New Approach for Understanding the Golden Section

Abstract

In nature, the golden ratio orchestrates objects as large as galaxies and as small as the DNA. It inspired artists and engineers since the ancient civilizations. However, a five century old question is still without answer. This question says; is the golden ratio a “divine” and mysterious ratio or it is a functional necessity in nature? This research provides a breakthrough in understanding why the golden ratio and the gold section exist in nature. Advanced 3D modeling was used in this research to prove that the golden section is in fact a logical and mathematical relation that governs the growth around a center, a growth that is essential in nature.

Key words; Golden Ratio, Fibonacci numbers, Growth, Nature.

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Introduction

Ancient Greek artists and philosophers are among the first civilizations who formed an independent branch of science to analyze the beauty in nature. Artists search for a true in beauty, and the scientist for beauty in a true. The question was always; is there a common rule behind the harmony in the ancient Egyptian pyramid, Leonardo da Vinci " the last super" painting, the human anatomy, and the space galaxies?. The well-known Italian theorist of architecture Leone Battista Alberti wrote ""There is something greater, composed from combination and connection of three things (number, limitation and arrangement), something that lights up all face of beauty. And we called it by Harmony, which is, undoubtedly, the source of some charm and beauty. You see assigning and purpose of harmony to arrange the parts, generally speaking, different under the nature, by certain perfect ratio so that they met one other creating a beauty … It encompasses all human life, penetrates through the nature of things. Wherefore everything that is made by Nature, all this is measured by the law of harmony" (Stakhov, 2007).

Harmony as defined in the encyclopedia as "coordination of the parts and the whole, coalescence of different components of the object in the unified organic whole. In the Harmony, the internal orderliness and measure of the Being get the external revelation" (Stakhov,2007) .

Ancient civilizations have discovered what was later called by Leonardo Davinci, " the golden ratio".

This ratio equals to 1: $\frac{1}{2}\left(1+\sqrt{5}\right)$ or approximately 1: 1.618033.

This ratio govern the harmony of most living elements in nature, and can be found in elements as small as the DNA and as large as galaxies. Great mathematicians throughout history have invested great amount of time studying the Golden Ratio. These mathematicians include Pythagoras and Euclid in ancient Greece, the medieval Italian mathematician Leonardo of Pisa, the Renaissance astronomer Johannes Kepler, as well as present-day scientists such as Oxford physicist Roger Penrose. Fascination with the Golden Ratio extended to Biologists, artists, musicians, historians, architects, psychologists, and even mystics. (Mario, 2002). In the absence of a scientific reasoning, Greeks referred to the center of the golden ratio as the " eye of God".   Numerous current publications and websites which were produced to link the existence of the existence of God. Current scientific research concluded a relation between the golden rectangle and the diliation of time intervals and the Lorents contraction of lengths as predicted by Einstein's theory of special relativity (Smith, 2006)

Scientists relate the golden ratio in nature to the so-called "Principle of the Least Action" acts. In this "Principle" the system permanently passes from the less steady state to the most stable state. Thus formed derived from the golden ratio provides the "minimum of surface energy compatible with external orientating forces" (Mario, 2002).

All inorganic materials are govern by "Principle of the Least Action". However, the biological and plant elements do not always follow this principle. Animals and plants seek to reach such morphological envelope, which is more suitable for reproduction and resistance to environment.

Animals and plants use the "Principle of Matter Economy" . This principle does not prevail in the inorganic world. A good example of this principle is the eggshell where form, structure and material are used wisely to achieve maximum strength with the least amount of material. In addition, living organisms must maintain the capability of growth with the least amount of effort. While inorganic crystals are increased by addition of the identical elements' the living organism grows by "immersion" going from a center to the outside (Stakhov,2007).

Another fundamental distinction between inorganic and organic matters is: the molecular elements of the inorganic matter do not change its character during the life time of that matter, while the living elements is gradually died, leave and are reintroduced, and at the same time save the general character of the organism form during growth. A famous example is the shell of a snail which grows, while saving the initial form despite what appears to be an asymmetrical growth (Stakhov,2007).

So far, no connection was established between the Principle of Matter Economy, the principle of Least Action or any other biological growth principle and the golden ratio.

1. Principles of Golden ratio

In order to understand the relation between the golden ratio and the principle of growth in nature, an in depth geometrical analysis was conducted in this research. As a result, the following Principles were obtained;

2. Growth around the center

The golden ratio represent the growth around the center. It is known that the golden section maintains its original ratio every time we take a square from it as shown in Figure 1. In order to prove that the golden section govern the growth around a center, let us draw circles around the squares in the golden section as shown in Figure 2. This figure shows that when drawing circles that circumscribe the squares in the golden rectangle, these circles will intersects around one point, which will be the center of the golden section. As we know, the golden section is generated from irrational number, so, it is not possible to draw the perfect golden section with a definite center. When the Greek first discovered that the golden ratio is generated by irrational number, they were very concern because nature (God) cannot make mistakes. However, when we look at the center of the golden section as a center of growth, then, the infinite nature of the golden ratio allows for an infinite number of circles to be generated around the center. Thus, continuous growth will be achieved.

3. Uniform growth around the center

If the center of the golden section represents a center of growth, then what is the mystery behind Fibonacci numbers and the golden section? To explain this relation let us do the following:

3.1 Draw a golden section and establish the spiral form by creating quarters of circles in each square as shown in Figure 3. There is nothing new in this figure.

3.2 Rotate the spiral around the center of the golden section so that the spiral intersects with the outside corners of the golden rectangle as shown in Figure 4. Interestingly, we will find that the ratio between the length of each segment and the successor segment of the spiral in this figure represents the golden ratio (1.618…)

3.3 Divide the spiral of the golden section into equal segments in which the total number of segments equal to one of Fibonacci numbers. For example, Figure 5 has a total of 87 segments which represents the total numbers of 8 stages in Fibonacci numbers (1, 2, 3, 5, 8, 13, 21, 34). When counting the number of line segments of each portion of the spiral that spans between two corners of the golden section, you will find that the number of line segments in each spiral portion is equivalent to that in the Fibonacci numbers; As shown in

Figure 5, the first section will have 1 segment, the second section will have 2 segments, the third section will have 3 segments, the fourth section will have 5 segments and the fifth section will have 8 segments and so on. This finding clearly shows that Fibonacci numbers are in fact a logical division of stages in the uniform growth from the center in the golden section. In particular, Fibonacci numbers are the numbers in a steady and uniform growth around a center at 90 ° intervals.

3.4 Consider a golden section with six consecutive squares similar to the one in Figure 2, and let us make a set of circles that circumscribed each of the six squares. If the diameter of the first circle that circumscribed the first square is 0.041594 , then the diameter of the circle that circumscribed the sixth square should be 0.873474 as shown in Table 1. If we apply Fibonacci numbers to this section, we will find out that the total number of segments in a spiral that matches the Fibonacci numbers should be 21 segments (1,2,3,5,8).

3.5 Now let us draw transitional circles in a uniform growth pattern. The diameter of each of these circles should be as follows:

Diameter of any transitional circle =.041594 +§(.0416-.873)/21).

Where § = the number of a circle in the Fibonacci numbers as shown in Table 1.

To draw the transitional circles, we will scale the base circle (the circles circumscribed the squares) in a scale factors shown in Table 1. The base point for scaling should be the center of the golden section.

3.6 Rotate each circle in a specific angle as in the; make the base point for rotation on the center of the golden section, and rotate the circles so that each circle tangents with the spiral. To determine the rotation angles let us draw ray lines between the center of the spiral and points of division of the spiral in Figure 5 ; When dividing the spiral of the golden section to total number of segments equal to the that of Fibonacci numbers, each segment of the spiral that spans between the two corners of the golden section will have point small segments equivalent to that in Fibonacci numbers (1, 2, 3, 5, 8, 13, 21, 34) as shown in Figure 6, and measure the angle between each two line. The result of rotating the circles is shown in Figure 8. Interestingly, these circles will tangent with the spiral of the golden section. In other words, when circles grow in a constant rate around a center, these circles will follow the path of the spiral in the golden ratio and will fit the Fibonacci numbers. Since the golden section generates a logarithmic spiral, Using measured angles from Figure 6, a line fit statistical analysis was used to calculate the rotation angle. The relation between the diameter of the circles and the rotation angle is found to be

$Y = a+b*\log(x)+c*\log(x)^2$

Where;

Y= rotation angle

X= Circle diameter

a= 85.61

b= -42.99

c= 5.74

The standard of error in the above relation is .79.

From the above discussion, we can conclude that the golden section is in fact a natural result of a constant and uniform growth around a center. This finding also suggests that the golden section represents the circles in the path of growth which are in a perpendicular angle from the center. Thus, Fibonacci numbers represents the transition between each bench marks of the golden ratio.

3.7 A question might arise: why is the Fibonacci numbers? And can the order of steady growth be on any number? To answer these questions, let us divide the spiral of the golden section to any number, say 100 segment as in Figure 7, then rotate to golden section in any angle and scale it to fit the spiral, we will find that the ratio between the number of segments on each side of the golden section will match the golden ratio (18, 29, 48). Thus, Fibonacci numbers explains the golden ratio in a specific condition in which the growth starts from 1, and when the spiral reaches 90 degrees in each quarter. However, Growth can be on any rate around the center. We can now say that any constant growth rate around a center will follow the spiral of the golden section.

3.8 Shortest path to the center of growth; to understand the concept of the growth around the center, let us analyze the golden ratio in a 3D environment as in Figure 9 , the circles in this figure where placed on constant increments of heights as shown in Figure 10. We can notice that all circles tangents the vertical line on the center of the golden section (the red line in Figure 9). The top view of Figure 9 is Figure 8. This shows that the center of the 3D spiral of the golden section is the vertical line on the center of the golden section, which is the shortest distance that connect all the circles in a growth around a center.

