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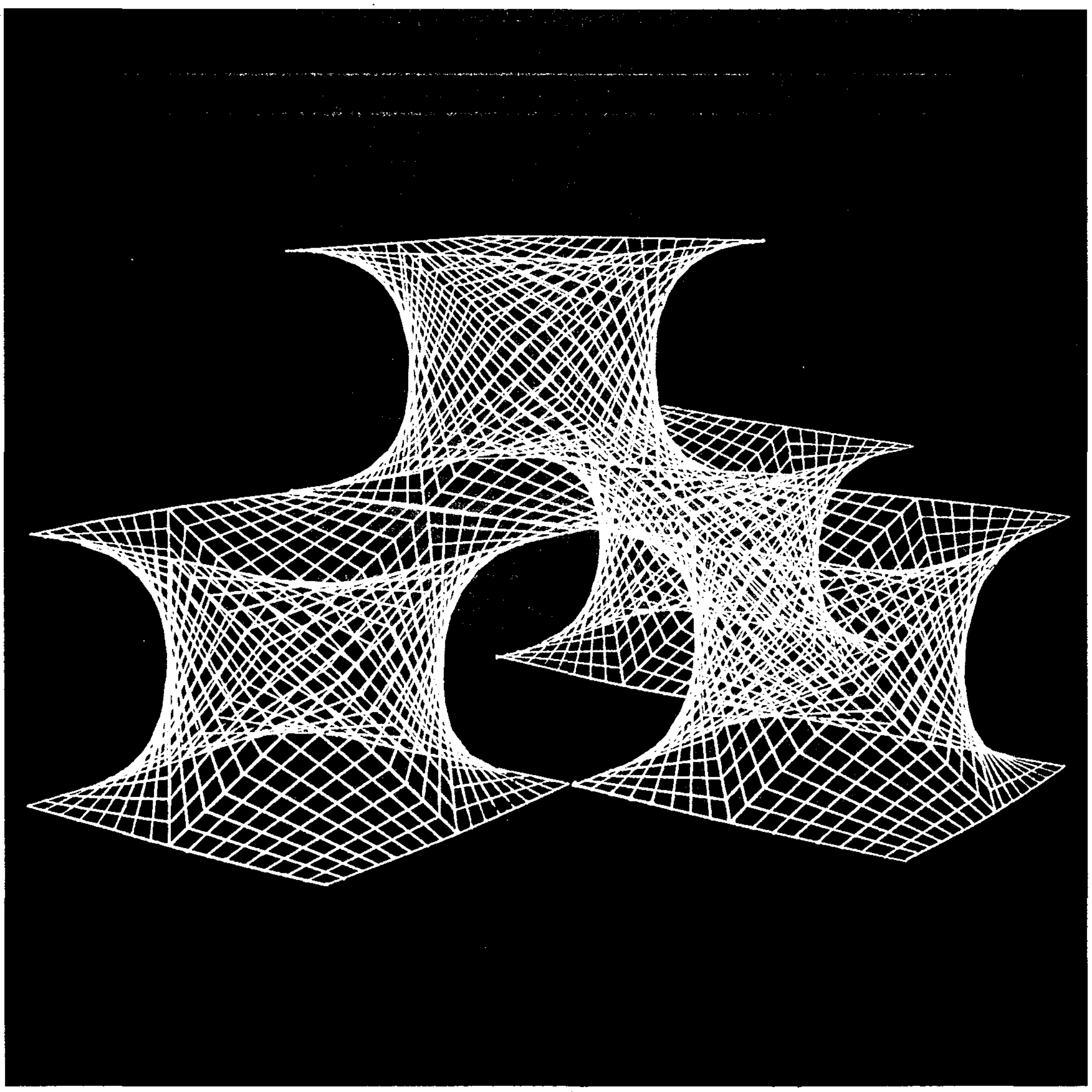

