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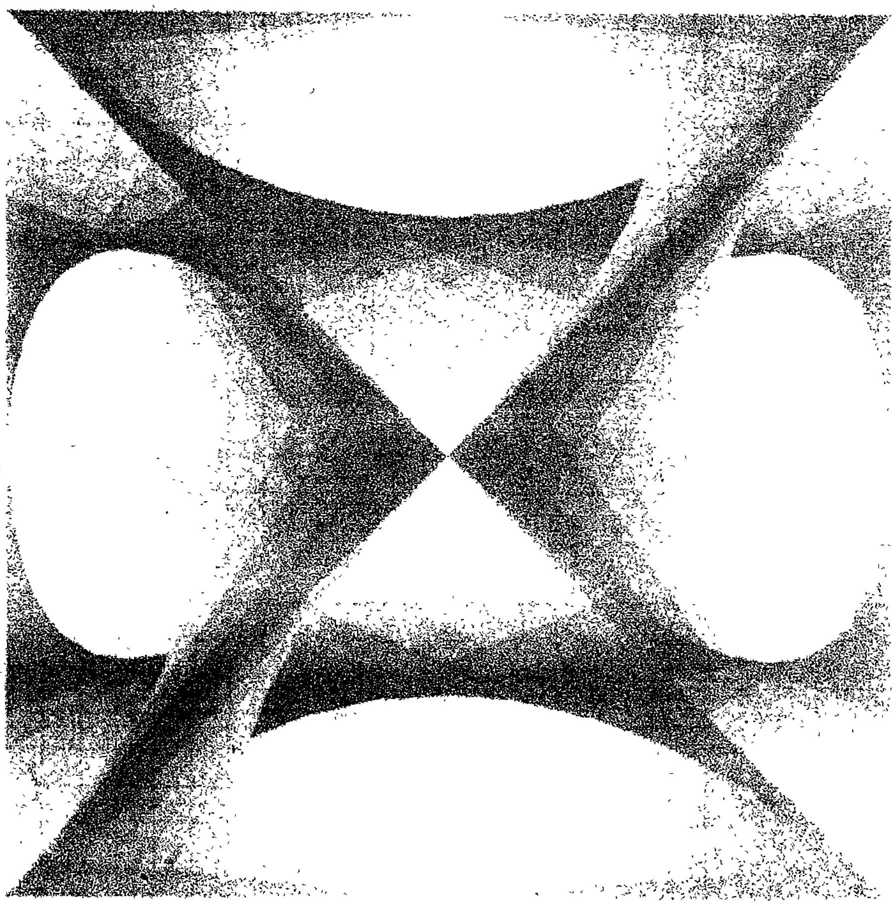

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MATHEMATICAL SYMMETRY: A MATHEMATICS COURSE OF THE IMAGINATION

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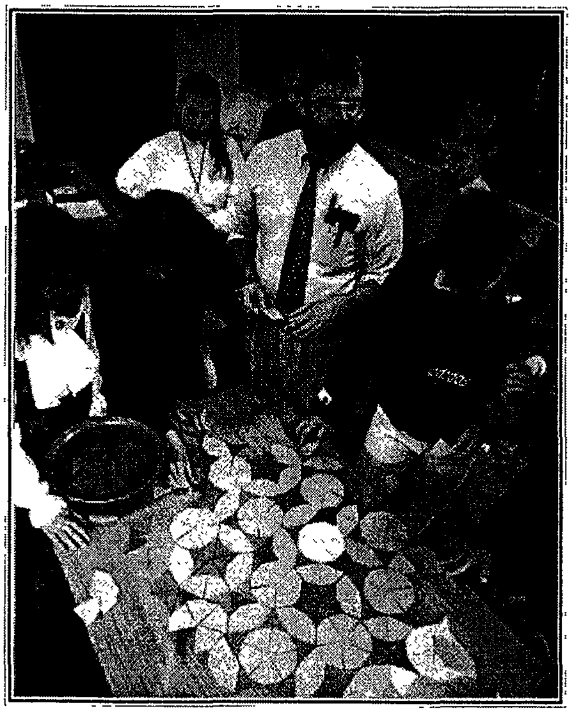
Fields of interest: Combinatorial group theory and topology, Euclidean and non-Euclidean geometry, Mathematical symmetry, Mathematical art, Creating mathematical pottery, Designing and teaching mathematics courses and art courses which involve connections between mathematics and art.