3.9 So far, we have looked at the growth in a form of circles. Let us now represent the growth in 3D environment and represent each circle in Figure 9 whith a sphere that has the same diameter as the circle in Figure 10. The result will be Figure 11 which shows the transition between each sphere in a 3D growth. The top view of Figure 11 is shown in Figure 12 which shows that all spheres tangents around the golden section center. Figure 13 is a close up view on the center of the golden section which shows that the spheres are closest together on the golden section center. A render image of the top view is shown in Figure 14.

3.10 When creating a cross section through the center of the golden ratio in Figure 12, the spheres tangents in the center of the golden section will create a close to a vertical line as shown in Figure 15. A close up look at what seems a vertical line is in fact tangents of all the spheres around the golden section center as shown in Figure 16 and Figure 17. This finding clearly shows that the golden section is the natural result of constant and linear 3D growth around a center. This pattern of growth provides the shortest path between each stage of growth to the center of growth.

4. Evidence in Nature;

If the golden section is the natural result of uniform and constant growth around a center, then how this is explained in nature. To answer this question, let us first take a look at the anatomy of the human body. Figure 18 shows that when applying the golden section on the human body, the heart of the human body is located on the center of the golden ratio. The same configuration applies to the human fetus as shown in Figure 20. This configuration allows the heart to be in the center of the body so the body can grow around the heart. Historically, the golden section was applied to the human body as shown in Figure 21. However, it was not referred to a specific functional reasons. In modern architecture, Le Corbusier also built his module around the proportion of the human body

The same rule applies to most living creatures such as the penguin (Figure 23) , the cow (Figure 24) and the dear (Figure 25).

5- Conclusion

This research proved that the golden section in nature is based on a logical and functional bases. The research shows that the golden section represents a uniform growth around the a center. Thus Golden section is the logical path of growth around a center , which provides direct and short distance between all growth cycles and the center of the golden section. This research also showed that the Fibonacci numbers represent certain locations on the growth spiral . Thus, wherever the golden section exists, Fibonacci numbers explains benchmark locations on the golden section spiral.

This research also explains how the golden section was utilized in nature to provide efficient growth pattern for the living creatures. Golden section is also a logical pattern for least effort blood circulation or body movement for most creatures. The centralized growth nature of the golden section inspired artists and scientists to produce comprehensive structures and art work that provide a unified composition.

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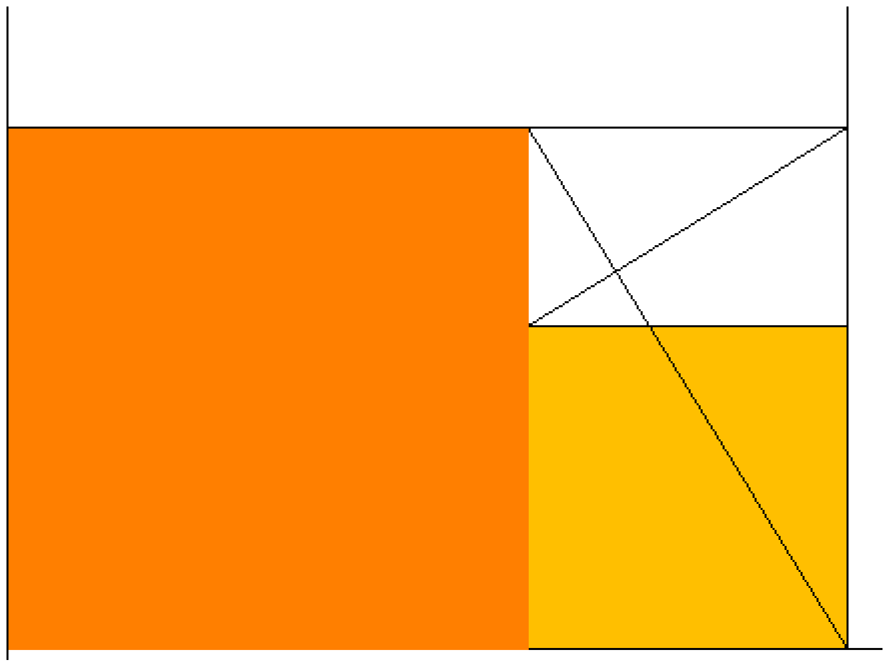


Figure 1 ; Golden ratio; The intersection of the diagonal lines can be onsidered the center of the golden section

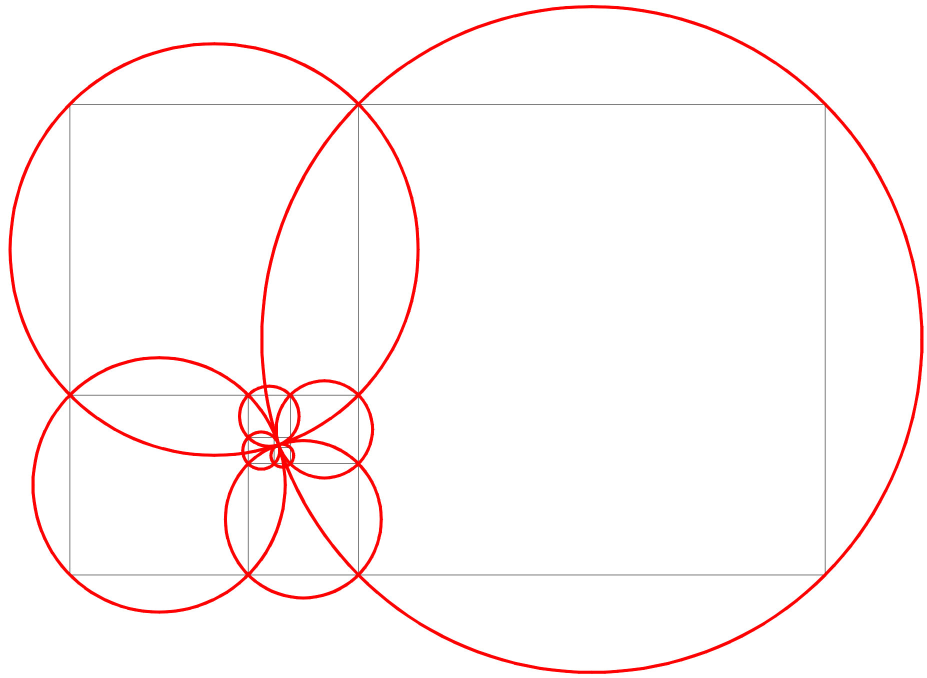


Figure 2 : when drawing circle to circumscribed the squares in a golden sections, these circle will meet around the center of the golden section

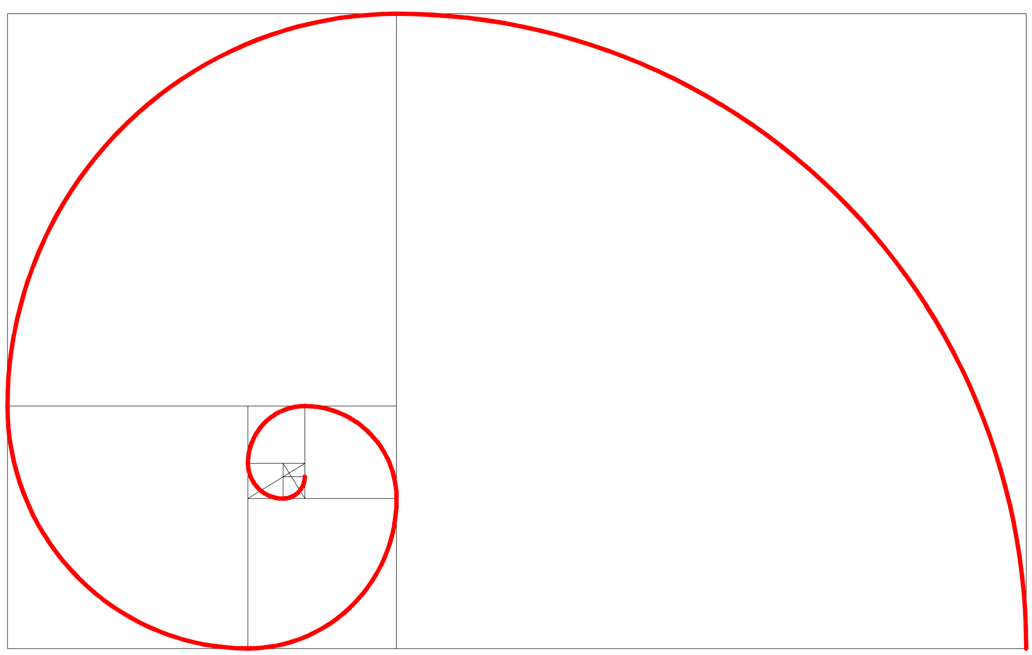


Figure 3 ;Generate the spiralshape in the golden section

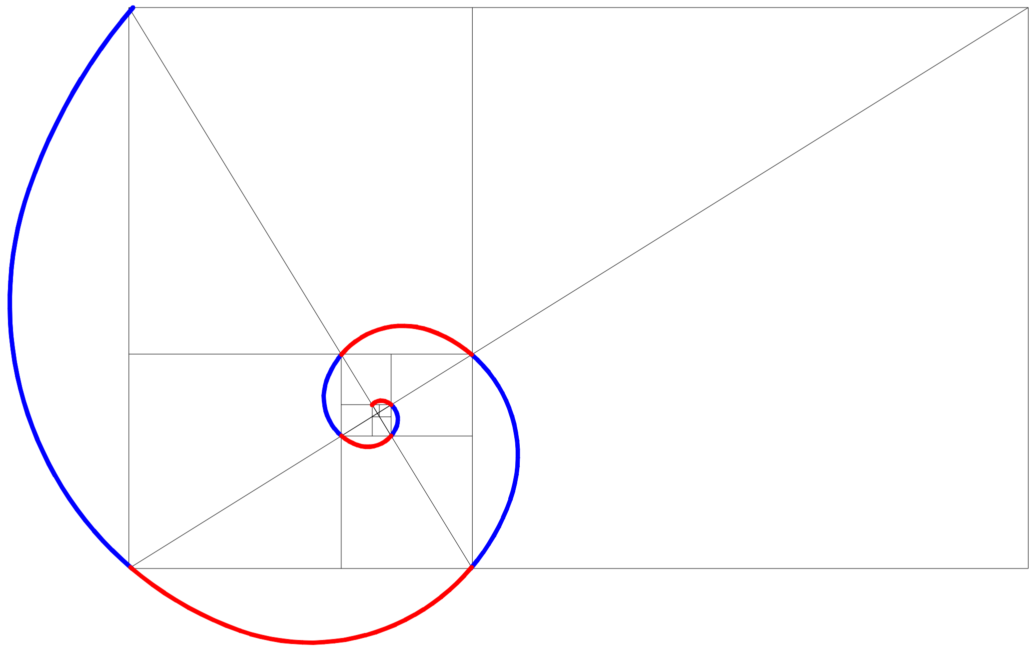


Figure 4 ; When rotating the spiral to intersects with the corners in the golden section, the ratio between the length of each segment and the successor segment of the spiral (the red and the blue) is the golden ratio (1.618…)