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A SYMMETRIC PATTERN IN FINANCIAL MARKETS

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Abstract: *A subtle pattern is hidden in the markets for financial assets which is due to the finite size of the "ticks" by which prices can change. The nearly symmetric pattern arises when the values of a time-series are plotted against their delayed values. This paper clarifies the reason for this phenomenon. High frequency data from a foreign exchange market are used to illustrate this phenomenon.*

1. INTRODUCTION

For generations so-called chartists and technical analysts tried to find structure in financial and economic data but, as the generally accepted "perfect market hypothesis" suggests, most of these endeavors proved futile. However, the study of chaotic phenomena in the physical sciences suggested new avenues in this research area. For example, Brock, Dechert, LeBaron and Scheinkman (1996) introduce a test for the whiteness of time-series observations (i.e., the absence of structure) based on the correlation integral, a tool originally introduced by Grassberger and Procaccia (1983) to determine the dimension of a chaotic system. Attempts to forecast time-series have been made using a technique that is related to the Grassberger-Procaccia method and have been successful in the physical sciences (Farmer and Sidorowich 1987). They failed in economics and finance, however, and Szpiro (1997) suggested using the method to measure the amount of noise that is present in economic data.

The so-called phase portrait, a well-known instrument in the theory of dynamical systems, is a tool that has been utilized to detect non-linearities in time-series.

This method requires embedding the data in two- or higher-dimensional space – in the most elementary version this simply means plotting x_{t+1} against x_t – and inspecting the resulting graph. It was generally thought that a non-uniform distribution of the plotted points indicates the presence of some underlying structure. The inference was that in such a case the data are, in principle, forecastable. However, rounding errors in the measurement process or the non-continuity of price quotations can significantly affect the above conclusion. Crack and Ledoit (1996, henceforth CL) described the effect that emerges when the returns R_t of a financial time-series are embedded in two-dimensional space. By plotting the return of a share in one period, against its return during the previous period, CL discovered that rays, originating at the origin, radiate in all directions. Major directions are more pronounced than minor directions, and CL named the figure a "compass rose". No predictive power can be drawn from the existence of this phenomenon, however, and, even more seriously, Kramer and Runge (1997) showed that the existence of the compass rose seriously distorts tests that are meant to detect the presence of chaos in time-series.[1]

In view of recent developments, for example NYSE's announcement to move the minimum increment in the price of traded stock (the tick size) from eighths of a dollar to sixteenths and then to tenths (NYSE 1997), or the emerging tendency to charge minute fractions of cents for services on the Internet, it is important to analyze the effects of rounding errors in a rigorous fashion. This paper – together with its predecessor (Szpiro 1998) – puts the analysis of non-continuities in market prices on a sound basis. In the following section the mathematics of a formal model is set out, and Section 3 uses data from a foreign exchange market to show that the symmetric patterns arise in actual economic time-series.

2. THE MATHEMATICAL MODEL

The return and the closing price of financial asset, say a stock, at date t will be denoted by R_t and P_t respectively, and h is the size ofthe tick by which the stock can rise. We have,

$$\frac{R_{t+1}}{R_t} = \frac{(P_{t+1} - P_t)/P_t}{(P_t - P_{t-1})/P_{t-1}} = \frac{n_{t+1}h/P_t}{n_t h/(P_t - n_t h)}. \tag{1}$$

[1] In a different context, it has been noted that finite measurements, or rounding errors, may cause interesting patterns in the physical sciences (Szpiro 1993)

The right hand side of the equation arises because P_{t+1} equals $P_t + n_{t+1}h$ and P_t equals $P_{t-1} + n_t h$ where n_t is the number of ticks by which the price of the stock increased in the time period t-1 to t. The behavior of this equation is analyzed in the following section.

