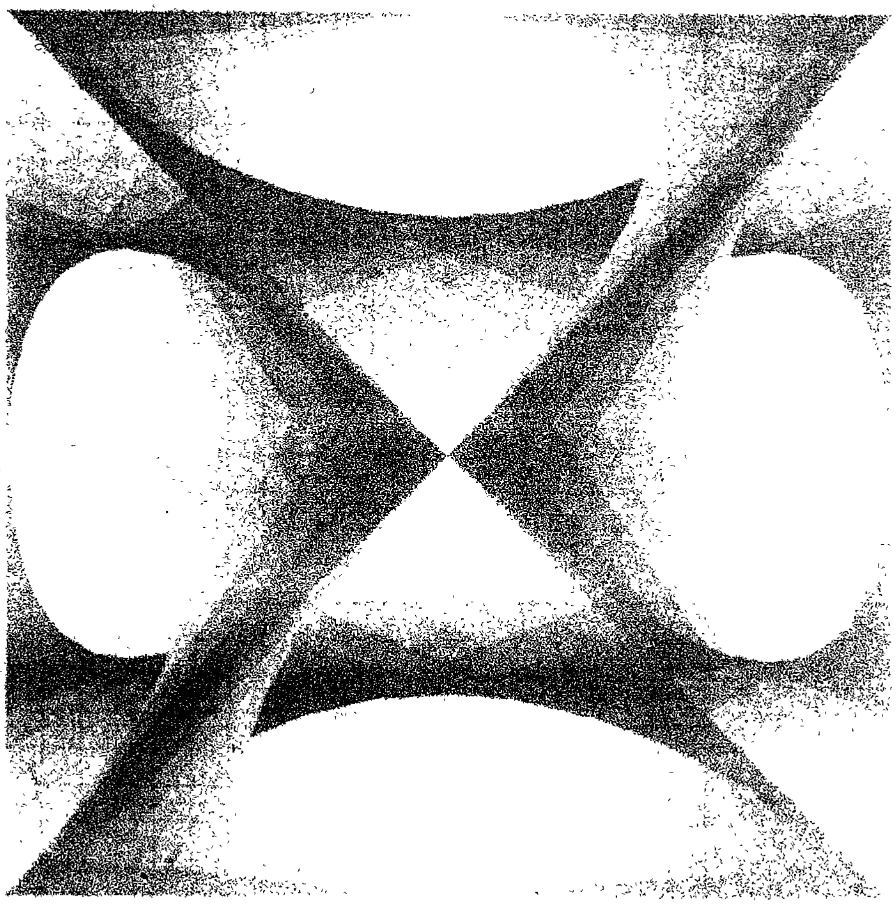
Symmetry: Culture and Science

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Volume 8, Number 3–4, 1997



Symmetry: Culture and Science
Vol. 8, Nos. 3-4, 369-393, 1997

SYMMETRY IN EDUCATION

ESCHER'S WORLD: LEARNING SYMMETRY THROUGH MATHEMATICS AND ART

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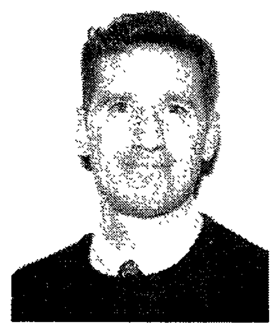


Fields of Interest: Mathematics, Mathematics Education, Computational Media, Design and Design Education.

Publications: Symmetric Intuitions: Dynamic Geometry/Dynamic Art, *Symmetry: Culture and Science*, 6 (1995), 3, 476-479; *Exploring Trigonometry with the Geometer's Sketchpad,* Key Curriculum Press, (1995) 100 pp, Thinking About Two Algebras, Invited paper for National Science Foundation *Algebra for the 21st Century* Conference (1992), Groton, MA, 10 pp.

Abstract: *In recent years, mathematics educators have begun to call for reforms in mathematics learning that emphasize open environments rather than the traditional pedagogy of exposition and drill. This paper explores one example of an open learning environment created by combining mathematics and design activities in a "mathematics studio." The paper looks at: (a) whether students can learn specific mathematics topics in a studio environment, (b) whether learning in such an environment will change the way students solve mathematical problems, and (c) whether learning in such an environment will change students' attitude towards mathematics. Two iterations of the mathematics studio experiment in a project at the MIT Media Laboratory known as Escher's World suggest that: (a) students can learn about the mathematical concept of symmetry in a studio learning environment, (b) students learn to use visual thinking to solve mathematical problems in a studio learning environment, and (c) students develop*

a more positive attitude towards mathematics as a result of working in a studio learning environment.

1 INTRODUCTION

The publication of the National Council of Teachers of Mathematics Standards in 1989 marked the emergence of a coherent reform movement in curriculm (NCTM 1989). Instead of a pedagogy based on exposition by the teacher, reformers propose moving from drill and practice to more open learning environments. They call for the introducion of extended projects, group work, and discussions among students. In many ways, the learning environment these reformers describe seems more similar to a studio course in design or architecture, where students work on extended projects exploring creative solutions to general problems, than to a traditional mathematics class. This paper describes one such mathematics studio, and evaluates the success of this approach in learning some mathematics concepts.