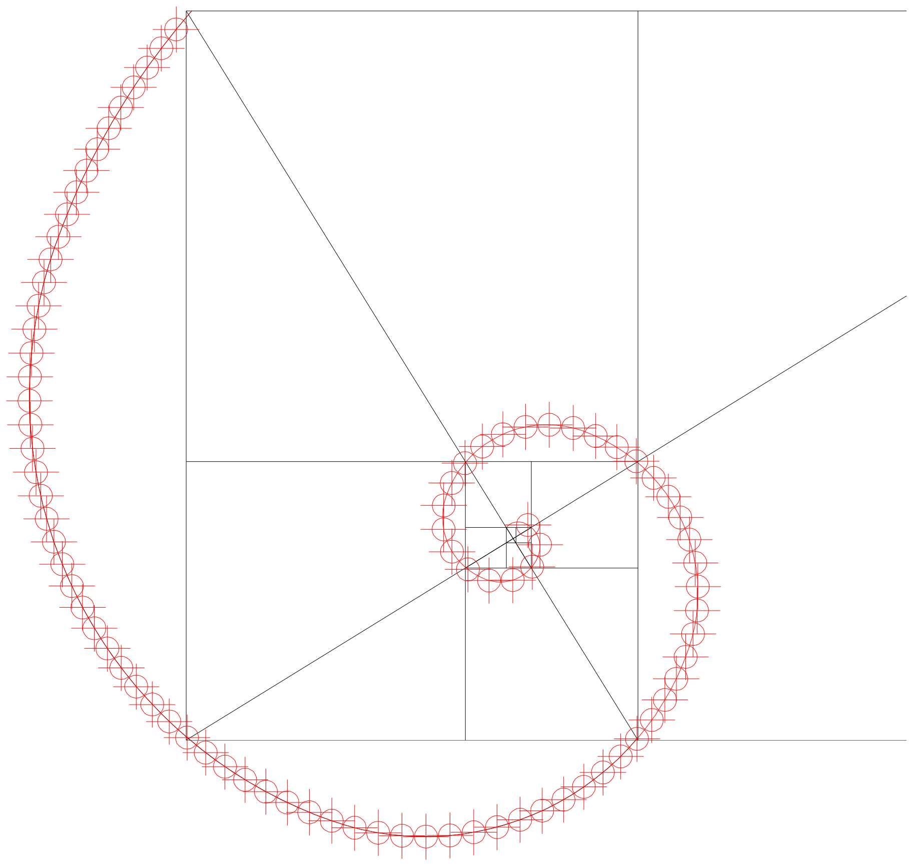


Figure 5 ; When dividing the spiral of the golden section to total number of segments equal to the that of Fibonacci numbers, each segment of the spiral that spans between the two corners of the golden section will have point small segments equivalent to that in Fibonacci numbers (1, 2, 3, 5, 8, 13, 21, 34)

Fibonacci number	The Number of the transitional circle (§)	Diameter of transitional circles = §(.0416-.873)/21)
1	**1**	**0.041594**
1	**2**	**0.083188**
2	**3**	**0.124782**
3	**4**	**0.166376**
5	**5**	**0.20797**
	6	0.249564
	7	0.291158
8	**8**	**0.332752**
	9	0.374346
	10	0.41594
	11	0.457534
	12	0.499128
13	**13**	**0.540722**
	14	0.582316
	15	0.62391
	16	0.665504
	17	0.707098
	18	0.748692
	19	0.790286
	20	0.83188
21	**21**	**0.873474**

Table 1 ; A table represents the diameters of transitional circles between a circle circumscribed the a first and sixth square in a golden section

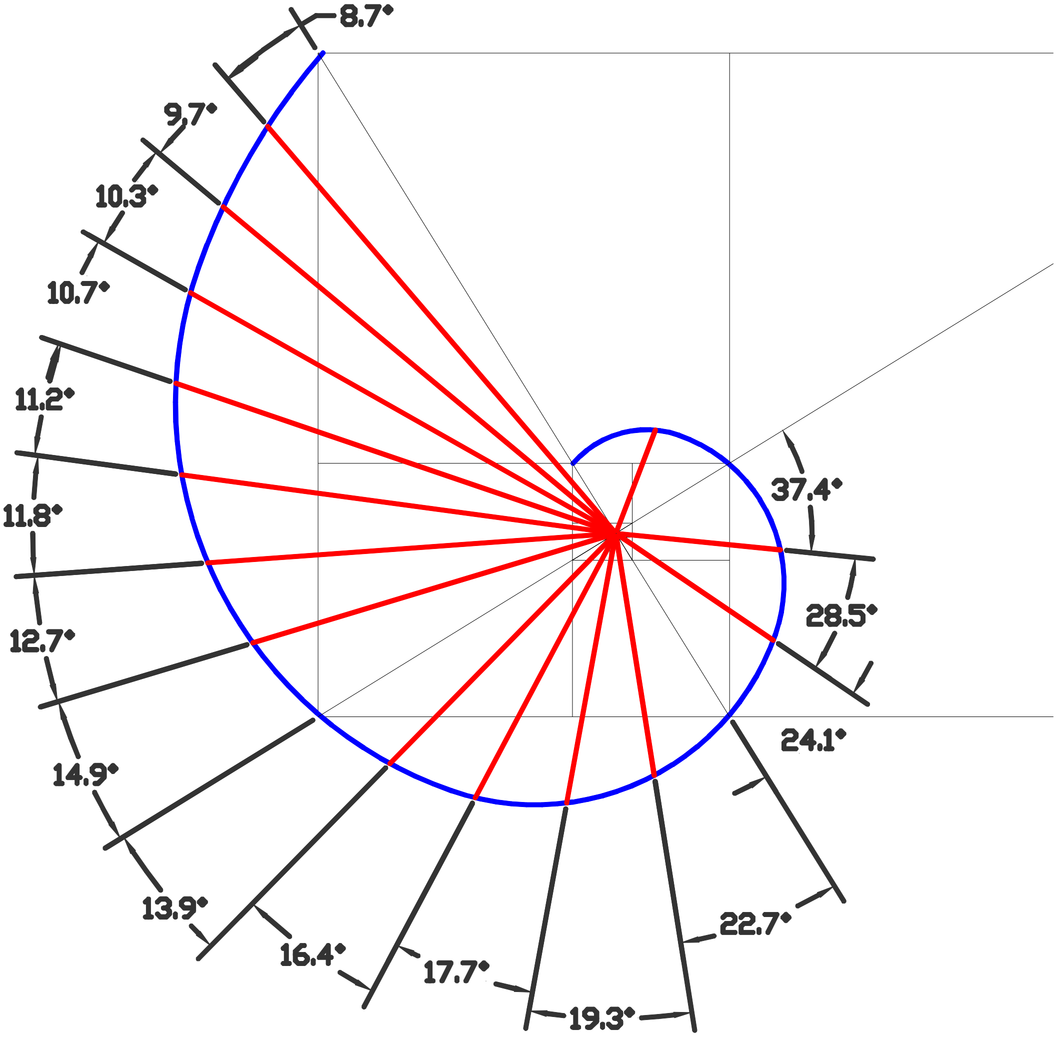


Figure 6; Determining the rotation angle of circles by dividing the spiral into the equivalent Fibonacci numbers and then draw lines from the end of each segment to the center of the golden section

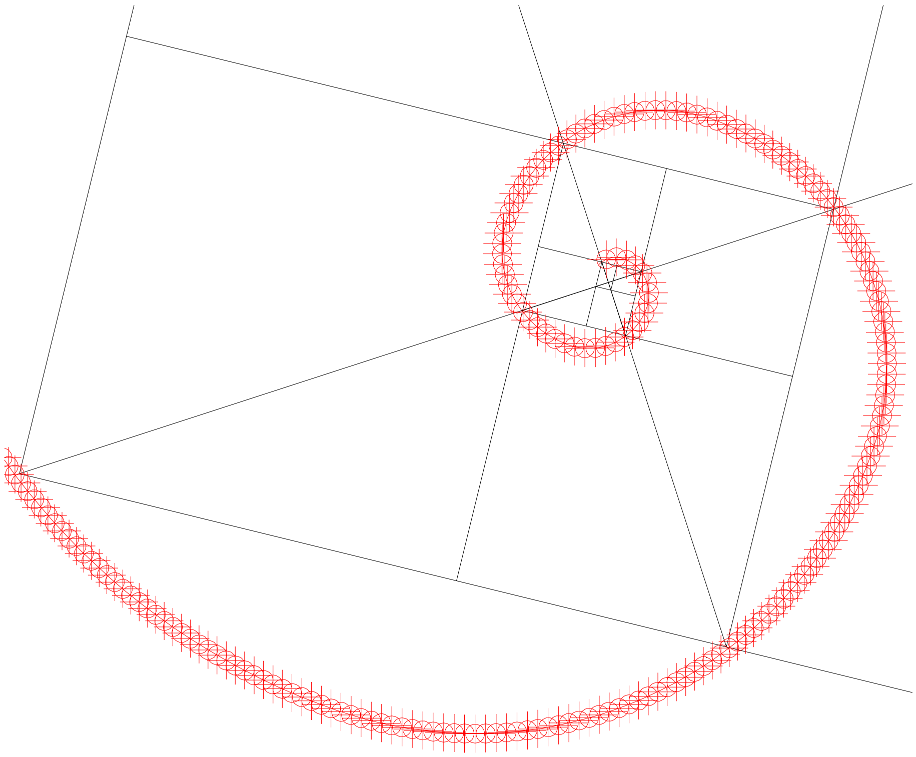


Figure 7 : This figure shows that when dividing the spiral of the golden ratio into any equal distances, then scale and fit the golden section on the spiral, then, the number of segment on each side of the golden section will match the golden ratio.