To investigate equation (1), the locus of

$$X:\ R_t = \frac{n_t h}{P_t - n_t h} \quad \text{and} \quad Y:\ R_{t+1} = \frac{n_{t+1} h}{P_t} \tag{2}$$

will be plotted in two-dimensional space. It should be noted that this plot, which is often called a phase-portrait in the physical sciences, is not single-valued, since different combinations of n_t , n_{t+1}, and P_t, may result in identical loci in the figure. Hence, to begin the analysis, I hold the price of the share constant (P_t = 100). (Throughout the paper the tick size h is set equal to 1.0, and the integers n_t, and n_{t+1} vary between -5 and +5.) As CL have pointed out, the system defines a grid (Figure 1). However, on closer inspection ofthe figure, one may note that this grid is not regular: as follows from equation (1) the distance between the vertical gridlines expands with n_t.[2]

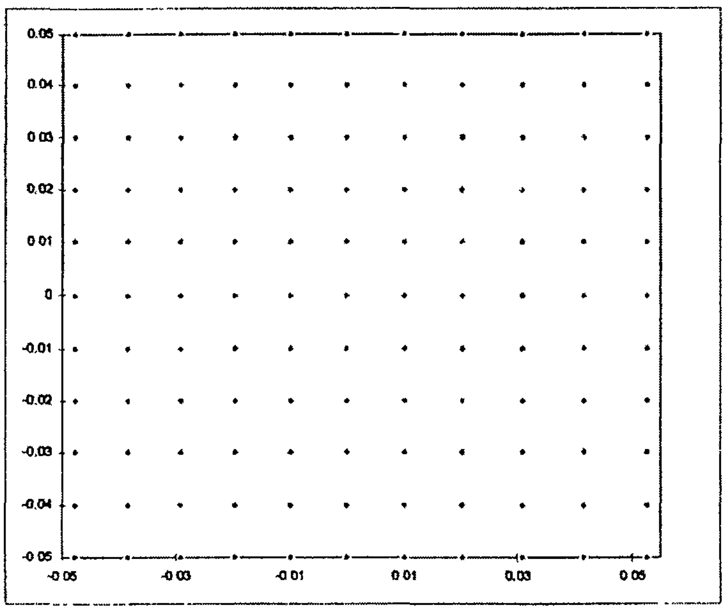

Figure 1: The grid (P_t = 100)

[2] This effect becomes more pronounced when the stock price is low in relation to the tick size (i.e., when the ticks are coarse).

Now in addition to n_t and n_{t+1}, let P_t also vary. CL showed that this results in a "smeared grid". If P_t varies, but not too much, the pattern begins to emerge. Figure 2 depicts the loci of system (2) for four prices (P_t = 100, 105, 110, 115). The most prominent feature of the figure is that the compass rose is actually made up of separate segments. For each combination of n_t, and n_{t+1}, the loci corresponding to a collection of P_t-values from a cluster which, in effect, is a smeared gridpoint. Note that smearing does not quite occur in the directions of the compass rose: since the distance between the horizontal gridlines expand horizontally the rays exhibits curvature. Again, this phenomenon becomes more pronounced if the tick size is coarse.