Publications: The swap conjecture, *The Rocky Mountain Journal of Mathematics*, 23, 3, Summer (1992), (with Edward C. Turner); Tessellations with hyperbolic squares, *The Journal of Recreational Mathematics*, 27, 2, (1995). Small study groups in the classroom, *The Canisius College Teaching Quarterly*, 1 1, September (1992), Constructing tessellations and creating hyperbolic art, *Symmetry: Culture and Science*, 3, 4, 367-383, (1992).



Members of the symmetry class at Canisius College, fall semester 1992, discussing a project involving Penrose's nonperiodic "kite and dart" tiles. (Left to right) M. Kearns, F. Bochynski, E. Caldero, J. Trost, Ray Tennant, D. Sherman, J. Krutz, and M. Henry.

Abstract: *Symmetry forms a natural bridge between the worlds of mathematics and art. It is this connection combined with the creative imagination of students that forms the basis for the mathematical symmetry course that is described below.*

THE COURSE - MATHEMATICAL SYMMETRY: A MATHEMATICS COURSE OF THE IMAGINATION

In his classic, *Symmetry*, Hermann Weyl describes symmetry as the "harmony of proportions". It is this notion that is the common thread running through the university mathematics course titled *Mathematical Symmetry: Connections between Mathematics and Art.*

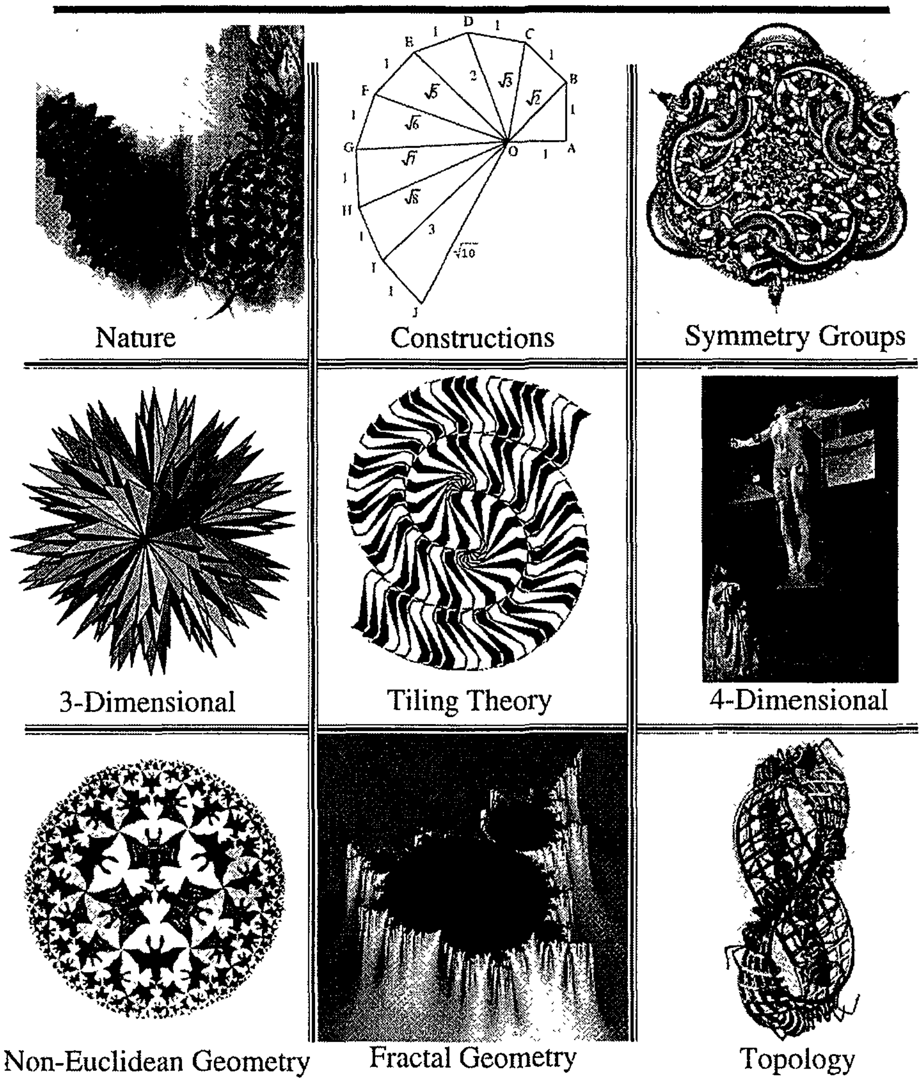
A main perspective of the course holds that students have strong visual senses and that they may be introduced more easily to complex ideas by appealing to their geometric intuition. The students explore about ten major topics, including tiling theory, geometry in nature, group theory, non-Euclidean geometry, 3-dimensional tessellations, and fractals This is done with the aid of straightedge and compass constructions, computer programs, polyhedral models, and examples drawn from the worlds of art and architecture. Each student creates a planar design project and writes a thesis paper involving the imagination. Earlier versions of the symmetry course have been popular with fine arts majors and prospective math teachers as well as students from other disciplines such as psychology, music, history, and engineering. In fact, the classes have been most enjoyable when the students who are participating are drawn from a variety of disciplines.