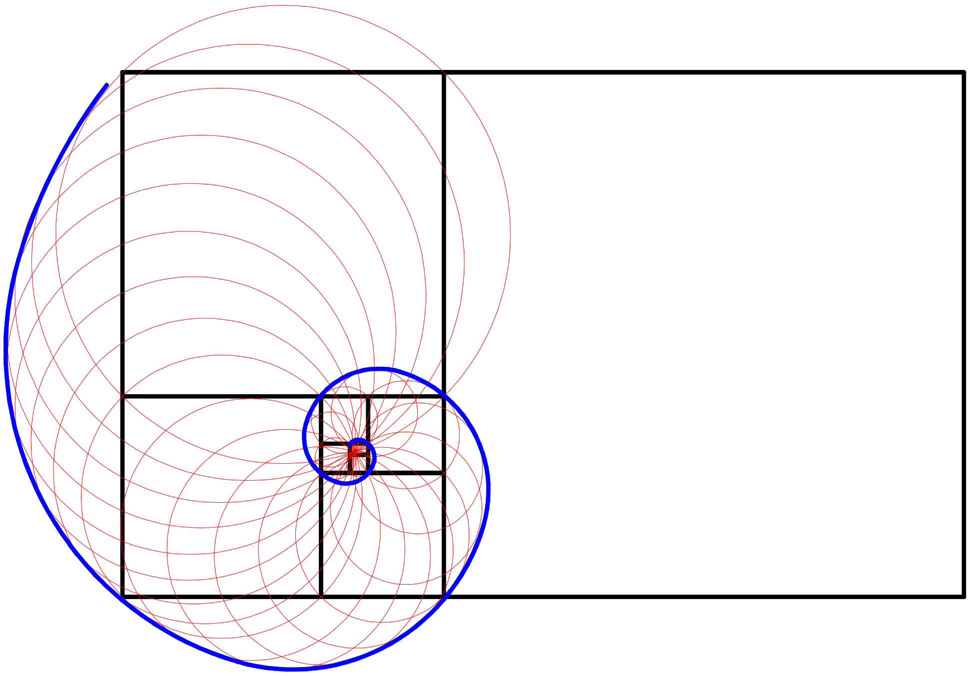


Figure 8 when drawing a number of circles equals to the equivalent Fibonacci numbers,

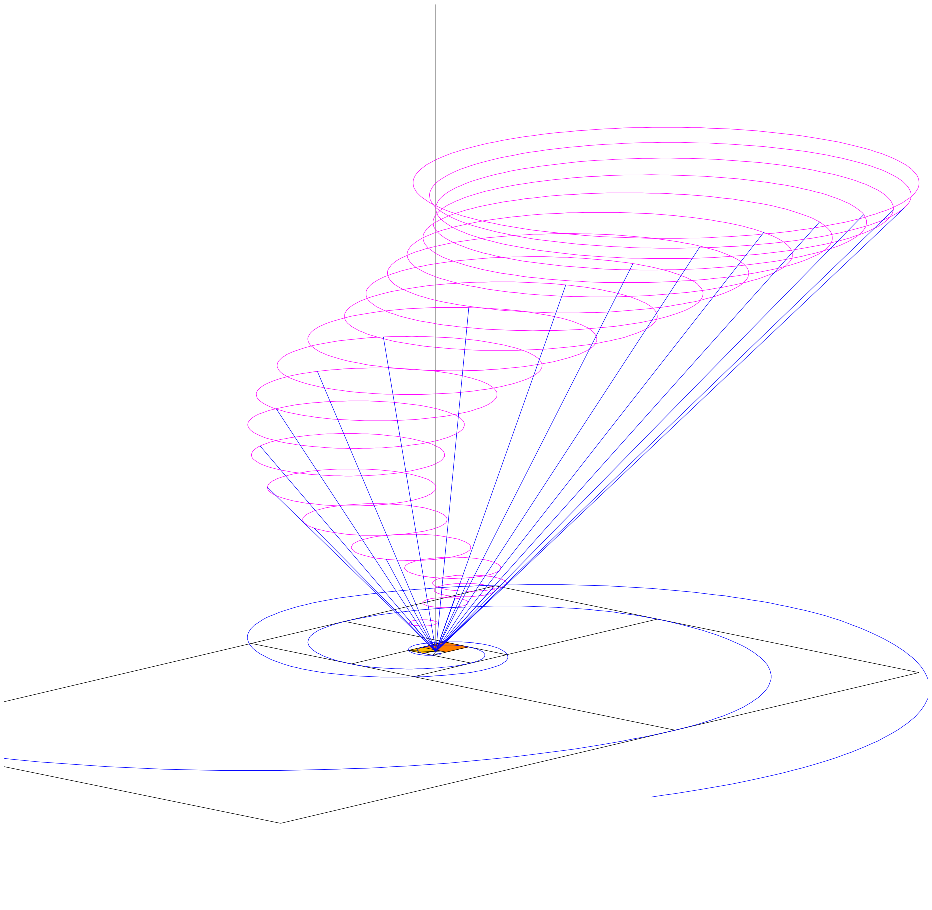


Figure 9 ; 3D configuration of the golden section. The circles represents a constant growth in the circles' diameter and a constant travel in the Z axis from the origin.

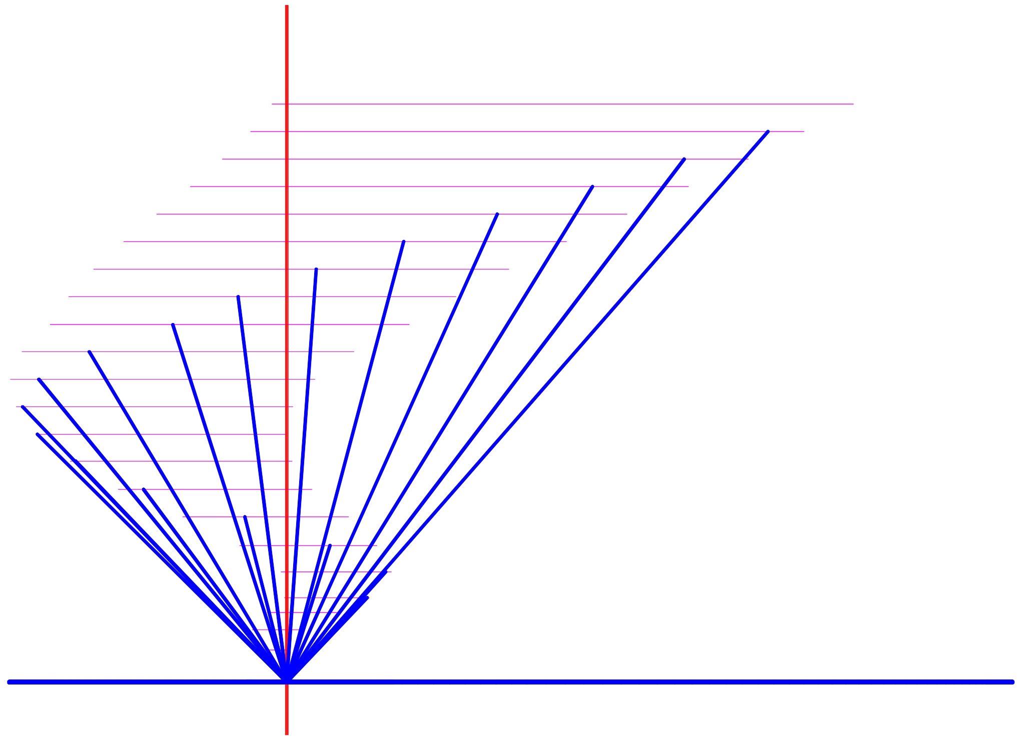


Figure 10 ; A front view of Figure 9 shows the constant growth of circles in the Z axis (Height).

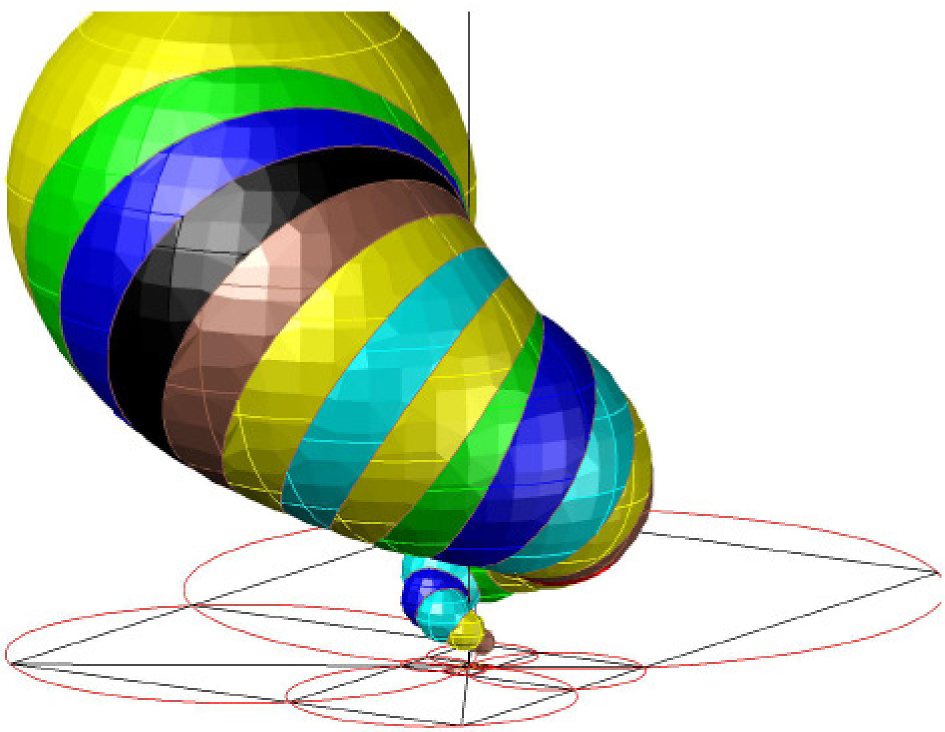


Figure 11 Spheres where drawn for each circles of Figure 9. The spheres show transitional growth between each twp circles.