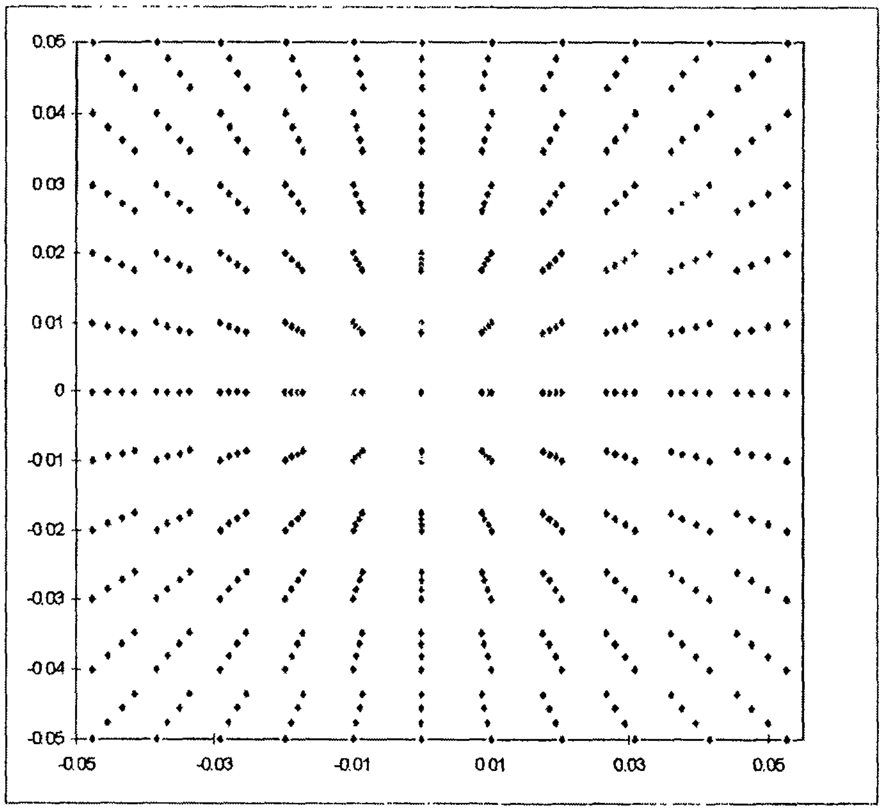

Figure 2: Smeared grid (P_t = 100, 105, 110, 115)

If the P_t-values vary over a sufficiently wide range the clusters overlap. To show this, it must be proved that the "endpoints" of the clusters connect, and that their slopes are equal whenever the loci coincide. Let us look at two clusters that lie on the same ray δ/ε, one belonging to $n_{t+1} = \lambda\delta$, $n_t = \lambda\varepsilon$, and the other belonging to the following cluster $n_{t+1} = (\lambda+1)\delta$ and $n_t = (\lambda+1)\varepsilon$. The corresponding loci are

$$X_1 = \frac{\lambda\varepsilon}{P} \qquad Y_1 = \frac{\lambda\delta}{P-\lambda\delta}$$
$$X_2 = \frac{(\lambda+1)\varepsilon}{Q} \qquad Y_2 = \frac{(\lambda+1)\delta}{Q-(\lambda+1)\delta} \tag{3}$$

Obviously , X_1 coincides with X_2, and Y_1 with Y_2 if

$$Q = \left(1+\frac{1}{\lambda}\right)P. \tag{4}$$

Hence when the share prices span values between P and $P(1+1/\lambda)$ the clusters connect; when they span a wider range the clusters overlap and the points belonging to them intersperse. Note that when P grows the higher-order clusters (those with large λ) are the first ones to connect. For all clusters to connect, the price must vary between P and $2P$. The slopes at a certain locus are identical even if the clusters belong to different rays. The slopes at the loci defined by price P in the first cluster and by price Q in the second cluster are

$$Slope_1 = \frac{\varepsilon}{\delta}\left(\frac{P}{P-\lambda\delta}\right)^2 \quad \text{and} \quad Slope_2 = \frac{\varepsilon}{\delta}\left(\frac{Q}{Q-(\lambda+1)\delta}\right)^2. \tag{5}$$

If $Q = P(1+1/\lambda)$, the slopes are equal.

If the P_t-values vary sufficiently during the observation period, curvature and interspersion cause the phase-portrait to become "smudged" (in addition to being smeared), and the compass rose seems to disappcar. Only the major directions remain delineatcd. In Figure 3 the range of share prices is larger than in Figure 2 (P_t = 100, 118, 136, 154), and only the major rays can be made out in the disarray. The question is why major directions stand out even after the compass rose has become smeared and smudged. As I will now show, the answer is based on an optical illusion.

