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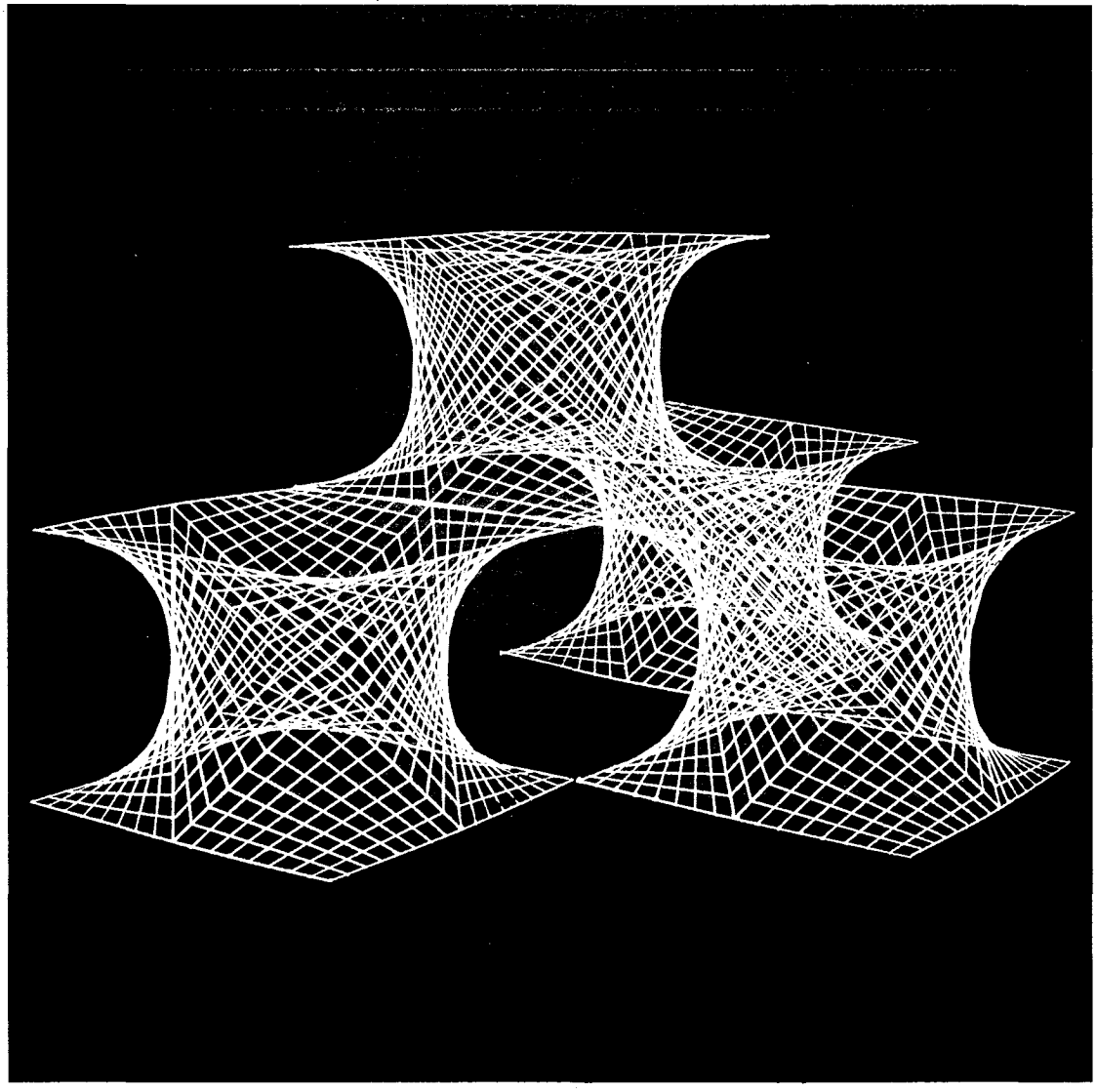

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ON WŁADYSŁAW STRZEMIŃSKI'S 13. UNISTIC COMPOSITION. PHILOSOPHICAL, PHYSICAL AND MATHEMATICAL ASPECTS

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INTRODUCTION

More than 30 years ago when I saw for the first time the Władysław Strzemiński's 13. unistic composition, its intellectualism struck me. It would not be easy for me to say why, for many years, I had not lost my inward union with the painting even though my professional interests were then rather far removed from art problems. The possibility of delivering a lecture on Strzemiński's work, in a seminar organized by Janusz Rębielak, under the auspicies of the ISIS-Symmetry, renewed my attention towards it with great pleasure. Therefore, the painting is the subject of our present contemplation.

Symmetry, which is so mightily manifested in Strzemiński's composition, is a rather specialized notion; it is understood in general, not only in a confined sense. Nevertheless, it is a commonly accepted view that the idea of symmetry permeates deeply and comprehensively the reality which surrounds us. Interdisciplinary studies meet the notion half-way.

If we adduce the two notions and concentrate our attention on Strzemiński's composition – that is, if we confine our interest to the art of painting which is named commonly as an abstract one – we detect a number of gateways some which lead us to other areas of creation and at the same time to language regions other than those which are used when we speak about artistic work. If we go through such a gateway and enter other scenery, it is not easy to stop on the path which inevitably directs our thinking to unexpected points of dispute.

The painting under consideration reveals two primary aspects, one deterministic, the other probabilistic. The first aspect should be named structural, while the second aspect is inseparably connected with the production of the work. The design of the work, in any art activity carried out in the mind, is, one may suppose, almost always structural, whilst the production of it embraces, however, unavoidably, random deviations from the contemplated structure. I disclose in what follows the mathematical, or, more precisely, the algebraic structure of Strzemiński's composition. The execution of the work is however a more comprehensive problem, involving as it does the probabilistic aspect of the painting. I will merely touch on the point to be solved by the work as it presents itself to our eyes.

Now, on the theoretical background I will elaborate my probability doctrine, which I submit for serious consideration. Because I had the painting in front of me in all its concrete material attributes, it would be inappropriate to refer to probability abstractly defined. So the definition I give in Chapter III will contain only such notions as have their referents directly in the world perceptible by our senses. This definition provided in one of the gateways which led me to fundamental physical problems in particular Born's probabilistic interpretation of the solution to Schrödinger's equation. The consequences of methodological failure in this interpretation are of course philosophical ones. I point out the other failure in modern physics which has philosophical consequences, namely the departure from classical differentiation of two language regions: in one of them the terms have their referents in the physical world, in the other, in the world of notions created in mathematics.