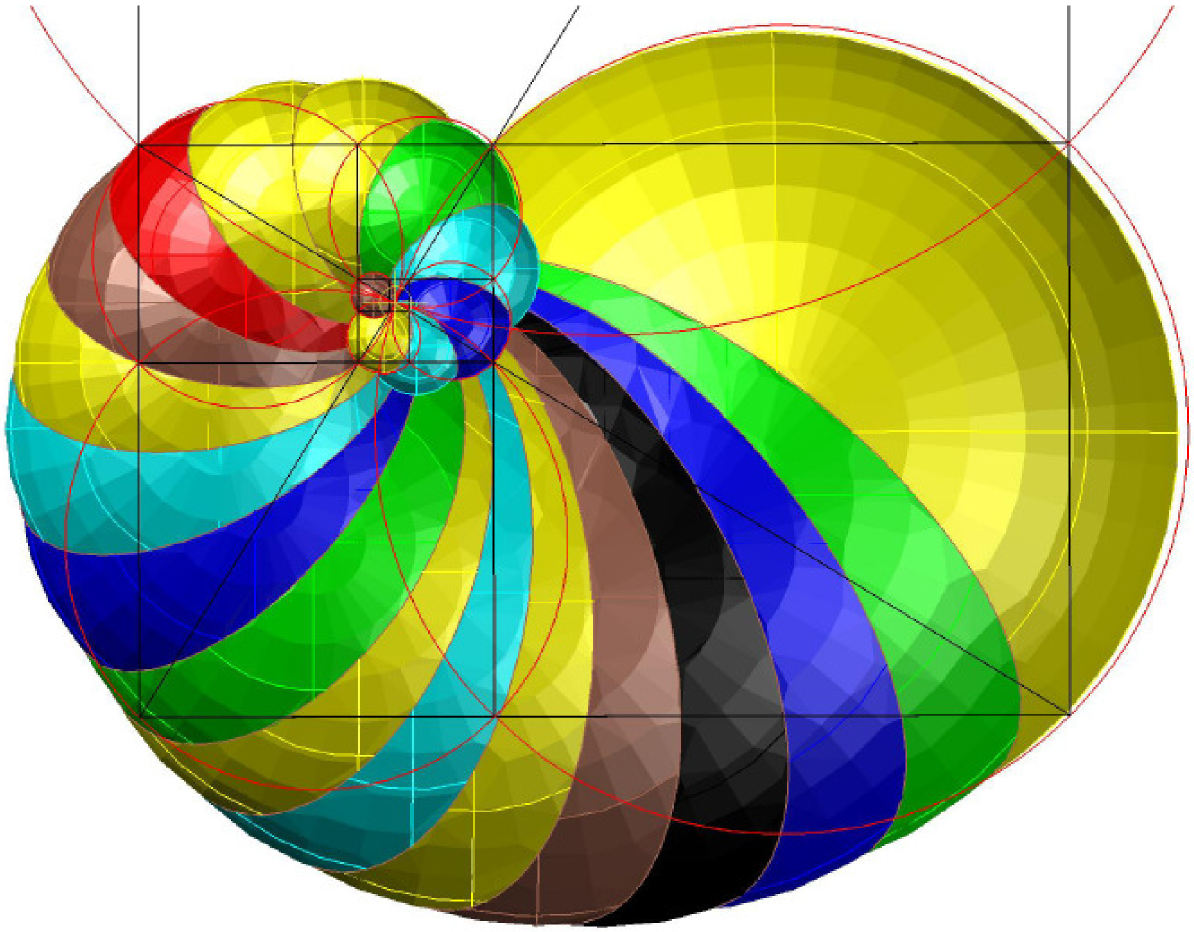


Figure 12 : Top view of Figure 11 which shows that each sphere tangents the center of the golden section.

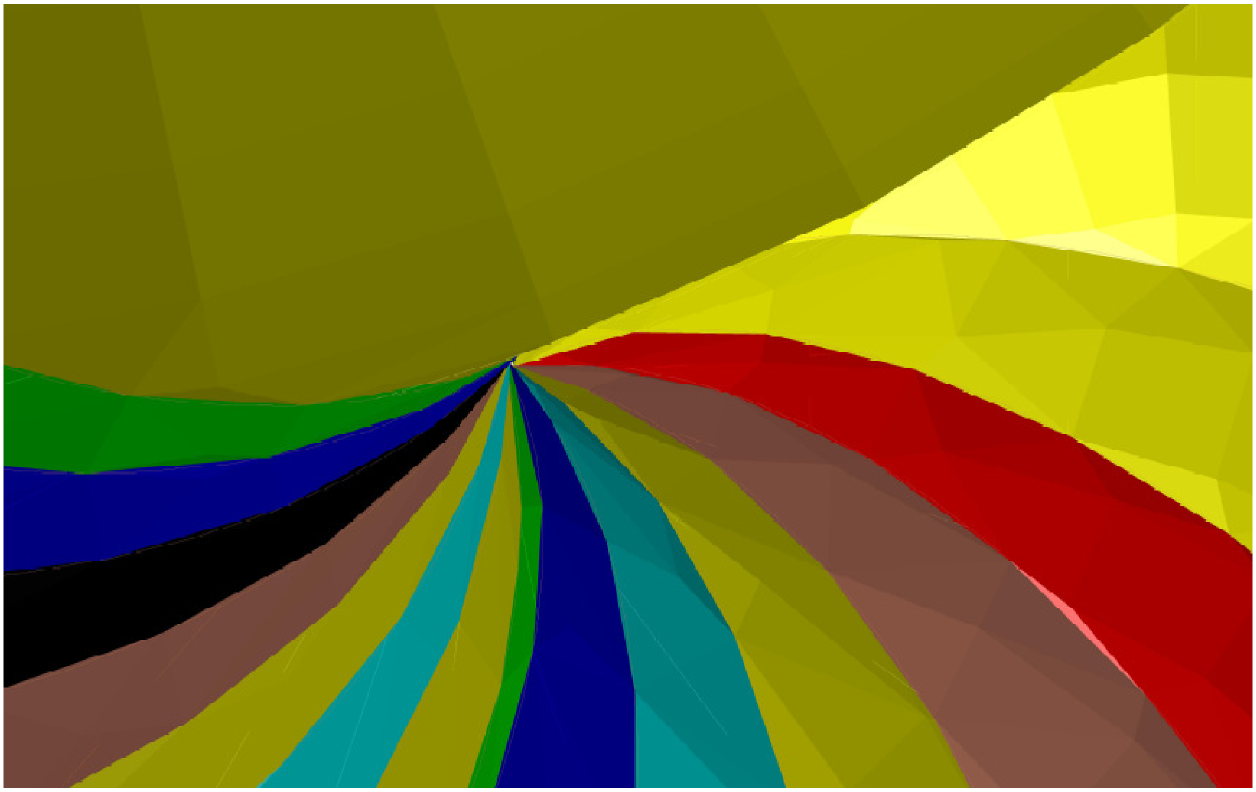


Figure 13 : a close up look inside the center of the golden ratio which shows that all spheres are meeting in the center of the golden section.