Goal of the course

The symmetry course covers a number of interesting topics from the world of mathematical symmetry. The class is encouraged to explore geometrical ideas using constructions, computer programs, models, and a variety of other investigative techniques. The spirit of the course is based on the Chinese proverb:

I hear and it helps a little.
I see and ideas begin to form.
I do and ideas become real to me.
I do, see, and hear and I understand.
I talk about it and I understand more.
I apply it and I see it's value.

Picture Syllabus - Mathematical Symmetry



Flow of topics

I. Geometry in Nature & Architecture
- The Golden Ratio & Fibonacci Numbers
- Phyllotaxis

II. Geometric Constructions
- Straightedge and Compass Constructions
- Pythagorean Constructions
- Polygon & Star Polygon Constructions
- Geoboard Constructions
- Impossible Constructions

III. Symmetry Groups
- Isometries & Groups of Motions
- Symmetry Group of a Pattern
- Isomorphism of Symmetry Groups
- Cyclic & Dihedral Groups, Group Tables
- Design in Hubcaps & Logos
- Leonardo's Theorem, Frieze Groups

IV. 3-Dimensional Symmetry
- Platonic Solids & Semiregular Polyhedra
- Euler's Formula & Schleigel Diagrams
- Symmetry Motions in Space

V. Tiling Theory
- Regular & Semiregular Tessellations
- Transforming Tilings into "Escher Designs"
- Crystallographic Restriction
- Wallpaper Groups
- Nonperiodic & Aperiodic Tilings

VI. 4-Dimensional Symmetry
- Flatland
- Salvador Dali's "Corpus Hypercubus"
- Hypercubes & N-Dimensional Symmetry

VII. Non-Euclidean Geometry
- The Sphere & the Poincare Disk
- Escher's "Circle Limit IV"
- Hyperbolic Tessellations

VIII. Fractal Geometry
- Von Koch's Snowflake
- Computer Generated Fractals

IX. Elementary Topology & Graph Theory Topics
- Surfaces & Nonoriented Surfaces
- Konigsberg Bridge Problem, 4 Color Problem

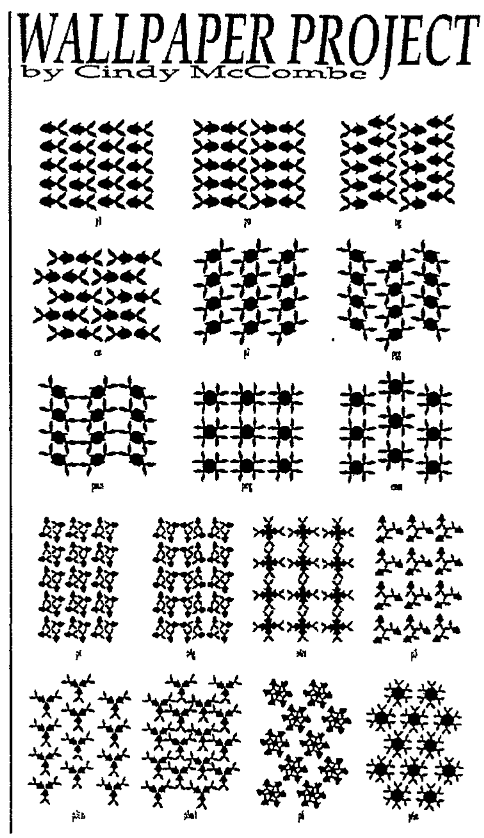
X. Geometric Perspective in Art
- Albrecht Dürer's "St. Jerome in His Cell"

The planar design project

For many students in the symmetry course, the 2-dimensional design project has proved to be an exciting and challenging endeavor. Students are free to choose a project from the various topics in the course, e.g. tiling theory, wallpaper groups, color groups, fractals, hyperbolic tessellations, etc. As it develops, each student discusses their project with the class.

Past design projects have included:

* Islamic Tiling Constructions
* Interlocking Ceramic Wallhangings
* A Study of Symmetric Music
* Creating Fractals with Mathematica
* Patterns for the 17 Wallpaper Groups, Using NFL Team Logos
* Color Groups in the Plane
* Straightedge & Compass Constructions for High School Teachers
* Poincaré Disk Constructions
* Symmetry Groups of Molecules
* Needlepoint Crystallographic Groups
* Buddhist Mandala Designs
* Symmetry in Flag Design



The design on the right (less than half its original size) was created by Cindy McCombe in a symmetry class at the University of Southern Colorado in 1994. It shows fish patterns for the 17 wallpaper groups and was created using Adobe Illustrator.