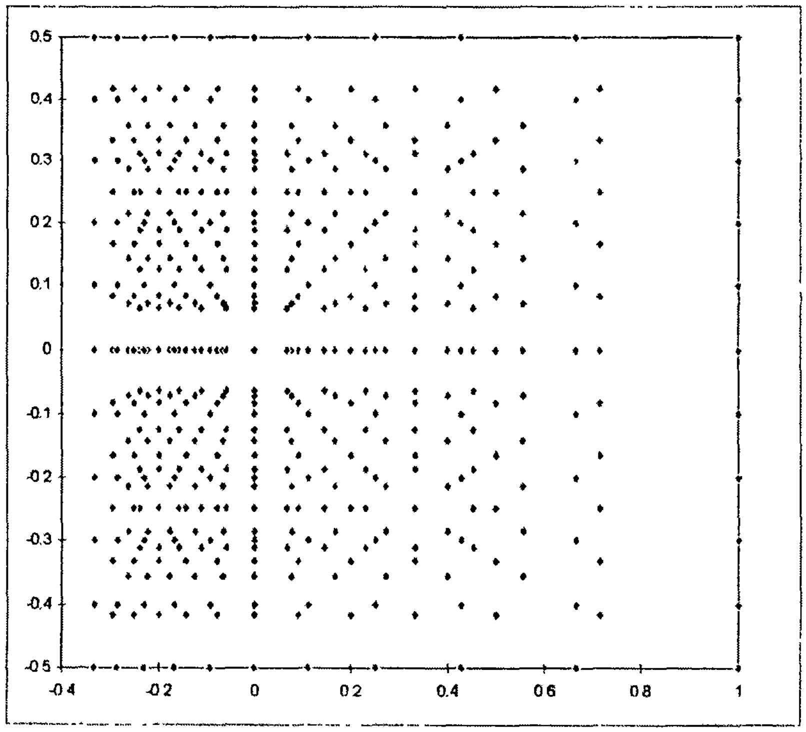

Figure 3: Smeared, smudged grid (P_t = 100, 118, 136, 154)

When stock prices span values between P and Q the points in the phase-portrait cover a distance of $\varepsilon(1/P - 1/Q)$ in the y-direction, and of $\delta(1/(P-\delta) - 1/(Q-\delta))$ in the x-direction. Hence, the larger ε and/or δ are, the more the points in the phase-portrait are pulled apart. As an example let us analyze the first two clusters on a major ray and on a minor ray, respectively. In the North-West direction the first two clusters belong to $\varepsilon/\delta = 1/1$ and to $\varepsilon/\delta = 2/2$. In a minor direction, say WWNW[3], the first two clusters belong to $\varepsilon/\delta = 1/4$ and to $\varepsilon/\delta = 2/8$. It follows from equation (7) that the clusters on each of the two rays connect when the stock prices span, say, the values from 100 to 200. However, in the phase-portrait the clusters on the WWNW ray are separated by a distance that is four times as large as the distance for the NW ray. Hence by increasing the stock prices from 100 to 200, the clusters of WWNW are pulled asunder to such an extent, that the human

[3] As CL pointed out, the rays do not correspond exactly to the compass rose. WWNW in the compass rose corresponds to an angle of $\frac{1}{4}(45°) = 11.25°$, while WWNW , as used in this paper , corresponds to $\tan^{-1}(\frac{1}{4}) = 14.03°$

eye no longer identifies them as lying on a ray.[4] The mentioned curvature would further obscure the rays. To make this point in another way, note that it is much more difficult for minor rays to get close to the origin in the phase-portrait than it is for major rays: in order for the NW ray to reach the point $x = 0.01$, a share price of 100 suffices, while a share price of 400 would be required for the WWNW ray to reach that point.