The other gateway which directed me to metaphysics is the problem of time. The point in dispute raises some inconsequences, similarly to the previous case, in the fundamentals of physics.

I would like to insist – in spite of the prevalent opinion – that metaphysics deals with *the reality*, i.e. something which exists, *exists* independently of whether or not can we grasp it, either with our senses, or intellect. This attitude of mind could not be unrecognized as

conformable with the common sense. We speak in this philosophy about *being as such*, i.e., about beings which can be grasped as a whole. And we assume that the reality is a whole of all beings, independently of whether we can grasp differences between them or not.

The production process of a work of art and its finished form belong to two worlds: to the one which surrounds us with concrete materialized matter and to the world which transcends matter. The aim of my paper is to throw some light both on the process, and on the finished work, in this case by examining is Strzemiński's 13. unistic composition. In Chapter I I renew one of the primary existential questions touching the world in which we are at the same time immersed and make ourselves its particular particles.

I. NINE PRIMARY QUESTIONS

The many-weftedness which I have outlined in the Introduction, does not relieve me from discussing some crucial notions with suitable accuracy. It occurs consequently as indispensable to allude to the fundamental problems investigated in contemporary physic, not only by these gateways which are commonly accepted, but by those not noticed, needful nonetheless I think, in order to better apprehend something that goes on in the microworld.

I formulate our questions in a general way.

Is the world
- 1° deterministic?
- 2° indeterministic? (probabilistic?)
- 3° a juncture of both these potentialities?

A positive answer to the first question would accept the view that all events are deterministic. It is unreasonable, I think, to be of the opinion that a positive answer to the second one would mean all events are random. It would rather imply that there exist random events. My deliberations, which I offer in this lecture, show, however, that the notion *random events* is, in a physical sense, vague. Not without good reasons, therefore, I have involved in the third question both notions mentioned in the two previous ones.

Let me repeat, however, that such a juncture is rather to point out the difficulties entangled in them than to penetrate their precise meaning.

I draw up in what follows the definition of the deterministic event; it will be useful, however, to precede the definition by some remarks. If we talk about *an instant* we appeal to the physically unrecognizable notion. Similarly, we can not avoid other physically unrecognizable notions too: *a point*, *a curve* (in special cases *a straight line*), a surface (in special cases *a plane*). Our considerations do not lose anything as to their generality if we use the term *plane-area*, being, if necessary, an approximation of *surface-area*.

When talking about an event, we have in mind however, such a distinguished and bounded space-area which "moves" in a well defined direction and inside of which something goes on. To be consistent we will use the term *process-event* occurring in a *time-interval*. Consequently, we have found ourselves in an unavoidable situation: when talking about physical process-events we are not able to omit four abstract (or, if you prefer, mathematical) notions mentioned above.

Their referents – an instant, a point, a curve, a surface – however, in the physical world are meant as: a small time-interval, a small spot, a pipe having a small cross-section, a flat and suitably thin part of space. When talking about *space* we have in mind at the same time three-dimensional space mathematically meant, and the space in which we are immersed; sometimes it will be more convenient to say, in the last case, *physical space*.

Let us consider:

1° a process–event, $\gamma(t, t')$, going on in a time-interval $[t, t']$, $t < t'$; by $\gamma(t)$ and $\gamma(t')$ let us denote the "ends" of it; they could be named plane-events; let us assume they are going on in a bounded space-area;

2° a process-area, $\Pi_\gamma(t, t')$ going on in the physical space: $\Pi_\gamma(t, t')$ "is swept" by the plane-event $\gamma(t, t')$;

3° the plane-area, $\Pi_\gamma(\tau)$, which is defined by $\Pi_\gamma(t, t')$ for any instant τ, $t \leq \tau \leq t'$; $\gamma(\tau)$ is the end of the process-event $\gamma(t, \tau)$.

Definition

A process-event, $\gamma(t, t')$, is said to be determined iff:

1° there exists in the physical space a limited plane-area $\Pi(\tau)$ which in a time-interval $[t, t']$ "sweeps" a process-area $\Pi(t, t')$ such that: (a) at every instant τ, $t \leq \tau < t'$, $\Pi_\gamma(\tau) \subset \Pi(\tau)$; (b) $\Pi(\tau) \div \Pi_\gamma(\tau)$ constitutesa surface-area which is homeomorphic with a ring;

2° there is not going on any other process-event in the process-area $\Pi(t, t')$;

3° when there occurs at the left end of the process-area $\Pi(t, t')$ a plane-event $\gamma(t)$, the process-event $\gamma(t, t')$ is the one and only.

The definition seems to be reasonable. If we agree that the notions *undetermined process-event* and *random process-event* are synonyms and then we would like to formulate the definition in such a way that the new condition instead of 3° would deny the existence of *the one and only* process-event, we would be, I think, in an unsolvable situation.

And what about *all* events? (The notion *all events* is equivalent to the notion *world* used in our triple fundamental question). The question puts us in limitless difficulties and at the same time in limitless possibilities. I therefore omit it and I go to further considerations.

As the next step in our deliberations let us begin with the following doubt: should we assert that intellect (I have in mind the intellect of the human being) and its function are components of the physical world or at least the function is something *beyond* it? I leave out epistemic questions yielded by this dichotomous query. A good deal of effort has been devoted by many thinkers to this problem.

I assume that intellect performs its functions when engaging its physical part and its other part being beyond the physical world. Consequently, I should reform the questions which have been formulated above.

The full version of our inquiring questions is therefore the following:

Is the reality { A = physical reality, i.e. the world which we observe and investigate by physical methods } { 1° deterministic? / 2° indeterministic? (probabilistic?) / 3° a juncture of both these potentialities? }

B = the rest

II. TWO REALITIES

Let us denote the whole of two realities, A and B, by R. The reality R contains all that exists. It is groundless to think that beyond the reality A there exists nothing. So, let us assume the reality B is not empty. As to the reality A let us assert that it is uncreative in the following sense. New beings arise in the reality A by transformation (one should say: it takes a time-interval) of other beings or by compounding (one should say too: it takes a time-interval) of same other ones being components of A previously.

