. E.g.: the book contains five words, but the student is only able to recall less than five words.

Consequently: productive learning is the most efficient, and the students should know the methods of productive learning. Productive learning is learnable and teachable.
Structure and Information from viewpoint of OxIPO-model. The structure of text influences the temporal and practical efficiency of learning. For example: From the aspect of the logical structure of a text, a text type can be chronological, conceptual hierarchy based, comparison, etc. Another hand, a text can be well or poorly edited, structured. One of the possible areas of information production is the restructuring (re-editing) of a poorly structured input text. So, a productive learner able to:

1. identify the logical structure of an input text,
2. evaluate the quality of editing of this text,
3. re-edit (restructure) the wrong edited text to well-edited text (this is the mental production of the student).

Summary: the structure of information impacts the effectiveness of school learning, and one of the tasks of learning development is to teach the methods of structuring the information. Seems, the asymmetry can be useful from the aspect of the info-economy of productive learning.
Interaction of (a)symmetry and structure: the restructuring of the input information necessarily results in an asymmetric relationship between input and output. The qualitative and/or quantitative characteristics of the output text may become better than those of the original text.

The OxIPO-model and the consequences of interactions above can apply in a number of researching areas. We can use this model for learning diagnostic and development, behaviour modifying, researching artificial intelligence, cultural history. One of the interesting questions in the last area: if you know the input information of Mendeleev, Galileo, Darwin, Pasteur, etc., do you able to discover their results? Can you restructure their original information? Can you create an asymmetry between previous and new knowledge?

QUALITY OF INFORMATION AND SYMMETRY

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$(1 + N) * N/2$ sums 1 to N, as each step of series: $1 + N$, $2 + (N - 1)$, $3 + (N - 2)$ ... returns: $1 + N$. Formula REUSES "$1 + N$" $N/2$ times. Cubism REUSES a nose in profile/front, M. C. Escher REUSES Arabic ornaments in illusions, XYZ space REUSES equation's value to define geometry… REUSAGE is a fundamental quality of information consisting of reusable units $(i) = i_1 + i_2 + i_3$... linked by IQ in series encoding information structure. Combining information adds new quality e.g., calculus unites geometry (g) and algebra $(a) = (g_1 + g_2 \ldots) * (a_1 + a_2 \ldots)$.
Information is so defined by its structure rather than its content/accuracy, but false information can't be intricate e.g., a wrong formula is unusable in calculus. Information accuracy increases its reusability and rareness (uniqueness) because accurate formulas are rarer than the wrong. The higher quality, the rarer (more intricate) and more reusable information implying higher probability of re-invention (for sufficient IQ). It explains why Newton and Leibniz could invent calculus independently. In contrast, it's far less likely that e.g., Ch. Baudelaire would invent the same poem as E. Poe or A. Pushkin, as poetry varies more - and so it is less unique than calculus. I will show: (1) more intricate chess compositions were re-invented more often; (2) why Arcimboldo's 16th century portraits of fruits or other things defined the modern art (creating unreal from real) to be reused in e.g., Dali's art; (3) How Velvet Underground influenced D. Bowie that influenced e.g., Argentinian Soda Stereo. At the same time, I'll show how social media or religions spread low quality information to maximize the power /profit.

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PERIODIC LAW IN THE CONTEXT OF COGNITIVE DEVELOPMENT

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According to one of the pioneers of quantum physics Wolfgang Pauli: "The process of understanding nature as well as the happiness that man feels in understanding – that is, in the conscious realization or new knowledge – seems thus to be based on correspondence, a "matching" of inner images pre-existent in the human psyche with external objects and their behavior" [01].

Regardless of whether archetypal images are innate or acquired at different stages of brain development, we talk about the establishment of interneuronal communications in the cerebral cortex [02]. On a conceptual verbal level, this is commonly referred to as an interdisciplinary approach.

Our view on the symbolic representation of Dmitry Mendeleev's Periodic Law of elements based on Pythagorean's figurate numbers conception suggests a new approach to teaching chemistry and quantum physics as academic courses.