There is some asymmetry about the y-axis which follows from equation (2) when n_t is taken once as positive and once as negative. The deviation from symmetry for two members of a pair of rays can be computed as,

$$\Delta = \frac{(n_t h)}{(P - n_t h)} - \frac{(n_t h)}{(P + n_t h)} = \frac{2}{\left(\frac{P}{n_t h}\right)^2 - 1}. \tag{6}$$

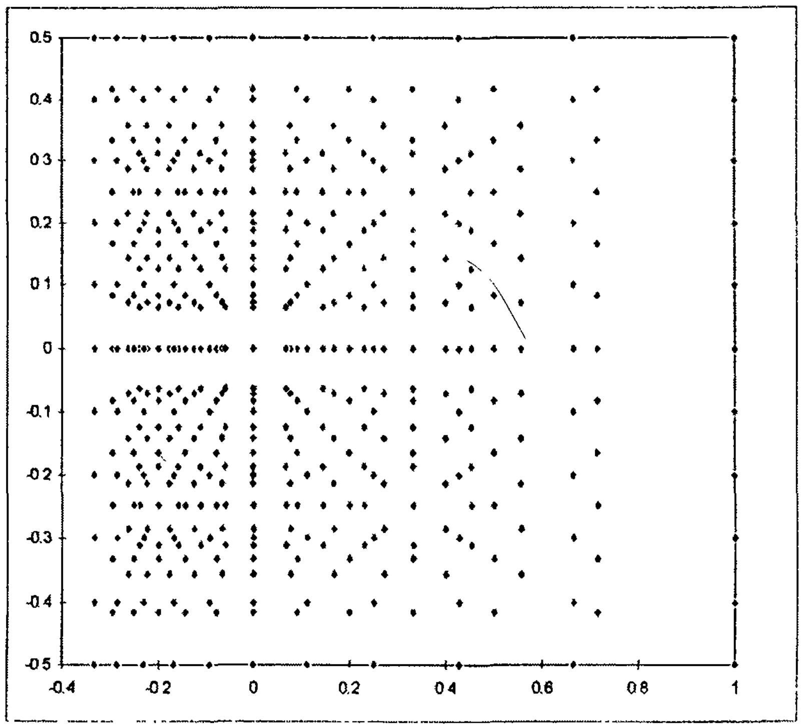

Figure 4: Coarse tick size (P_t = 10, 12, 14, 16)

[4] Furthermore, for large ε and/or δ and/or small variations in the price of the stock the connections often lie outside the range that is depicted in the figure.

To portray the discussed features in a salient manner, I now plot the situation with a coarse tick size (P_t = 10, 12, 14, 16). In Figure 4 the compass rose, curvature, asymmetry and the disappearance of the minor rays can be made out.[5]

3. EMPIRICAL EVIDENCE

In order to illustrate the phenomena that were predicted in the previous section, we use data from the Dollar-Deutschmark foreign exchange market. The data consist of a high-frequency time- series of bid and ask quotes, whose arithmetic means are taken as proxies for actual trades. Nearly 1.5 million observations were collected for the period between October 1st 1992 to September 31st 1993, and the series was made available to the academic community.[6] During the observation period the exchange rate varied from a low of 1.3950 DM/$ to a high of 1.7455, that is, the rate fluctuated by about 25 percent. This implies, by equation (4), that the fourth, fifth and higher clusters connect.