The distinction of these two realities, A and B, is essential from the physical point of view. It seems that it was Einstein who first used, in the Special Theory of Relativity, the notion *observer*. In spite of the suggestive term, however, observer belongs to the reality B not to A. Let us say more exactly: the term *observer* is a synonym for the *observer's senses*. The observer is, therefore, identified with instruments recording signals which are dispatched or received. There is, however, involved the distinction in the Theory of Relativity, though unspoken, which we have made in our scheme of questions. It is easily to see that both Theories of Relativity, Special and General, could not be presented in any other way as only by a *narrator*; "he" looks on what is taking place in the reality A, i.e., what had been observed by instruments. The narrator's point of "theoretized observation" belongs to the reality B. Let us imagine, for example, that an observer directs his eyesight – using a telescope – to a star; the sun's position is at the same time such that the starbeam bends because of the influence of the sun's gravity field. The observer locates the star in a position which is imaginary. The notion *imaginary position* could be understood, however, only by the narrator and not by the observer.

The above necessity, resulting from the point of view of physics sciences, *forces us* – that is, I think, the right word in this place – to direct our attention to metaphysics. The remark leads to the following generalization. I do not see any reasons to uphold that the reality B contains the narrator's intellect only. The gateway opens our thinking, of course, to the metaphysics.

I consider in this paper a materialized object: Strzemiński's 13. unistic composition. It is a part of the reality A. One could not agree that the painting is not a fruit of the author's spiritual powers. Therefore, the scientific attitude of mind – what I have stressed above – and, as well, the attitude of an artist's genius when creating his work implement our conviction that the transcendental reality does exist. The ninefold question formulated at the end of Chapter I is, therefore, justified.

III. EXPECTATIVE PROBABILITY

The judge casts a coin upwords on a football field. All football players and all spectators know that the coin will fall down some seconds later on the field. They know too that the probabilities of the two expected self-evident possibilities are equal. It is for them quite needless to attach to any of the two expected occurrences the number 1/2. It is meaningless too if anybody ever anywhere, in a similar or unsimilar situation, had been casting a coin upwards and estimated chances of the two interested parties. Consequently, there is *no set of events* connected with the notion *expected event's probability*. In other words: to speak of a space of events is needless. Actually, at this very moment, we are awaiting for one and only one event which will occur at the end of a time–interval. As a principle, as in the above example, we are interested in some two versions, *a priori* marked out, of the expected event.

This is another example. Let us consider an arrangement similar to a continuous roulette wheel. The rod turns round freely on its axis. The rod's end moves over the ring having the same axis as the center. There are marked on the ring n unequal sections. One of them, e, is indicated by a distinguishing mark; the section e covers the p-th $(0 < p < 1)$ part of the ring. Let us give the rod an impetus, sufficiently strong, so that it turns round tens or more times. At the beginning of the experiment the expected event, E, is the stop of the rod. The event has in respect to our interest two versions: stopping over the section e, and the other, e', stopping beyond the section e. When the rod still turns round but sufficiently slowly we are able to attach to the expected version e the probability nearer and nearer to zero or to one. Before the experiment is being undertaken, or when the rod moves sufficiently quickly, we attach to the version e the probability equal to p and to the expected event E the probability equal to 1. I repeat the comment made on the occasion of the previous example: it is of no significance whether anybody, ever or anywhere, had performed such an experiment, or if the impetus to the rod more had been given than once.

Definition

Let us assume that we expect at an instant t, $t < 0$, the event E having to be realized at $t = 0$. To the expected event E let us attach the integer 1 called the expected probability of it. Let two versions, e and e', be distinguished in E, $e \cup e' = E$: if it realizes the version e, then it does not realize the version e' and vice versa; let us attach the number p,
$0 < p < 1$, to the version e; consequently, the number $1 - p$ to the version e'; they are called respective expected probabilities of the version e or of the version e'. (Compare [2]).

I make an obvious remark, but not without implications. The use of symbols in the definition does not break our essential goal: to formulate definition which terms have their referents in the physical world. Let me say this another way: semantic meaning of the terms used in the definition refer *a priori* to the world A and not to the area of mathematical language. If one talks, however, about *a probability of an event* and not about *a probability of an expected event* the referent of the term *event* belongs to the area of mathematical language and it denotes an element of an abstract set and nothing more.

The notion *respective expected probability* used at the end of the definition connects it with its applications. There are two sources of knowledge about expected events probabilities. We have in mind the knowledge to be possessed of by the intellect evaluating the probability at an instant t, which anticipates the expected event. The two sources are the following: (a) the understanding of the process' mechanism yielding to the expected event; (b) information in general necessary on the same – or rather sufficiently similar – events which had taken place in the past. The first one let us name *technological*, the second one is usually named *statistical*. To evaluate the probabilities mentioned in our two examples statistical methods are needless. In the case when it is indispensable an adequate statistical model is built; suitable parameters of it and parameters of suitable tests of them are not probabilities.

Let us say in the philosophical language: expected probability is a property of the relation rooted in the intellect and in the future (expected) event. The definition formulated above attaches to the relation – let us name it *uncertainty relation* – a measure. It seems that the expected probability is the only measure of this relation.

Is it possible to define probability – of a future event, obviously – if it is not expected by any intellect? In other words: is there a reasonable thing to say about an objective probability? Let us consider this questions.

Let $K(t)$, $t \in [\tau, 0)$, be a set of all phenomena which anticipate an event E including two possibilities, e and e'. Both $K(t)$ and E belong to the same bounded region. Let us consider in a *mental experiment* two situations. In one of them the version e of E had been realized. In the other one all phenomena $K(t)$ run exactly the same way as they were previously run but the version e' is realized. Realization of two such situations would affirm the existence of a random event in a physical sense in the physical world A. Our understanding of physical laws causes, however, that the above mental experiment grows to be internally contradictory. In every time-interval there *come to light* exactly the same physical laws relevant to the phenomenon, separated from all other ones, which is the same as in any other time-interval. Using the words *come to light* I would like to emphasize that physical laws have another ontic status than the observed phenomenon or the phenomenon which is not observed by any intellect. Therefore, random events do not exist in the reality A.

Another reasoning leads to the same conclusion. Let us assume that in a time-interval $[t_1, t_2]$ ($[t_1, t_2] \subset [t, 0]$) the run of the phenomena $K(t)$, considered in the above mental experiment, had been disturbed by an interference coming from the reality B. If the version e' would be realized at the instant $t = 0$, we would not have any reason to maintain the hypothesis announcing that the same running of the phenomena could result in two different versions. The disturbance caused, of course, that there occurred an another set of phenomena anticipating the event E, $K'(t)$ instead of $K(t)$. The reasoning belongs to the borderland of science and metaphysics languages.