Specifically this paper addresses three aspects of mathematics learning:

- Content knowledge: Can students learn to understand a specific mathematics topic in a design studio?
- Skill acquisition: Will learning in a design studio affect the way students solve mathematics problems?
- Change in attitude: Will students feel differently about mathematics after learning mathematics in a digital studio?

The results suggest that indeed a studio learning environment can be used effectively and profitably in mathematics education.

2 SETTING

This paper reports results from the Escher's World research project at the Massachusetts Institute of Technology Media Laboratory. During the spring and summer of 1995, the project brought high-school students in grades 9 and 10 (age 14-16) from public schools in Boston, Massachusetts to the Media Laboratory. Students attended brief but intensive workshops where they created posters and worked on other design projects using mathematical ideas of mirror and rotational symmetry.

3 METHODS

3.1 Workshop Activities

The Escher's World project ran two workshops for students. In each workshop, six or seven students from Boston public schools came to the Media Lab for twelve hours of workshop activities spread over two or three days. The workshops were divided into two sections, one about mirror symmetry, the other about rotational symmetry. Each section had two sets of activities: investigations and explorations.

3.1.1 Investigations

Investigations lasted approximately one hour. Students worked on a series of short problems relating to symmetry on their own or in small groups, wrote entries in their workshop journals, and discussed their observations with a workshop leader as facilitator. In the first day of the workshop, for example, students began their investigation of mirror symmetry by making name-tags that read normally when viewed in a mirror. This was followed by a search for words that look the same when viewed in a mirror, and from there to the classification of the letters of the alphabet by their mirror lines.

3.1.2 Explorations

Based on their investigations, students spent three to four hours working on extended projects in design on their own or with a partner. Students worked on one shorter project (approximately one hour), and then presented their work to the group for discussion, questions, and comments. Following this "peer review," students began a more ambitious project (approximately two hours), integrating ideas about symmetry, principles of design, and feedback from their presentation. In the first day of the workshop students followed their classification of the alphabet by creating a design of their own choosing that had mirror symmetry. After discussing their designs, students worked for the remainder of the day creating designs that had mirror symmetry but did not place the focus of the composition in the middle of the design (see Figure 1).

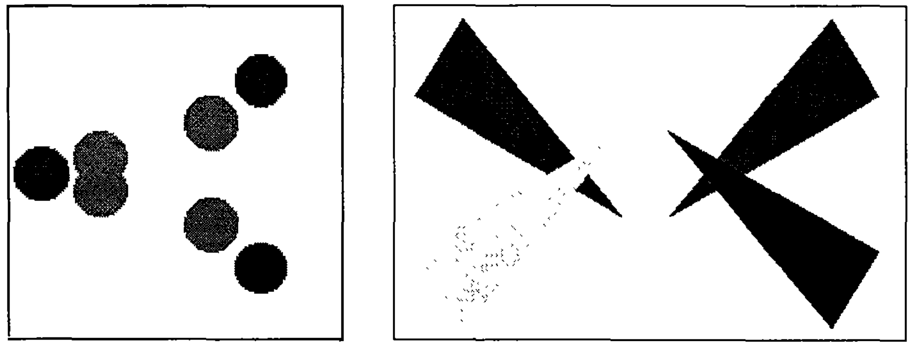


Figure 1: Student work from Escher's World: learning about composition and symmetry

3.2 Workshop Resources

3.2.1 Workshop Space

The workshops took place in a single conference room approximately 15' by 25' that had been modified to resemble an art studio. Works of art by students and professional artists were placed on the walls, and a variety of artistic media available for students' use. In addition to the author, who acted as workshop leader for both workshops, there were one or two other adults in the studio as a resource for students.

3.2.2 Equipment Used

Macintosh computers were available for student use during the workshops, with one computer available for every two or three students. Computers were connected by a network to flatbed scanners, color printers, and a large format color plotter. Computers were equipped with Aldus Superpaint and Adobe Photoshop (commercially-available drawing and image-manipulation programs; Aldus Corp. 1993, Knoll et al. 1993) and with the Geometer's Sketchpad (commercially available educational software for mathematics; Jackiw 1995). During the investigation portion of the workshops, students were introduced to some of the basic functionality of these programs (particularly the Geometer's Sketchpad). Students were able to work on the computers or with traditional materials during their explorations; all of the students chose to use a computer for some portion of their work.