The written project of the imagination

Required reading

Flatland: A Romance of Many Dimensions by Edwin A. Abbott, 1884.

Other possible reading (discuss with instructor)

Sphereland, a Fantasy about Curved Spaces and an Expanding Universe by Dionys Burger and Cornelie Reinboldt, 1965.

Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions by Thomas Banchoff, 1990.

Hyperspace: A Scientific Odyssey through Parallel Univrerses, Time Warps, and the 10th Dimension by Michio Kaku, 1994.

The Mathematical Tourist: Snapshots of Modern Mathematics by Ivars Petersen, 1990.

Goal of the Assignment

To use writing and the imagination as a way of gaining insight into an abstract land in the world of mathematics (e.g., 4-dimensionland, or even *n*-dimensionland, the land of fractals, non-Euclidean worlds, topology landscapes, and others.

The research for the paper may involve reading, discussion, the study of a painting, a computer investigation or some other method of discovery. The student will then use imagination to describe an aspect of their abstract world or to create a fictitious story which extends their new world in some way.

Assignment Steps

1. All students read Flatland and are given written assignments to describe aspects of Flatland and then to use their imagination to extend Flatland to areas which are not covered in the book.

2. Each student chooses an abstract world that they wish to investigate.

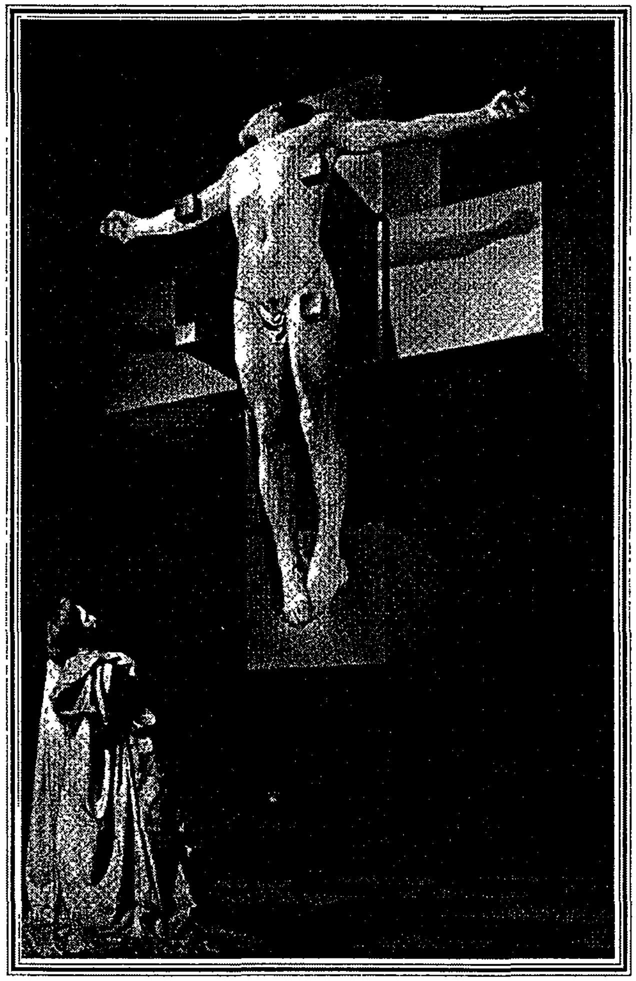
3. Each student finds a resource (possibly a book, a painting, a computer program, etc.) as the motivating thesis for their paper.

4. Through discussions and rewrites, each student writes a thesis paper about their abstract land. The paper can range anywhere from being scientific to being pure fantasy.

5. The writing projects evolve over the semester and students discuss their progress during class.

Some writing projects of the imagination from past symmetry classes are listed below.

* A Flatland Person Discovers "Up" or "Upward not Northward".
* Police and the Law in Flatland.
* Salvadore Dali's "The Crucifixion", subtitled "Corpus Hypercubus" and Hyperspace.
* Salvadore Dali's "Clocks", and Mappings in Topology.
* Can 4D-man turn 3-D man inside out?
* The Poincare Pool and Hyperbolic Fish.
* Picasso, Cubism, and the 4th Dimension.
* Fractal Dimension: Between Lines and Planes.
* Magic Squares and Polyhedra in Albrecht Dürer's "Melancholia"



Salvadore Dali's *Corpus Hypercubus* 1954