Against the background of the above deliberations let us make two historical remarks. The well-known saying of Einstein: "God does not play dice with the universe" more exactly means: God does not play these dice which belong to the reality A. And at the same time He does not play till the end, i.e., till the end of time interval $[t, 0]$. The second remark is the following: if one assumes that in the reality A there do exist random events (whatever this means), then denying Einstein's saying one accepts at the same time the existence of the reality *non-A*.

Consequently, let us state positively: anything you like which is happening in the reality A has a determined character until there occurs the inference of a force which come from the reality B.

IV. ON THE INDETERMINACY PRINCIPLE. PRELIMINARY REMARKS

Most physicists are convinced that the view that the world has a probabilistic character is well-founded on the Heisenberg Indeterminacy Principle. Conclusions, however, supposedly arising from the Principle's formalization have been involved in such reasoning which is not deprived, at least, of inconsequences.

I develop an analogy; it will be quite perspicuous and shows, I think, an error – other ones will be mentioned in the further chapters – in the argumentation on Indeterminacy Principle. Let us consider a moving point, p, on a plane. We observe it, but we cannot come to an agreement as to its position at an instant, t, until we choose three points which do not move, and the positions of which are well-defined. In other words, until we fix a coordinate system, let us say, the rectangular Cartesian one, XOY. Let us place a theodolite in a suitable position with its object-glass directed perpendicularly to the OX-axis. We take, of course, the reading of the abscissa of p. When taking this reading we cannot say anything about its ordinate and anything about its velocity. A point is an integral notion; it is something by itself. Fixing of a coordinate system is something arbitrary. Giving the point two coordinates is something arbitrary too. The surveying of them has two limitations: the measurement instrument (it is such as it is, nothing else) and the theory (it is such as it is, not any other). It does not result therefore that, if we are not able to define both coordinates at the same time, and define one of them only, the other one does not exist. The transparency of the experiment and the simplicity of the theory assure us that the theory and practice are complete. It would be worth to add: not only because of the practice, e.g., location of rockets, we do not meet failure.

The simplicity of the experiment has other aspect too. Notions referring to the objects under consideration belong to such two language regions, separation of which would be unnecessary. The notions of one of them have their referents in the reality A, the notion of the other one in the mathematical terms-region. The mutual attaching of these referents – even when they have been expressed by the same words – is well-known.

Similar correspondence – in the case of the physical phenomenon called *electron* with its theory – is more complicated. Let us say without hesitation that because of unimaginability it is rather mysterious. The above developed analogy convinces us, however, I think, that Heisenberg's interpretation of the theory referring to a moving electron is not persuasive. In the further considerations I show that it is methodologically faulty. I concentrate the attention on some notions treated metaphysically: *existence*, *being*, *relation*, *movement* and *time*.

V. EXISTENCE, BEING AND SOME REMARKS ON METHAPHYSICS

Two notions, *existence* and *being*, belong to the fundamentals of every thinking. It is not the question if the being, which is the subject of a talk, or meditation, does exist, but how it exists. When we use a language to be considered as scientific it is rather needless to refer to these notions in an overt way. When a discussion touches, however, phenomena in which physics of the micro-world is interested, the notion *existence* is allowed to be spoken of.

In methaphysics understood as the Aristotelean *philosophia perennis* a being is called *something* or also a *thing*. If we say *this thing does exist* – and we use a known name for it – we have in mind something with which we are previously not unacquainted at least in some (maybe one) aspects. How does the thing we have named exist is not an ontic problem but an epistemic one. The statement that *the thing does not exist* is, therefore, senseless; it means nothing.

Every branch of science searches into some occurrences of a phenomenon distinguished from other ones and applies an appropriate methodology; reflection on observation is a part of the methodology, the other one is speculative reasoning. The results of scientific searches are *ex definitione* local. To make the last notion plain I characterize it by referring to mathematics. In differential geometry there are considered local properties of topological spaces provided locally with a metric form. In algebraic topology there are searched global properties of them. Methods used in one of them are quite different than in another one. Indirect inference from local properties of a metric topological space to its global ones would be a failure and even impossible. This analogy exemplifies in some respect the difference between science and methaphysics and at the same time validates metaphysics; the last one is such a field of human thought which comprehends the reality and its fragments as a whole. One of the methods of metaphysics is speculative reasoning aiming directly – this means: without intervention of measurement instruments and without building any mathematical model – to grasp the reality A and, consequently, the reality beyond A. The speculative thinking rests upon rigorous, coherent, i.e. intrinsically close, reasoning, upon a characteristic way of generalization yielding insight into the reality B and upon intuition.

VI. THE COPENHAGEN INTERPRETATION OF THE QUANTUM MECHANICS. LANQUAGE AND NOTION PROBLEMS

Friedrich von Weizsäcker [4, p. 10] comments briefly the essence of the Copenhagen interpretation of Heisenberg's discovery. He writes: "(...) die Messung zerstört den stabilen Zustand [des Elektrons; R.K.] und schafft einen neuen. Das sie [Ort und Impuls des Elektrons; R.K.] nicht zugleich existieren können, spiegelt sich darin, dass sie nicht zugleich messbar sind. Aus diesem Bruch mit der klassischen Ontologie folgt der Bruch mit dem klassischen Determinismus. Die Messergebnisse sind *prinzipiell* nur mit Wahrscheinlichkeit vorhersagbar".

The last sentence – though the word *prinzipiell* is rather vague in it – is correct; it does not result from the idea that the classical determinism should be rejected. It is not so. Weizsäcker says more categorically: "Die »Kopenhagener Deutung« der Quantentheorie, Bohrs und Heisenbergs gemeinsames Werk, gilt heute im Unterricht als orthodox und wird, wie alle grosse Orthodoxien, von der Mehrzahl ihrer Bekenner kaum verstanden [...]."

There are physicists who are not believers of the orthodoxy. R. Eisenberg and R. Resnick [1, p. 88] write:

"Among the critics of the Bohr–Heisenberg view of a fundamental indeterminacy in physics is Louis de Broglie. In a forward to a book by David Bohm [...] de Broglie writes: »[...] The construction of a purely probabilistic formulae that all theoreticians use today was completely justified. However, the majority of them, often under the influence of preconceived ideas derived from positivist doctrine, have thought that they could go further and assert that the uncertain and incomplete character of the knowledge that experiment at its present stage gives us about what really happens in microphysics is the result of a real indeterminacy of the physical states and of their evolution. Such an extrapolation does not appear in any way to be justified [...]«. (From *Causality and Chance in Modern Physics* by David Bohm, © 1957 D. Bohm; [...] D. Van Nostrand Co.)."

