Symmetry: Culture and Science

ORDER / DISORDER
Proceedings, 4th Congress

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Volume 9, Numbers 2 - 4, 1998
A SYMMETRIC PATTERN IN FINANCIAL MARKETS

George G. Szpiro

Address: The Israeli Centre for Academic Studies, Kiriat Ono, Israel (affiliated with the University of Manchester), P.O.Box 6278, Jerusalem, Israel. Fax ++972-2-6230201. E-mail: george@netvision.net.il.

Abstract: A subtle pattern is hidden in the markets for financial assets which is due to the finite size of the “ticks” by which prices can change. The nearly symmetric pattern arises when the values of a time-series are plotted against their delayed values. This paper clarifies the reason for this phenomenon. High frequency data from a foreign exchange market are used to illustrate this phenomenon.

1. INTRODUCTION

For generations so-called chartists and technical analysts tried to find structure in financial and economic data but, as the generally accepted “perfect market hypothesis” suggests, most of these endeavors proved futile. However, the study of chaotic phenomena in the physical sciences suggested new avenues in this research area. For example, Brock, Dechert, LeBaron and Scheinkman (1996) introduce a test for the whiteness of time-series observations (i.e., the absence of structure) based on the correlation integral, a tool originally introduced by Grassberger and Procaccia (1983) to determine the dimension of a chaotic system. Attempts to forecast time-series have been made using a technique that is related to the Grassberger-Procaccia method and have been successful in the physical sciences (Farmer and Sidorowich 1987). They failed in economics and finance, however, and Szpiro (1997) suggested using the method to measure the amount of noise that is present in economic data.

The so-called phase portrait, a well-known instrument in the theory of dynamical systems, is a tool that has been utilized to detect non-linearities in time-series.
This method requires embedding the data in two- or higher-dimensional space – in the most elementary version this simply means plotting $x_{t+1}$ against $x_t$ – and inspecting the resulting graph. It was generally thought that a non-uniform distribution of the plotted points indicates the presence of some underlying structure. The inference was that in such a case the data are, in principle, forecastable. However, rounding errors in the measurement process or the non-continuity of price quotations can significantly affect the above conclusion. Crack and Ledoit (1996, henceforth CL) described the effect that emerges when the returns $R_t$ of a financial time-series are embedded in two-dimensional space. By plotting the return of a share in one period, against its return during the previous period, CL discovered that rays, originating at the origin, radiate in all directions. Major directions are more pronounced than minor directions, and CL named the figure a “compass rose”. No predictive power can be drawn from the existence of this phenomenon, however, and, even more seriously, Kramer and Runge (1997) showed that the existence of the compass rose seriously distorts tests that are meant to detect the presence of chaos in time-series.\footnote{In a different context, it has been noted that finite measurements, or rounding errors, may cause interesting patterns in the physical sciences (Szpiro 1993)}

In view of recent developments, for example NYSE’s announcement to move the minimum increment in the price of traded stock (the tick size) from eighths of a dollar to sixteenths and then to tenths (NYSE 1997), or the emerging tendency to charge minute fractions of cents for services on the Internet, it is important to analyze the effects of rounding errors in a rigorous fashion. This paper – together with its predecessor (Szpiro 1998) – puts the analysis of non-continuities in market prices on a sound basis. In the following section the mathematics of a formal model is set out, and Section 3 uses data from a foreign exchange market to show that the symmetric patterns arise in actual economic time-series.

2. THE MATHEMATICAL MODEL

The return and the closing price of financial asset, say a stock, at date $t$ will be denoted by $R_t$ and $P_t$, respectively, and $h$ is the size of the tick by which the stock can rise. We have,

$$\frac{R_{t+1}}{R_t} = \frac{(P_{t+1} - P_t)/P_t}{(P_t - P_{t-1})/P_{t-1}} = \frac{n_{t+1}h/P_t}{n_t h/(P_t - n_t h)}$$

\[(1)\]
The right hand side of the equation arises because $P_{t+1}$ equals $P_t + n_{t+1}h$ and $P_t$ equals $P_{t-1} + n_t h$ where $n_t$ is the number of ticks by which the price of the stock increased in the time period $t-1$ to $t$. The behavior of this equation is analyzed in the following section.

To investigate equation (1), the locus of

$$X: \frac{n_t h}{P_t - n_t h} \quad \text{and} \quad Y: \frac{n_{t+1} h}{P_t}$$

will be plotted in two-dimensional space. It should be noted that this plot, which is often called a phase-portrait in the physical sciences, is not single-valued, since different combinations of $n_t$, $n_{t+1}$, and $P_t$ may result in identical loci in the figure. Hence, to begin the analysis, I hold the price of the share constant ($P_t = 100$). (Throughout the paper the tick size $h$ is set equal to 1.0, and the integers $n_t$ and $n_{t+1}$ vary between -5 and +5.) As CL have pointed out, the system defines a grid (Figure 1). However, on closer inspection of the figure, one may note that this grid is not regular: as follows from equation (1) the distance between the vertical gridlines expands with $n_t$.²

![Figure 1: The grid ($P_t = 100$)](image)

² This effect becomes more pronounced when the stock price is low in relation to the tick size (i.e., when the ticks are coarse).
Now in addition to \(n_t\) and \(n_{t+1}\), let \(P_t\) also vary. CL showed that this results in a "smeared grid". If \(P_t\) varies, but not too much, the pattern begins to emerge. Figure 2 depicts the loci of system (2) for four prices \((P_t = 100, 105, 110, 115)\). The most prominent feature of the figure is that the compass rose is actually made up of separate segments. For each combination of \(n_t\) and \(n_{t+1}\), the loci corresponding to a collection of \(P_t\)-values from a cluster which, in effect, is a smeared gridpoint. Note that smearing does not quite occur in the directions of the compass rose: since the distance between the horizontal gridlines expand horizontally the rays exhibits curvature. Again, this phenomenon becomes more pronounced if the tick size is coarse.