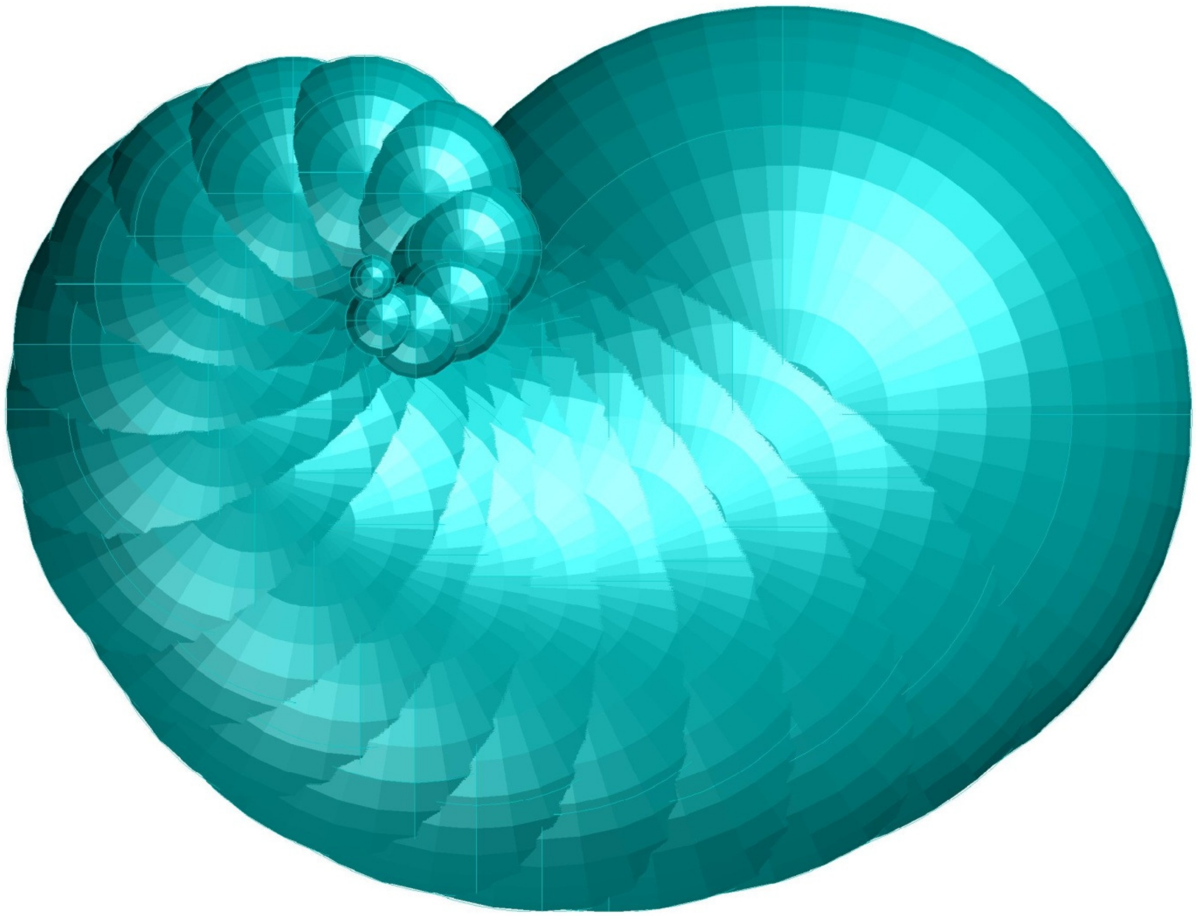


Figure 14 : A render of the spheres in Figure 12.

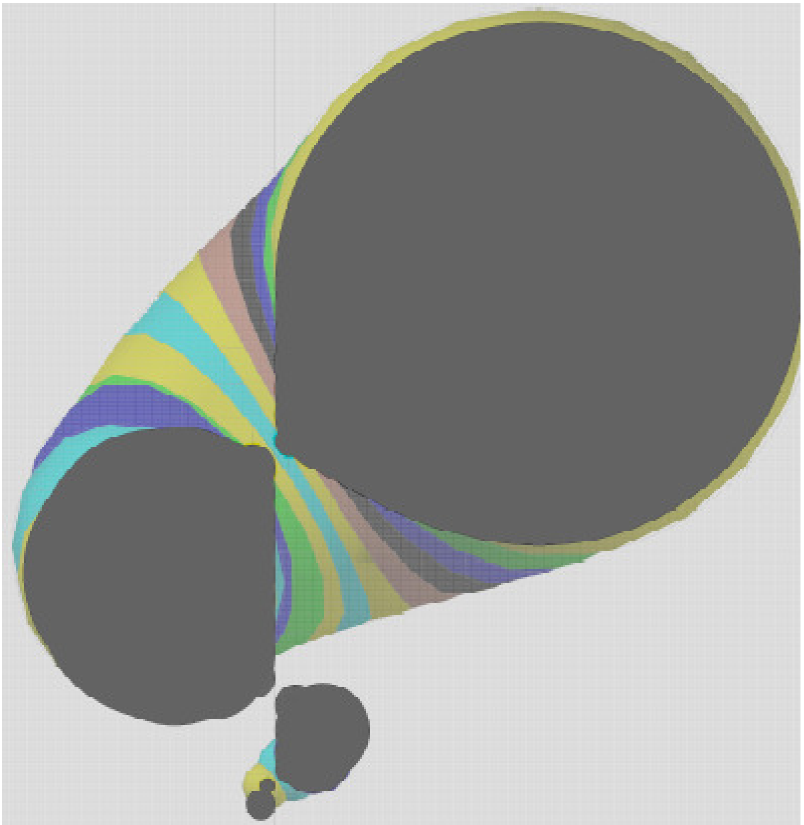


Figure 15 cross section through the golden

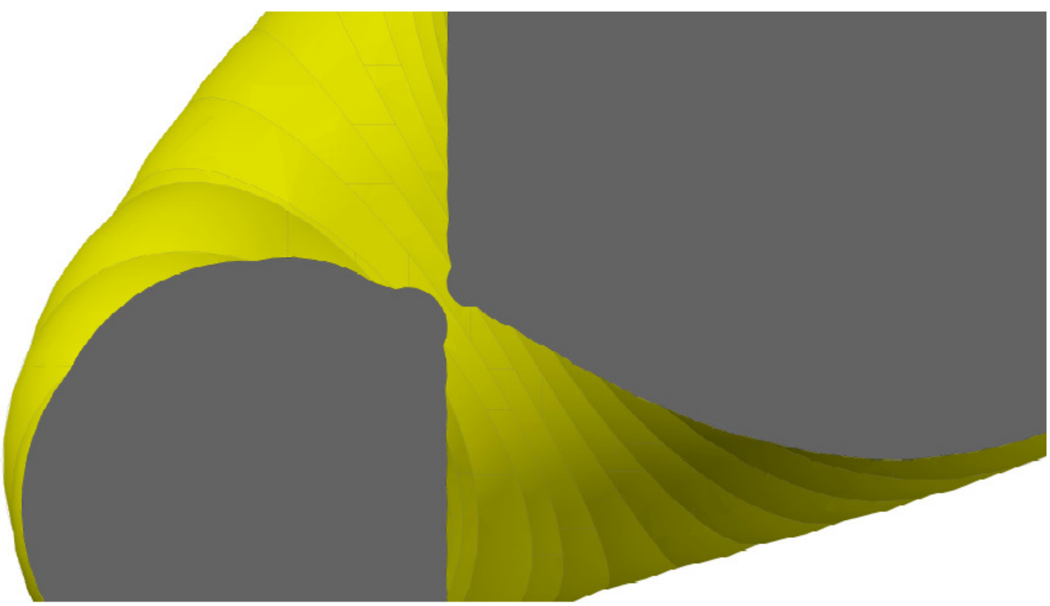


Figure 16 : close up look at the cross section line which shows that the strait line in the middle consists of tangents of the spheres.

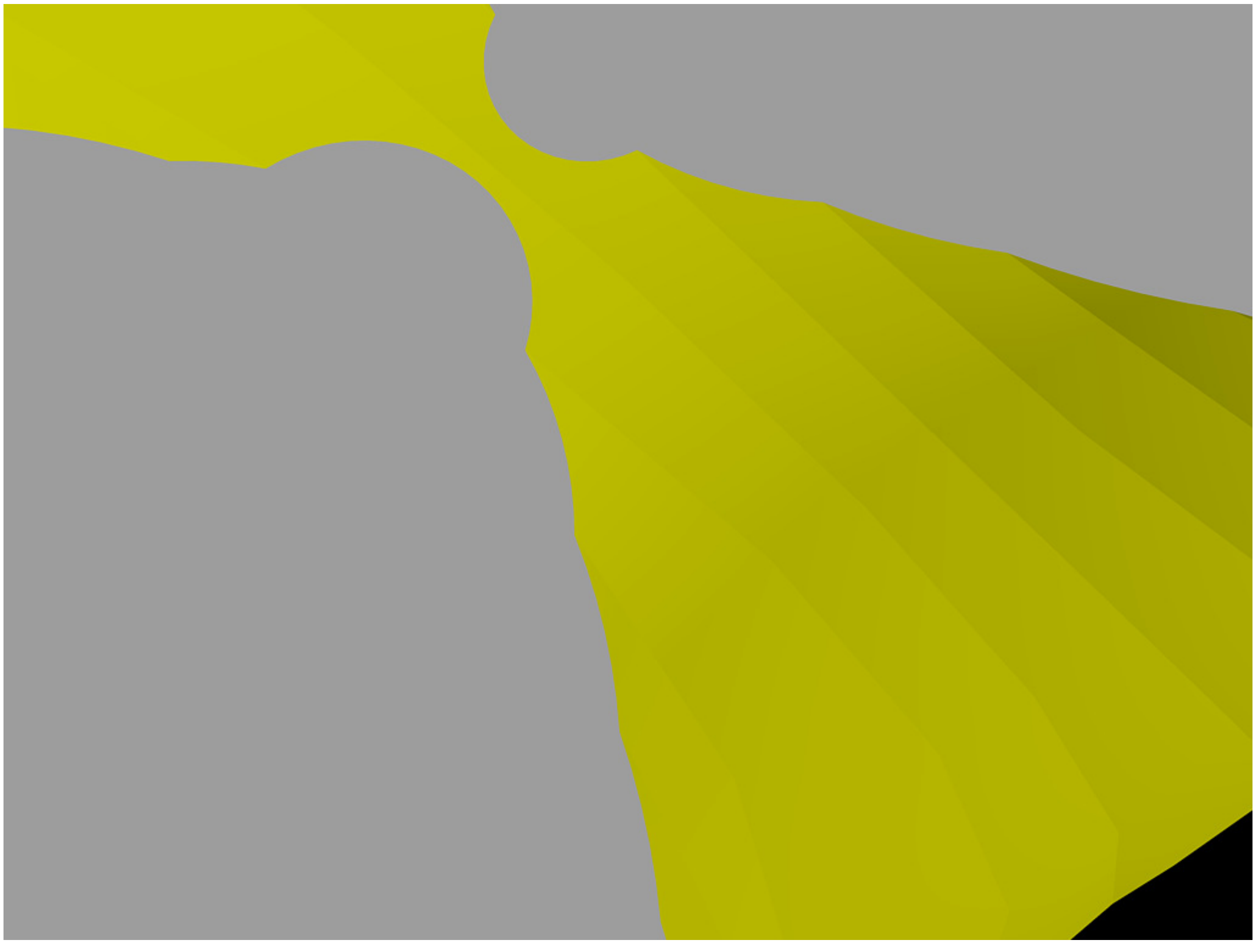


Figure 17 a close up look at the cross section line which passes through the center of the golden section. The cut line on the center is composed of tangents of spheres around the center.