The inference, emphasized by von Weizsäcker, let us express in its full reading: from the fact that the position and the momentum are not measurable at the same time one should conclude that both aspects of the electron do not exist at the same time.

Apart from the opposite opinions mentioned above I would like to present a reasoning more convincing, I think, than those. It will occur that fundamentals of physics are deeply penetrated by metaphysical notions.

We should speak about the electron whose property is *position-momentum*, like the property of a coin is two-sideness: *obvers-revers*. The method of measurement is something arbitrary and is restricted by possibilities of our devices and, therefore, the result includes either *momentum* or *position*; similarly, we see either the *obvers* or the *revers* of a coin. A general remark is the following: any measurement made by man is ontologically somewhat different from what happens in the reality *A*.

The view that the structure of the reality *A* is indeterministic is believed to have its warrant in the conformability of experiment results with the probabilistic interpretation of the Schrödinger equations solution. Two methodological misunderstandings are implied in this feeling of certainty. One of them is known. The conformability mentioned above justifies the following inference only: experiment results do not negate the probabilistic interpretation of Schrödinger's theory. No more. The second misapprehension has its source in the language groundwork. I mentioned in the Introduction that, thanks to the genius of classical physics founders, the process in which man's intellect assigns the notions, used in the physics theories, to their referents in the reality *A*, grows to the unnoticeably routine. In modern physics such a clear distintion has been lost.

Let us consider Newton's equation

$$F = m\frac{d^2x}{dt^2} \tag{*}$$

and the Schrödinger one

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}. \tag{**}$$

Let us divide symbols which occur in these equations into five groups:

1° Constants: *m* and *h*. Their physical meaning is exactly determined – in other words: it is the determined assignment of these symbols to the suitable »phenomena« occurring in the reality *A* – and at the same time they represent numbers.

2° Constant: $i = \sqrt{-1}$. The occurring of i in Schrödinger's equation and its lack in Newton's equation is one of the reasons which make the difference between them not only a formal one. The equation (*) is »comprehensible« for the observer and all the more for the narrator: all factors of it could be measured. The equation (**) is on the other hand, for the sake of i, »incomprehensible« for the observer but comprehensible for the narrator. The constant i is a link, in this case, between the realities A and B. No wonder, therefore, that any solution of Schrödinger's equation could not be applied to the reality A indirectly. The repeated interference of the narrator is indispensable; for the first time it took place during the construction of the equation.

The plausibility arguments, presented in a particularly clear way, leading to Schrödinger's equation are to be found in R. Eisberg's and R. Resnick's *Quantum Physics* [1]. The narrator's arbitrary, though necessary, interference went together with the calling into being of the equation. In other words: it went together with the translation of language directed to the reality into the mathematical language. This procedure brought out the arbitrary inference in the suitable translation of the mathematical solution into the language »comprehensible« by the observer. This is the contraposition which throws a light on the calling into being not only Born's interpretation, but the philosophical and methodological judgement about the interpretation too.

3° Parameters: t and x. Their referents in the world A are: a small time-interval and a small spot, respectively. The size of them depends on the question which is under investigation and on the implements being at our disposal. It is quite an obvious matter how they are understood in mathematical language.

4° Functions: $F(x, t)$, $\frac{d^2x}{dt^2}$ and $V(x, t)$. Mathematical comprehension of them and referring them to the world A are classical problems.

5° Functions: $\Psi(x, t)$, $\frac{\partial\Psi(x,t)}{\partial t}$ and $\frac{\partial^2\Psi(x,t)}{\partial x^2}$. From the point of view of »the comprehensive faculty« of all factors occurring in Schrödinger's equation those mentioned in this item have the same ontic status as constant i; I do not repeat, therefore, the commentary made in that case.

Here is Born's interpretation of the wave function $\Psi(x, t)$ being the solution of Schrödinger's equation [1, p.147]:

If, at the instant t, *measurement is made to locate the particle associated with the wave function* $\Psi(x, t)$, *then the probability* $P(x, t)dx$ *that the particle will be found at a coordinate between* x *and* $x + dx$ *is equal to* $\Psi^*(x, t)\Psi(x, t)dx$.

The sentence, or to use a specific term *the postulate*, we could treat as such which reveals a physical law independent of any experiment: formulation of the postulate leaves out the momentum's measurement of the particle; accordingly, the observer's and the narrator's points of view come together. Let us concentrate our attention on some aspects of the sentence.

If we agree that the postulate refers to the physical reality, all notions involved in it should be referred to this reality too. Probability should be understood physically as well. After the definition of the expectative probability (Chapter III), i.e. of the one which refers to the physical world, there should be taken into account a time-interval, let us say $[\tau, t]$. The assumptions which precede the construction of Schrödinger's equation do not justify talking about any well-defined time-interval which would be involved in the going on of the phenomenon called electron or other particle. In the light of this aspect the future tense, used in the postulate, from the point of view of the particle's behaviour, i.e., from the point of view of an observer, is »incomprehensible«.

Let us assume that the particle leaves its initial spot (x_0, y_0, z_0) at the instant τ. Let E denote the expected event that it will reach the axis X at the instant t; let e be the version of E which denotes that the particle will be found inside an interval $[x, x + dx]$. Then and only then the following statement could be comprehensible: the expected "probability $P(x, t)dx$ that the particle will be found at a coordinate between x and $x + dx$ is equal to $\Psi^*(x, t)\Psi(x, t)dx$". It should be, however, asked: what does the trajectory of the particle depend on? Let us consider two aspects of this question. The impacts, which the particle is submitted to on its way, are responsible for the random run of it. Let us leave aside this aspect and concentrate our attention on the other one. There are no reasons to think that the particle is spherically shaped. The possible trajectory of it, therefore, depends on what position, in respect to the X-axis, it assumes at (x_0, y_0, z_0). A pencil, of course, of possible trajectories of the particle is determined, therefore, at (x_0, y_0, z_0) at the instant τ. Thus, there are determined parameters of the probability distribution assigned to the particle having a well-defined situation at the instant τ.

In the experiments which preceded Born's postulate and then in the ones which supposedly confirm it there had been observed many particles. We say, from the point of view of the expected probability *many events*. Let me make an obvious remark. There are observed in these experiments event frequencies in various intervals of X. The experiments and then the histograms justify the following interpretation of them. If we could mark at random (if it were possible) a particle which just begins its trajectory directed to X we could say: the probability that it will be found at the instant t in the interval $[x, x + dx]$ is equal to $\Psi^*(x, t)\Psi(x, t)dx$. The inference is, of course, worthless.