\[ \begin{array}{cccc}
0.05 & 0.04 & 0.03 & 0.02 \\
0.01 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \\
-0.01 & -0.02 & -0.03 & -0.04 \\
-0.05 & -0.05 & -0.05 & -0.05 \\
\end{array} \]

\( -0.05 \quad -0.03 \quad -0.01 \quad 0.01 \quad 0.03 \quad 0.05 \)

**Figure 2**: Smeared grid \((P_t = 100, 105, 110, 115)\)
If the $P_t$-values vary over a sufficiently wide range the clusters overlap. To show this, it must be proved that the “endpoints” of the clusters connect, and that their slopes are equal whenever the loci coincide. Let us look at two clusters that lie on the same ray $\delta \varepsilon$, one belonging to $n_{t+1} = \lambda \delta , n_t = \lambda \varepsilon$, and the other belonging to the following cluster $n_{t+1} = (\lambda + 1)\delta$ and $n_t = (\lambda + 1)\varepsilon$. The corresponding loci are

\[
X_1 = \frac{\lambda \varepsilon}{P} \quad Y_1 = \frac{\lambda \delta}{P - \lambda \delta} \quad (3)
\]

\[
X_2 = \frac{(\lambda + 1)\varepsilon}{Q} \quad Y_2 = \frac{(\lambda + 1)\delta}{Q - (\lambda + 1)\delta}
\]

Obviously, $X_1$ coincides with $X_2$, and $Y_1$ with $Y_2$ if

\[
Q = \left(1 + \frac{1}{\lambda} \right)P. \quad (4)
\]

Hence when the share prices span values between $P$ and $P(1+1/\lambda)$ the clusters connect; when they span a wider range the clusters overlap and the points belonging to them intersperse. Note that when $P$ grows the higher-order clusters (those with large $\lambda$) are the first ones to connect. For all clusters to connect, the price must vary between $P$ and $2P$. The slopes at a certain locus are identical even if the clusters belong to different rays. The slopes at the loci defined by price $P$ in the first cluster and by price $Q$ in the second cluster are

\[
\text{Slope}_1 = \frac{\varepsilon}{\delta} \left( \frac{P}{P - \lambda \delta} \right)^2 \quad \text{and} \quad \text{Slope}_2 = \frac{\varepsilon}{\delta} \left( \frac{Q}{Q - (\lambda + 1)\delta} \right)^2. \quad (5)
\]

If $Q = P(1+1/\lambda)$, the slopes are equal.

If the $P_t$-values vary sufficiently during the observation period, curvature and interspersion cause the phase-portrait to become "smudged" (in addition to being smeared), and the compass rose seems to disappear. Only the major directions remain delineated. In Figure 3 the range of share prices is larger than in Figure 2 ($P_t = 100, 118, 136, 154$), and only the major rays can be made out in the disarray. The question is why major directions stand out even after the compass rose has become smeared and smudged. As I will now show, the answer is based on an optical illusion.
When stock prices span values between $d$ and the points in the phase-orthogonal cover a

**Figure 3:** Shaded triangle and $d_f = 100.118.136.159$
The equation for the change in the size of the pool is given by:

\[ \frac{1}{2} \left( \frac{y'u}{d} \right) = \frac{(y'u + d)}{(y'u)} - \frac{(y'u - d)}{(y'u)} = \nabla \]

Where \( \nabla \) is the gradient which follows from equation (2) when \( n \) is the number of rays used.

To make the rays more difficult to observe, the number of rays used is much more difficult. The next equation would further
To portray the discussed features in a salient manner, I now plot the situation with a coarse tick size \( P_t = 10, 12, 14, 16 \). In Figure 4 the compass rose, curvature, asymmetry and the disappearance of the minor rays can be made out.\(^5\)

3. EMPIRICAL EVIDENCE

In order to illustrate the phenomena that were predicted in the previous section, we use data from the Dollar-Deutschmark foreign exchange market. The data consist of a high-frequency time-series of bid and ask quotes, whose arithmetic means are taken as proxies for actual trades. Nearly 1.5 million observations were collected for the period between October 1\(^{st}\) 1992 to September 31\(^{st}\) 1993, and the series was made available to the academic community.\(^6\) During the observation period the exchange rate varied from a low of 1.3950 DM/$ to a high of 1.7455, that is, the rate fluctuated by about 25 percent. This implies, by equation (4), that the fourth, fifth and higher clusters connect.

---

\(^5\) Some new patterns also emerge, which are not germane to the discussion, however.

In Figure 5 the values of $R_{t+1}$ are plotted against $R_t$ for a sample of 250,000 observations. A smeared and smudged grid-like system, resembling the compass rose, is visible. When inspecting the blown up part of the figure (Figure 6), some of the features that were discussed in the previous section become evident. Especially the clusters are clearly visible. On the NW ray it can be seen that the fourth and the fifth cluster do, in fact, connect, while clusters of lower order remain separated.

Figure 6: Detail of Figure 5
REFERENCES

correlation dimension, Econometric Reviews 15, 197-235.

Crack, T. F. and Ledoit, O (1996) Robust structure without predictability: the 'Compass Rose' pattern in the
stock market, J. of Finance 51, 751-762

848.

346-349.


255.

Szpiro, George G. (1998) Tick Size, the Compass Rose and Market Nano-Structure, Journal of Banking and
Finance, forthcoming.