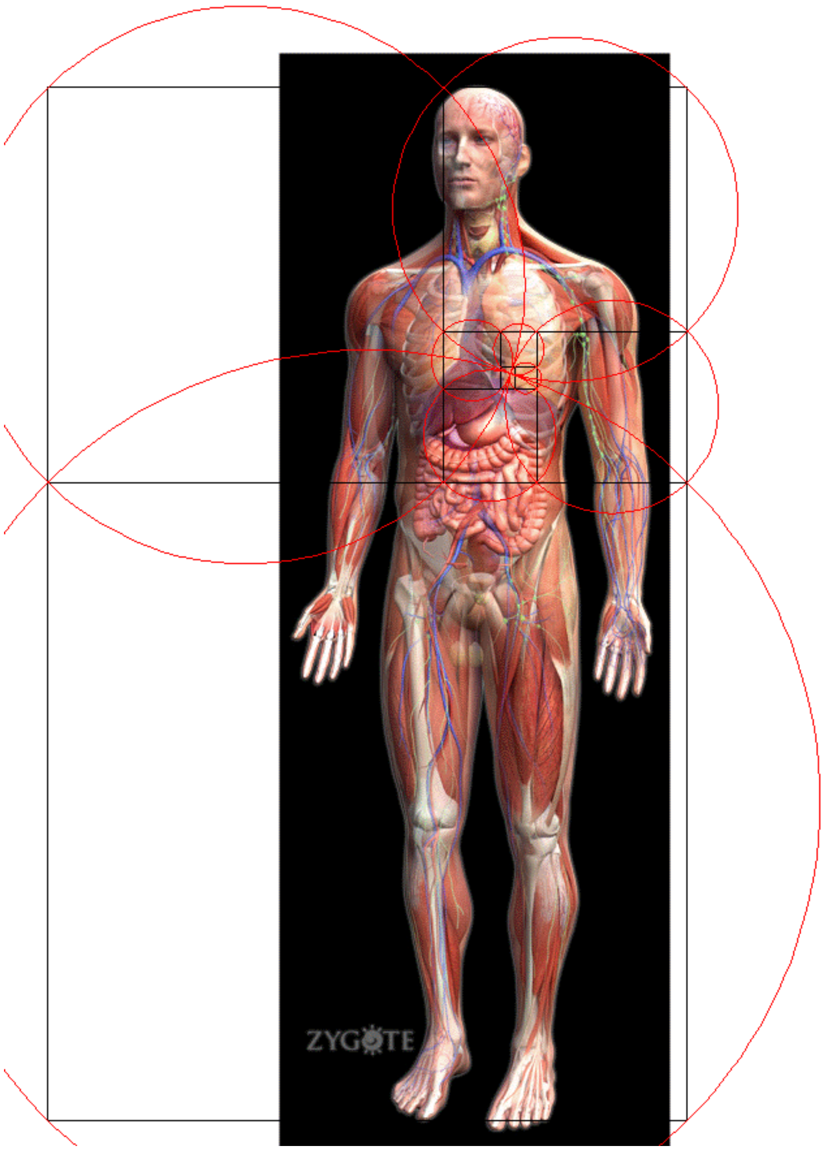


Figure 18; The heart of the human body is located exactly on the center of the golden section.

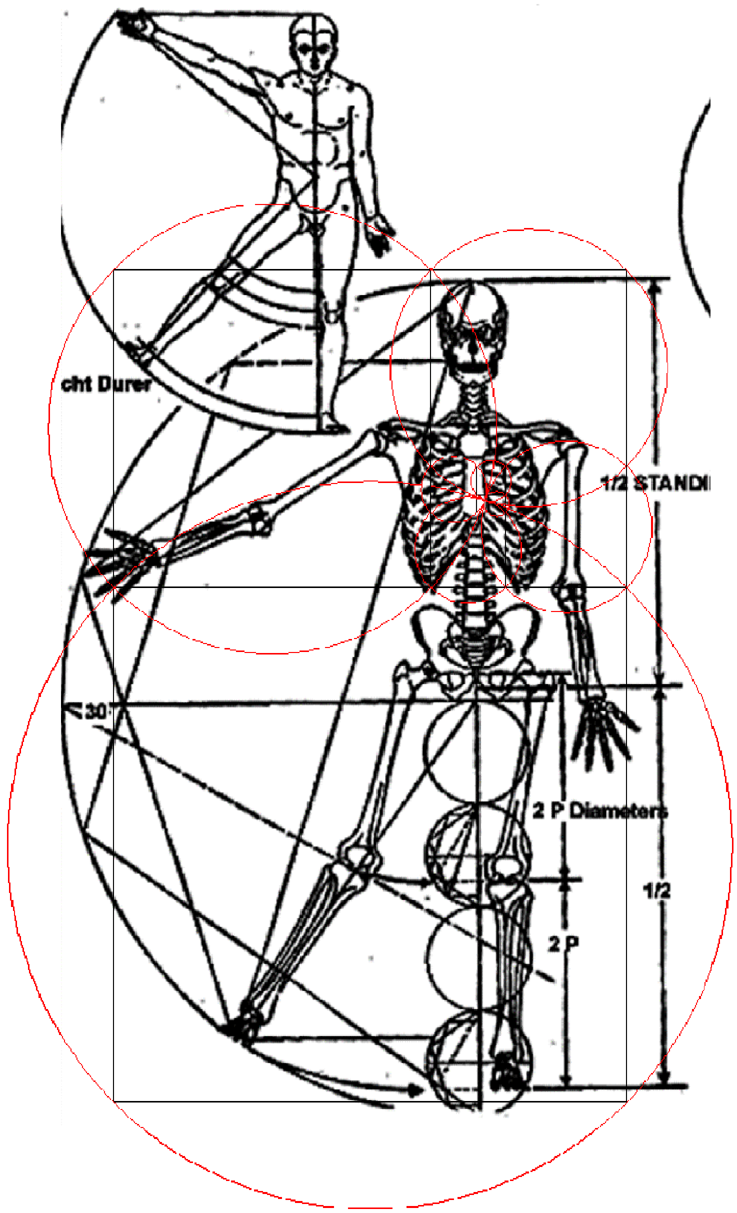


Figure 19 ; The golden section is applied on the human skeleton and shows that the center of the body intersects with the center of the golden section.

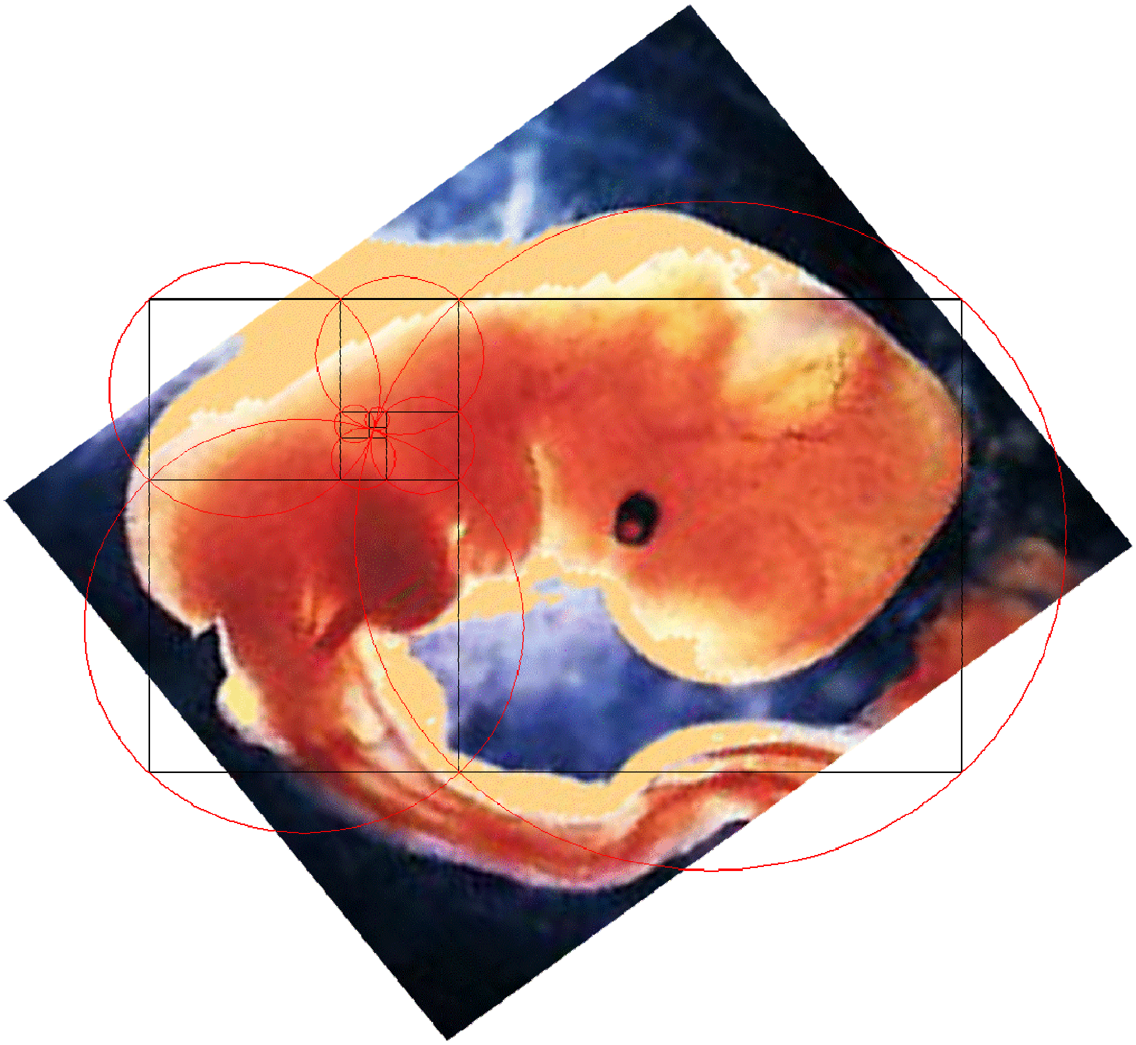


Figure 20 : the heart of a human fetus is located on the center of the golden section.