Experiments in which there would be used one and only one particle were impossible till now. Inference, however, which refers to the behaviour of a single particle, bases on, let us say, *a global experiment*. It is the methodological fault.

There should be brought out into relief other methodological fault implied by Born's statement. Time's duration, or time-interval, of an experiment – this interval determines frequencies – has other ontic status than the one in which a particle runs its trajectory. The first one is settled by the narrator and the other one is independent of him.

It is rather a common belief that the physical world is *a probabilistic* one. This philosophical inference resulting supposedly from Born's postulate has precarious premises. On the other hand, our speculative reasoning reveals that solids and particles run their trajectories – if they are not subjected to the impelling force having its source in the world B – in a determined way.

VII. POET'S INTUITION

As a particle and time are conceived by someone, who does not submit to the orthodoxy, so a grain of sand is conceived by Wisława Szymborska. The poetic language of her poem *View with a Grain of Sand* describes the world which surrounds us in a way which is not so precise as is made by physics but, I am ready to admit, more fundamentally from the point of view of our world's perception. I think, that not without reason, I quote this poem below [3, p. 247].

We call it a grain of sand,
but it calls itself neither grain nor sand.
It does just fine without a name,
whether general, particular,
permanent, passing,
incorrect, or apt.

Our glance, our touch mean nothing to it.
It doesn't feel itself seen and touched.
And that it fell on the windowsill
is only our experience, not its.
For it, is no different from falling on anything else
with no assurance that it has finished falling
or that it is falling still.

The window has a wonderful view of a lake,
but the view doesn't view itself.
It exists in this world
colorless, shapeless,
soundless, odorless, and painless.

The lake's floor exists floorlessly,
and its shore exists shorelessly.
Its water feels itself neither wet nor dry
and its waves to themselves are neither singular nor plural.
They splash deaf to their own noise
on pebbles neither large nor small.

And all this beneath a sky by nature skyless
in which the sun sets without setting at all
and hides without hiding behind an unminding cloud.
The wind ruffles it, its only reason being
that it blows.

A second passes.
A second second.
A third.
But they're three seconds only for us.

Time has passed like a courier with urgent news.
But that's just our simile.
The character is invented, his haste is make-believe,
his news inhuman.

VIII. TIME – PHILOSOPHICALLY COMPREHENDED

Time is "imagined" – when the word is used in physics – as a number axis and as such it does not belong to any reality: time is in physics an element of a suitable mathematical model. Man's intellect assigns – at the same time – to the notion *time* used in such a model, a position or position change of physical objects. (In Born's postulate such an assignment fails; it is a consequence of the methodological fault I have mentioned in Chapter VI). Thanks to the simplicity of this assignment the question *what is time* escapes the physicist's attention. I present in what follows a metaphysical comprehension of time. I am convinced that the conception I develop, is not without weight for physics fundamentals and for our understanding of the world.

Let us take the *motion's* notion as a starting point. Motion exists at least in the reality A; motion is, therefore, a being. We will distinguish motion as a being and motion's measure. It is rational to distinguish motion assigned to two or more beings. In the first case we say that the motion is a property of the being under consideration, in the second one that motion is a relation.

Now, let me sketch briefly the theory of relation. In order to be consequent in this display of my metaphysical outlook on what we name *reality*, I assume that every two or more beings are ontologically related: they are connected by a relation which is *a substantial being*. I say more expressively: the relation takes root, or is rooted, in the beings which the relation constitutes. It should be accentuated that every relation – i.e. beings involved in it and the relation as such – is subjected to a process or, we can say, to a metamorphosis. The most expressive example of a relation is marriage. Marriage is not two individuals only, but an institution as well, having its characteristic properties; they appear as something new, something which did not exist earlier. Maybe the most unexpressive relation is the distance between two beings. Let us emphasize that motion in which we are interested in should be treated, in our theory, as a relation taking root in two or more beings. Such an understanding of motion is compatible with the understanding of relative motion in the Special Theory of Relativity. Distance is a statical notion, while motion is a dynamic one. Every relation which connects organic beings, e.g. of some human beings, has some, maybe many, components. The relation of two beings which belong to the reality A, has at least one component: the distance. Every component of a relation is to be treated as a substantial being. I think the following hypothesis should be assumed: the only one common component, connecting no matter what beings under consideration, is motion. Motion exists independently of human intellect and, let us say, it does not recognize itself. That is just what Szymborska's

poem proclaims. Reality being left to itself is what it is and has neither past nor future. The two notions, *past* and *future*, are, of course, differentiable from the point of view of the intellect. We are interested in this paper in human intellect only. We have revealed, therefore, the essential source of *time's* notion: time meant in the human sense, in which we are interested, does exist as much as there exists the human intellect. We will enter into the idea of time if we define it as *the relation which takes root in the human intellect and in the motion*; just that is its ontic status. I say again: the measure of time is an attribute of it and nothing more. Here is the reason why we are inconsistent when we talk about time: the instant has no referent which would be something real in the reality *A*.

IX. STRUCTURAL SYMMETRY OF STRZEMIŃSKI'S 13. UNISTIC COMPOSITION

Let us fix our attention on Strzemiński's 13. unistic composition as it offers a view to our eyes of. Two aspects of it are in a remarkable way brought into prominence. I express the first one algebraically when creating an idealized schema and defining a transformation group of it. The second aspect, an analytical one, has undeniably a probabilistic character. One should call the first a deterministic one, not only to stay in the conventionality of symmetry, but in order to accentuate the creative inspiration of the Symmetry Principle.