In using the Pythagorean approach before the modern scientific study of the Periodic Law, one can see an analogy with the biogenetic law, also called the theory of recapitulation [03], which, in turn, is associated with the concept of cognitive development [04].

The biogenetic law - often expressed using Ernst Haeckel's phrase "ontogeny recapitulates phylogeny" - is a historical hypothesis that the development of the embryo of an animal, from fertilization to gestation or hatching (ontogeny), goes *through* stages resembling or representing successive adult stages in the evolution of the animal's remote ancestors (phylogeny).

The author of the concept of cognitive development Jean Piaget, believed that people move through stages of development that allow them to think in new, more complex ways. Piaget proposed four stages of cognitive development.

The relationship between Haeckel's biogenetic law and Piaget's Cognitive Development has been criticized [05]. Nevertheless, we refer to Herbert Spencer's opinion, who compactly expressed the basis for a cultural recapitulation theory of education in the claim:

If there be an order in which the human race has mastered its various kinds of knowledge, there will arise in every child an aptitude to acquire those kinds of knowledge in the same order education should be a repetition of civilization in little [06].

Such a view of learning allows transfer of the beginning of the study of very abstract scientific disciplines, according to Piaget, from the fourth - formal operational - to the second - preoperational period.

The teaching is divided into two stages. At the first stage, which, according to Jung, can be called *unconscious*, the child at the sensual level is introduced to the geometric archetypes of Periodic Law [07]. Educational games with cubes, balls, pyramids, mosaics, and other objects in an entertaining and safe form will strengthen the intellect and prepare the child for the second, *conscious* stage of obtaining academic knowledge. Thus, the age threshold for the beginning of chemistry and quantum physics studying will be reduced from 14 to 1.5 –3 years old.

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A COMBINED RESEARCH PLATFORM OF STRUCTURAL MORPHOLOGY, DEALING WITH THE ORDERED 3D SPACE

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The architecture of the human habitat and the whole domain of physical space structures, from the nano-molecular to the mega-galactic space, is mostly concerned with few basic *n*-dimensional spatial features and their diverse interrelations:

- Networks (Space Lattices), arrays of interconnected vertices, representing physical or imaginary-abstract entities, that might stretch to infinity, to occupy n-dimensional space.
- Polyhedral finite or infinite volumetric solids that might be associated with compact close packing of space.
- Space dividing partition surfaces, finite, or infinite, between any conceivable spatial features.

The number of morpho-topological, structures of these spatial features reach to infinity and their order - periodicity characteristics stretch from the most accidental to the most ordered, symmetrically uniform. Throughout history many attempts were performed to control intellectually this structural morphology domain. It was realized by the author that: dual network pairs, their associated close packing polyhedra and the associated hyperbolic space partition surfaces subdividing between dual network pairs are complementary 3D space features, termed by the author as, *The Quintuplet Phenomenology of 3D space*. Its principal characteristics are:

1. Determination of one constituent of the five makes possible to determine the remaining four.
2. All five constituents of the *Quintuplet* share same symmetry regime.

Limiting research inquiries to 3D space and to the most periodic, symmetrically uniform characteristics of the a.m. Quintuplet features, it was soon realized that *exhaustive enumeration of 3D symmetry space groups* should come first.

The most critical observation and statement of this presentation is that *the exhaustive enumeration of 3D symmetry space groups***,** claimed by Fedorov & al. in the late 19th century, *is not complete* and reaching far beyond the claimed 230 items (!).

The paper suggests a combined research platform, aiming at exhaustive enumeration of the most symmetrically uniform features concerning the following:

1. 3D symmetry space groups.
2. Uniform (equi vertex-edge) 3D space networks.
3. Self-packing polyhedra of 3D space.
4. Hyperbolic surfaces subdividing between uniform dual network pairs.
5. Hyperbolic surfaces subdividing between *self-dual* network pairs

Geometrical-topological solution and exhaustive enumeration of the above might contribute considerably to our present-day perception and understanding of 3D space related structural morphology at large.

It might significantly promote many research issues within the domains of chemistry and crystallography, morphology of physical space structures and technology concerned with close packing and associated physical space dividing partition surfaces.