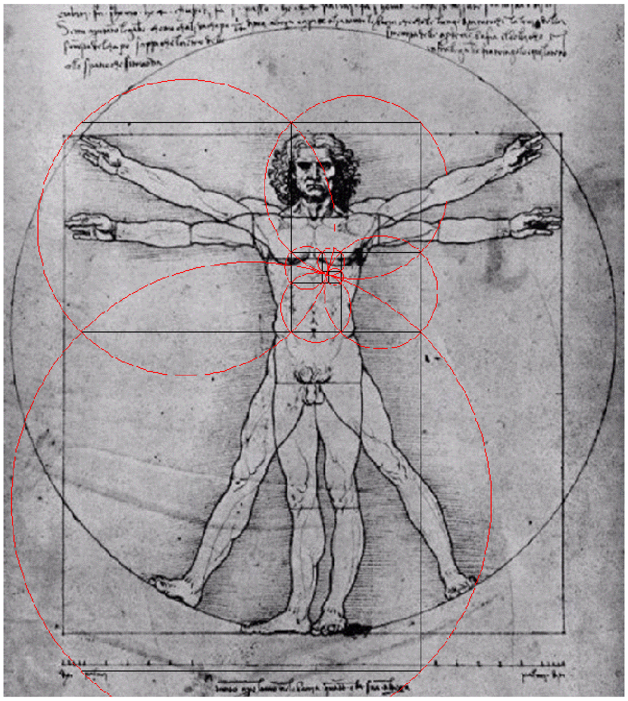
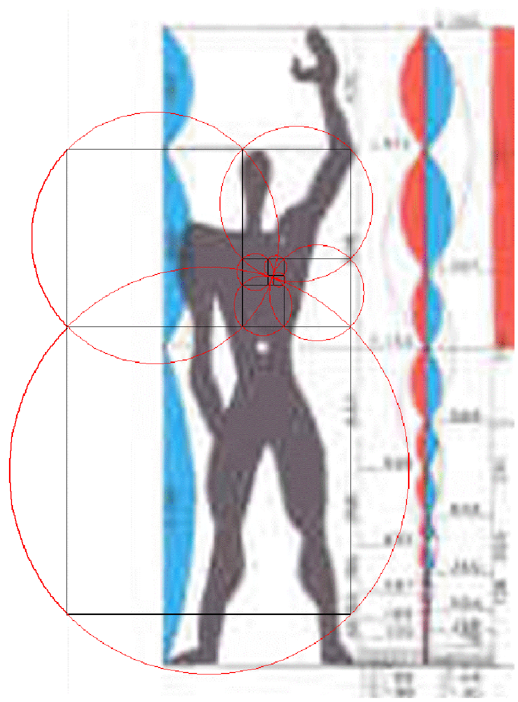


Figure 21 : When applying the golden section on Leonardo De Vinci human figure. The center of the golden section is located on the heart of the human figure.

Figure 22: To the left is the golden ration in Le Corbusier's module is in the heart of the human figure.



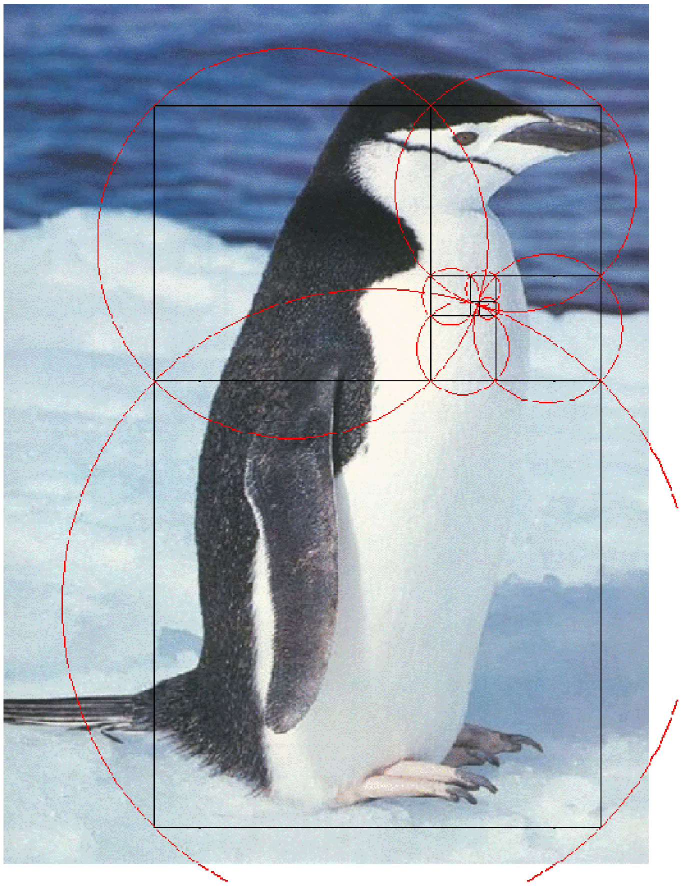


Figure 23 The heart of the penguin is located on the center of the golden section.

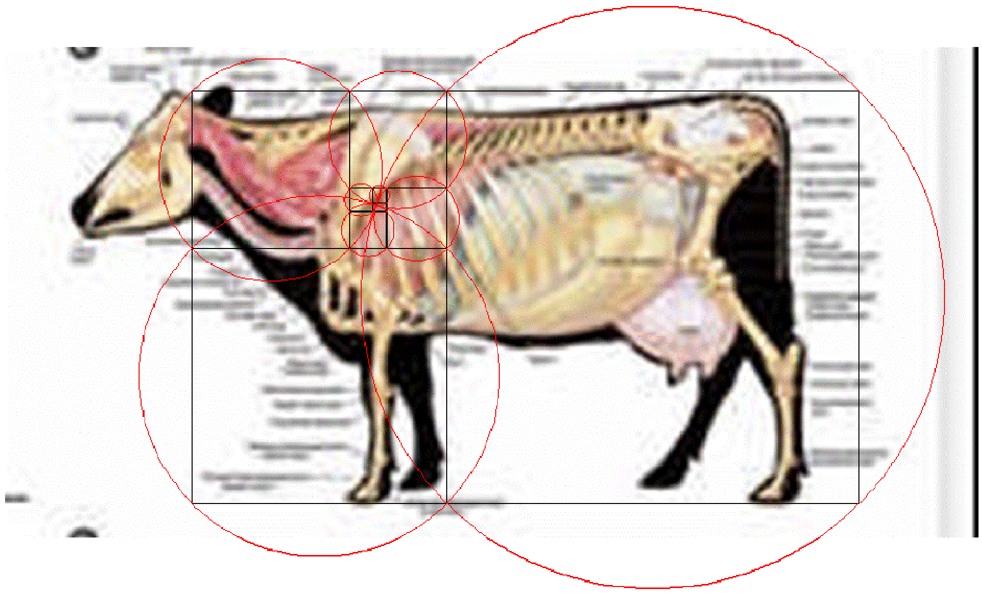


Figure 24; The center of the golden section is centered on the heart of the many animals

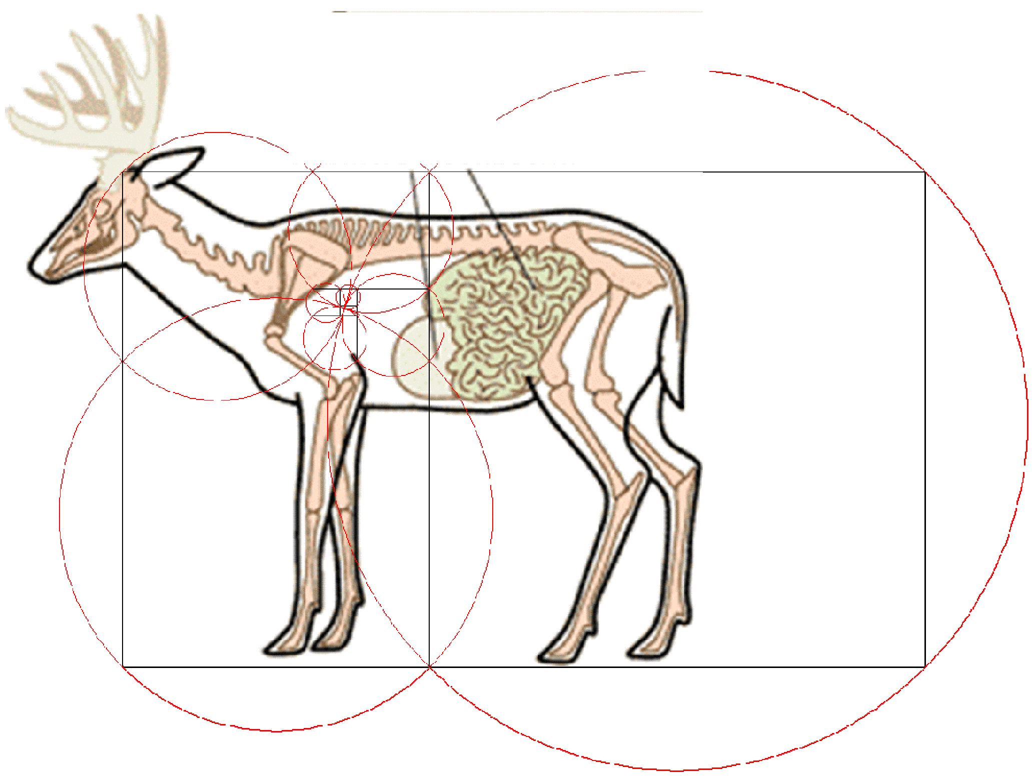


Figure 25 The center of the golden ratio is centered on the heart of the dear.