We take into account the shaping of curves and pay no heed to the groundwork's colour. Let us devide the drawing into four parts (Fig. 1). The axes, *X* and *Y*, are perpendicular, parallel to the painting's borders and intersect in its symmetry centre. Let us draw the rays, having their initial points on *Y*, in such a way that they would approximate the wave lines. Let us denote them with numbers: $n = 1, 2, \ldots$ Let us acknowledge that the lower part of the drawing is a mirror reflexion of the higher one. The points marked on the *Y*-axis have – in suitable choosen units – the following coordinates:

$$y_n = 17\sqrt{n} + 13n + \frac{2}{3}[9 - (n-3)^2], \quad n = 1, 2, \ldots$$

whereas the points on the borders have the coordinates:

$$y_n = 7\sqrt{n} + 13n + \frac{2}{5}[34 - (n-4)^2], \quad n = 1, 2, \ldots$$

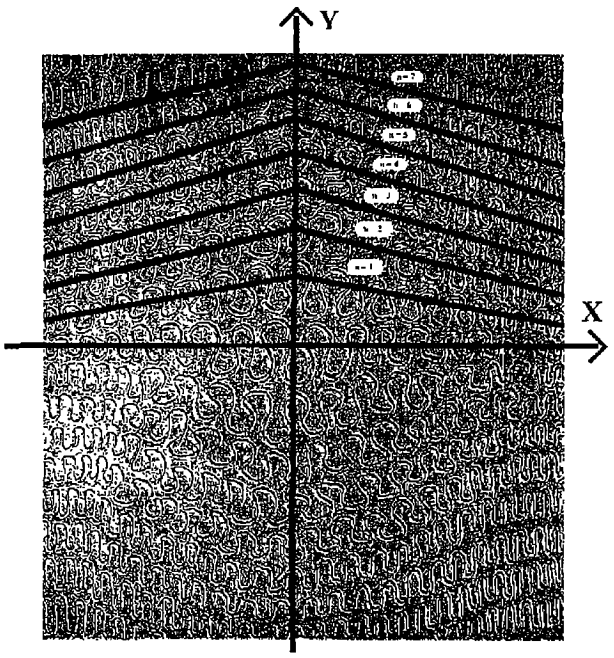

Figure 1

The first two components form the first approach of the approximation, the third one forms the first correction. The detection of further components would make our approximation more subtle. If someone does attach an importance to the number's magic, here, I think, he finds its confirmation.

I define now a transformation group and next we ponder on what part the group played in the creative act. Let there be given on the Euclidean plane three straight lines: horizontal axis X; axis OY vertical to OX intersecting the previous at the point O and directed upward; axis O^*Y^* parallel to OY intersecting the previous at the point O^* and directed upward (Fig. 2).

There is given on the positive half-axis OY: a well-ordered sequence of different points upward: $C_0 = O$, C_1, C_2, ...; well-ordered sequences of different points downward, $C_0^* = O$, C^*_{-1}, C^*_{-2},... The distances between neighbouring points are arbitrary. There is given on the positive half-axis O^*Y^*: a well-ordered sequence of different points upward, $C_0^* = O^*$, C_1^*, C_2^*, ...; a well-ordered sequence of different points downward, $C_0^* = O^*$, C_{-1}^*, C_{-2}^*, ... The distances fulfil the following conditions: $C^*_0C^*_1 < C_0C_1$, $C^*_1C^*_2 < C_1C_2$, ..., $C^*_0C^*_{-1} < C_0C_{-1}$, $C^*_{-1}C^*_{-2} < C_{-1}C_{-2}$, ...

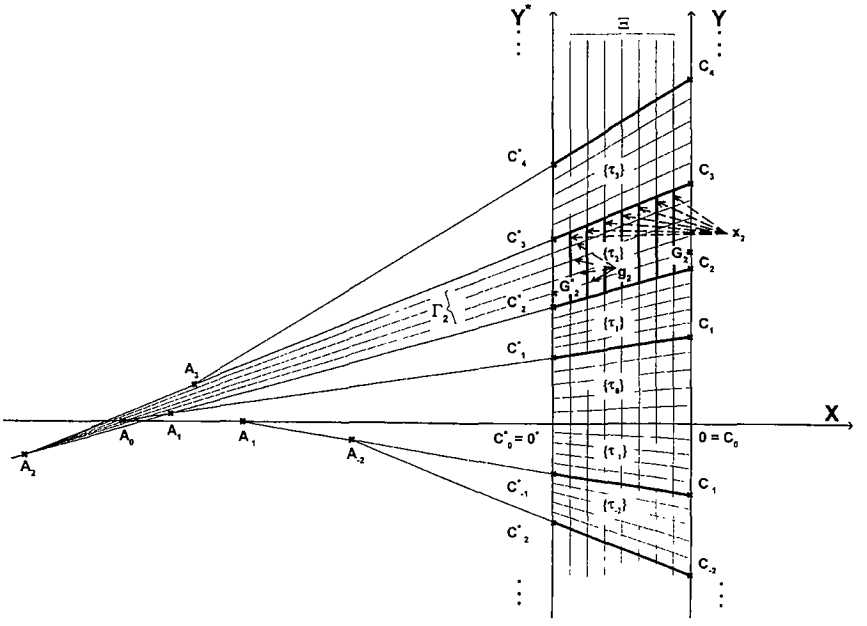

Figure 2

Let us denote in succession by A_0, A_1, A_2, ..., A_{-1}, A_{-2}, ..., the points: $X \cap C_1C^*_1 = A_0$, $C_1C^*_1 \cap C_2C^*_2 = A_1$, ..., $X \cap C_{-1}C^*_{-1} = A_{-1}$, $C_{-1}C^*_{-1} \cap C_{-2}C^*_{-2} = A_{-2}$, ... Let us denote the quadrangle $C_nC_{n+1}C^*_{n+1}C^*_n$, $n = 0, \pm1, \pm2, ...$, by τ_n. Let τ_n be covered by suitable segments of two pensils of straight lines: one of them, Ξ, is the family of straight lines parallel to OY which lie between OY and O^*Y^*; the other one, Γ_n is the pencil of straight lines having the common point A_n. Then, every τ_n is covered by a net; let us denote it by $\{\tau_n\}$. Let us denote suitable segments of Ξ by x_n and suitable segments of Γ_n by g_n. End points of g_n let us denote by G_n and G^*_n respectively (see Fig. 2; all these symbols are marked for $n = 2$).

Definition

The set of transformations

$$\sigma^k(\{\tau_n\}) = \{\tau_{n+k}\},\ k = 0, \pm1, \pm2, ...$$
$$n = 0, \pm1, \pm2, ...$$

such that for arbitrary n and k the net $\{\tau_n\}$ is transformed into the net $\{\tau_{n+k}\}$ is said to be *Strzemiński's translation* iff:

1° segments x_n and x_{n+k} are on the same straight line;

2° $|C_nG_n| / |G_nC_{n+1}| = |C_{n+k}G_{n+k}| / |G_{n+k}C_{n+k+1}|$, where the symbol $|\cdot|$ denotes the length of the suitable segment.

From the Tales theorem it follows that $|C^*_nG^*_n| / |G^*_nC^*_{n+1}| = |C^*_{n+k}G^*_{n+k}| / |G^*_{n+k}C^*_{n+k+1}|$; the segment g_n is transformed, therefore, into the segment g_{n+k}.