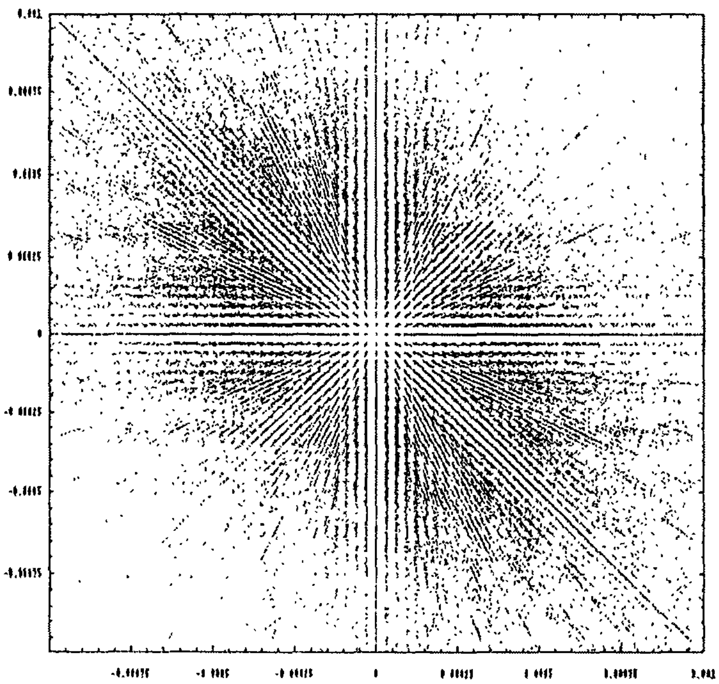

Figure 5: Compass Rose of the $/DM market

[5] Some new patterns also emerge, which are not germane to the discussion, however.
[6] Olsen & Associates, Institute for Applied Economic Research, Zürich.

In Figure 5 the values of R_{t+1} are plotted against R_t for a sample of 250,000 observations. A smeared and smudged grid-like system, resembling the compass rose is visible. When inspecting the blown up part of the figure (Figure 6), some of the features that were discussed in the previous section become evident. Especially the clusters are clearly visible. On the NW ray it can be seen that the fourth and the fifth cluster do, in fact, connect, while clusters of lower order remain separated.

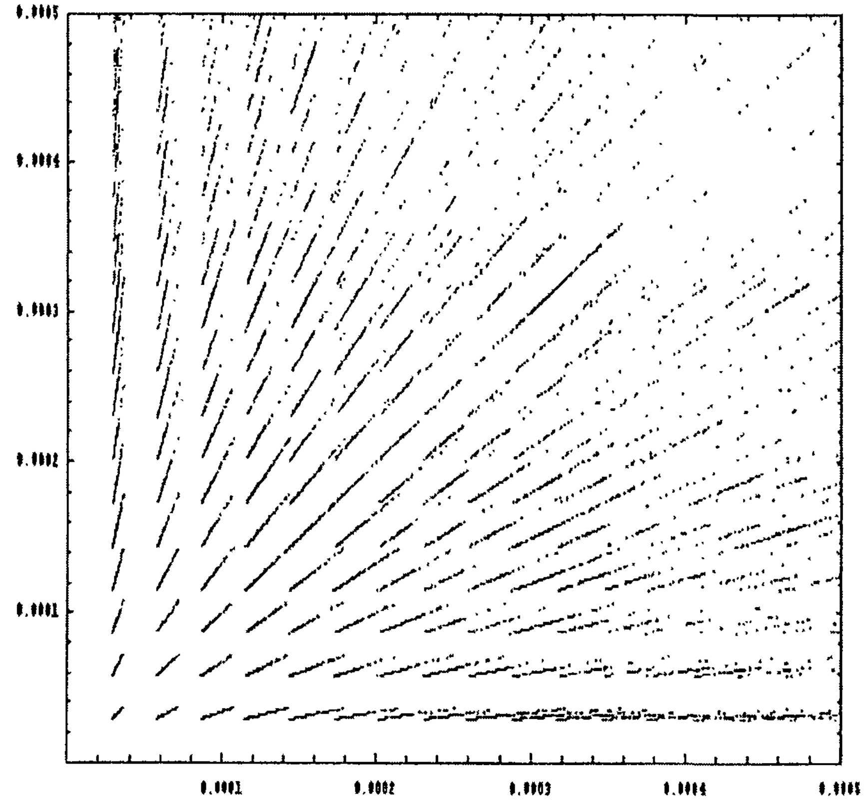

Figure 6: Detail of Figure 5

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