The set of Strzemiński's translations forms a group of transformations. Indeed:

1° σ^0 is the unit element of the set;

2° there exists for any element, σ^k, of the set the reciprocal element, σ^k, which belongs to the set;

3° the composition – defined as it is usually defined in the theory of symmetry – of two elements of the set forms a transformation which belongs to the set;

4° the composition of three elements, σ^k, σ^l and σ^m, of the set is associative:

$$\sigma^k(\sigma^l\sigma^m) = (\sigma^k\sigma^l)\sigma^m.$$

Similarly as any bounded pattern – in the space or in a plane – we talk about, that it realizes a group of symmetry, so we say that Strzemiński's 13. unistic composition "realizes" *in respect of its structure* the above defined group. Quotation marks used here stress that the realization touches the idea not the ultimate shape.

X. BETWEEN CERTAINTY AND UNCERTAINTY

The painting under consideration, apart from the group structure, has a structure which may be called the probabilistic one. Its intentional manifestation is, if I may say so, exceptional. I would like to throw some light – to some limited extent only – on the matter.

When accomplishing the intention to describe Strzemiński's 13. unistic composition from, so to say, the performance point of view, we stand face to face with a paradox. On one side one may not not perceive the certainty with which the painter drives the brush – as it is the master does it always – on the other side one may not avoid notions referring to random phenomena or, it is rather better to say, notions among which the uncertainty belongs to the essence of the matter. It is not, however, an apparent paradox. It is a piece of evidence of the deep difference which we disclose between the artist's certainty and

the uncertainty of the language which his works describe. One could look in various ways on the "ungeometriness" – let us use the naive word – of the curves which fill the painting. I have chosen the simplest one. More refined methods would give more interesting results.

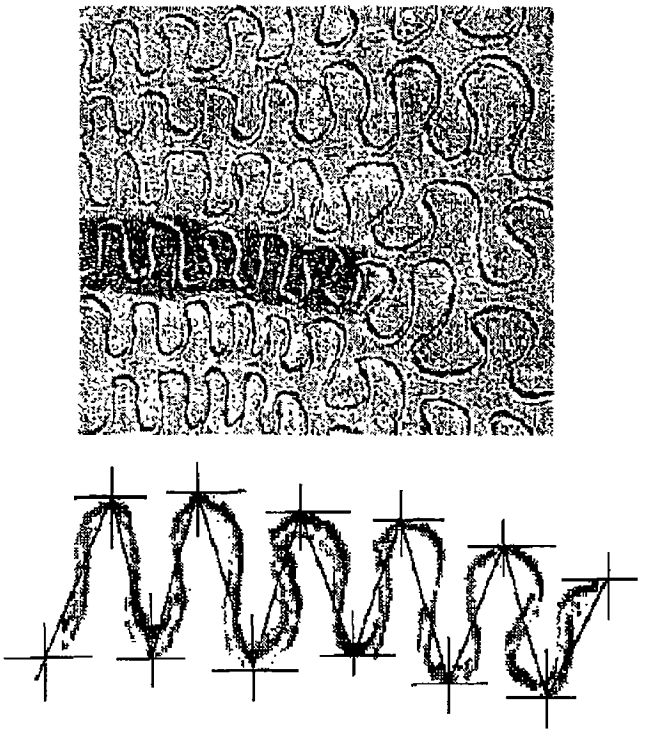

Figure 3

Let us assume that the amplitudes of all the curves of the painting have random sizes. In order to draw a suitable histogram (Fig. 4) amplitudes have been measured as is shown in Fig. 3. It is easy to put together the analytic outlook at Strzemiński's composition with the outlook at nature subjects when there are investigated some of their homogeneous subjects. Such a bringing of these two phenomena face to face, the artist's handwork and nature's "work", yields two questions which I present below. I will not attempt to answer the first one, while the other question not more than partly, i.e., on such a scale as it justifies what I have said in this paper.

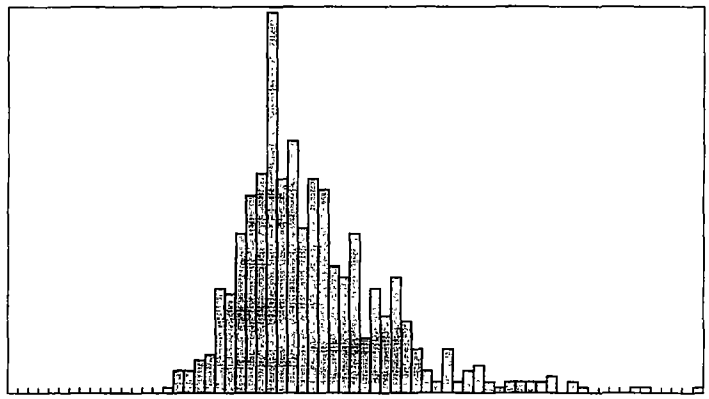

Figure 4

1° In what law strata, in both cases, i.e., in human and nature creativeness, there are hidden the same roots?

2° Has the paradox, we have talked about, its analogy in nature phenomena?

We ask in the second question, if the nature laws "work" at random. If the reality *B* does not exist in a real way, then – as it results from our previous considerations – we should assert that nature "works" with the entire certainty, without any hesitation and without any randomness. Our considerations, however, induce us to be more exact and we should complete the last statement as follows: nature "works locally" with... There arises then the next question: does nature "work" as a whole with unhesitating certainty?

EPILOGUE

Our considerations have been subordinate to the nine questions put in Chapter I. Their arrangement in the table given below seems to be natural and transparent:

A1°	A2°	A3°	B1°	B2°	B3°	AB1°	AB2°	AB3°

All these questions contain the fundamental and vexed points in dispute about the structure of the world which surrounds us, treated either locally or as a whole.

The attitude of my mind to them is the following:

+	–	–	?	?	?	–	–	+

Maybe the symmetry of this table is in some respect, or let me rather say, in a slight respect, a confirmation of the answers' correctness »yes« and »no«.

Our considerations have run parallel, either evidently or in their deeper stratum, along two currents. In one of them we reveal creative, persistently living and persistently reviving energy of the sources having their origins in the reality *B* and – we can state – being realized in the reality *A*. Along the second current we revealed the creative human genius. We contemplated it having in front of us Strzemiński's 13. unistic composition. It had taken its origin in the reality *B* and had been realized in a way approachable to our senses in the reality *A*. Crossing over the borderland – which connects the two realities, visible and unvisible – is the evidence of the human being's spiritual power.

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