3.3 Data Collection

3.3.1 *Kinds of Data Collected*

Escher's World uses a qualitative model of research (Geertz 1973, Glesne and Peshkin 1992, Maxwell 1992, Weiss 1994). Qualitative research attempts to understand phenomena by gathering a rich set of data for a limited number of instances. The main source of data for the Escher's World workshops was full, structured pre- and post-interviews conducted with each of the workshop participants immediately before and after the workshop, as well as an additional "affect interview" with each student from two to five months after the completion of the workshop. The format of these interviews and their subsequent analysis was guided by 14 preliminary interviews conducted with students, mathematics and art teachers, and experts in the field of symmetry. Interviews were supported by videotapes of the workshops and field notes from workshop leaders and other facilitators. Student sketches and designs from the workshops and student journals were preserved for review and analysis. Students in the second workshop were also given a brief survey about their feelings towards mathematics, art, and computers immediately before and after the workshop.

3.3.2 *Structure of Interviews*

The structured pre- and post-interviews were divided into three components. The first component was a series of affect questions about mathematics and art, focusing particularly on attitudes towards these disciplines. The second section of each interview was a detailed discussion of four works of art from a set of 7 images (see Appendix for images). The works of art were reproduced in standard size and format, and students were given three prompts: (1) How would you describe this to someone who had not seen it? (2) Would it be difficult to make something like this? (3) Do you like this piece? The final section of the interviews consisted of two to four mathematics problems from a set of 16 problems (see Appendix for problems). Students were asked to solve the problems, and to describe their thought process as they worked. Affect interviews (conducted two to five months after the workshops) were similar in structure to the first section of pre-and post-workshop interviews. In affect interviews, students described their attitudes towards mathematics and art.

3.3.3 *Structure of Survey*

The surveys given to students before and after the second workshop asked students to rate how strongly they agreed or disagreed with a series of statements about mathematics, art, and computers (see Appendix for survey questions). Ratings ranged from 5 (agree strongly) to 1 (disagree strongly).

3.4 Data Analysis

3.4.1 Coding of Interviews

Each section of the interviews (affect questions, image descriptions, and word problems) was coded seperately. In order to provide consistency across the interviews, excerpts were mixed randomly before coding, and coding within each secton of the interview questions (affect questions, image descriptions, and word problems) was done by the same person and checked for accuracy.

3.4.1.1 Codes for Image Descriptions

Design texts divide the basics of design education into "elements and principles" (Johnson 1995), where elements generally refer to particular physical portions of the design image and principles refer to formal or systematic relations between elements. Excerpts about images were assigned codes for elements of "form" and "color" and principles of "symmetry" and "composition". These categories were based on preliminary interviews with students, symmetry experts, and mathematics and art teachers, where novices used elements of form and color in their descriptions of images and experts used principles of symmetry and composition.

People tend to go through stages in their development of visual and aesthetic understanding across a range of topics (Parsons 1987). That is, there are stages in their understanding of color, or in their interpretation of forms or composition. Codes for the topics identified in preliminary interviews were subdivided into two stages for the purposes of this analysis: "general" comments and "analytical" comments.

For elements (form and color), general comments referred to excerpts that contained catalogs of shapes or colors ("a lot of circles and lines"). Analytical comments about elements were those that referred to specific relationships between elements of the picture ("the green is too dark for the bright blues and purples"), that distinguished between similar elements based on a specific criteria ("the sun is blue, and then [there is] another sun, smaller—it's red"), or that combined elements into larger descriptive units ("diamond shapes all together combined into like a star").

Similarly, general comments about principles referred to excerpts that contained informal descriptions of formal concepts such as symmetry and composition. For instance, one student's comment that in an Escher print "the heads are all together everywhere you look... so it's like they're standing right beside each other in different [places] " was coded as "symmetry, general. " Analytical comments about principles were those that used formal or mathematical descriptions of symmetry ("it's four time radial symmetric") or composition ("it doesn't really have a focus—or it has multiple focuses").

Thus, the coding matrix for images had 8 cells:

	Elements		Principals	
	Form	Color	Symmetry	Composition
General				
Analytical				

Many excerpts were coded in more than one category. The following description, for example, was coded for "form, analytical," "color, general," and "symmetry, analytical,":

> I noticed these little blue lines coming out of these little red designs, and I realized that it was angular symmetry. It looks like... whoever made this could have just started out with a block that had on two sides the red and blue base, and then on the rest of it just made the yellow, and these blue dots and red dots and the little line right there, and then made part of the circle, or one fourth of those circles—and then... made four versions of it with different angles and... moved them together.

3.4.1.2 Codes for Word Problems

Word problems were coded for students' use of a visual representation during some portion of the problem-solving process (usually some form of sketch of the problem). Following Rieber, the term visual representation was used broadly to refer to "representations of information consisting of spatial, non-arbitrary (i.e., 'picture-like' qualities resembling actual objects or events), and continuous... characteristics," including both internal and external representations (Rieber 1995). Problems were also coded for correct or incorrect answers to the problem, where "correct" answers included answers that fit the stated conditions of the problem even if a student's solution was not the "expected" answer.

3.4.2 Statistical Analysis

When coding was complete, frequencies were tallied for each code. Pre- and post-interview totals were compared overall, between workshops, and for individual students. Student responses for affect interviews were included in the analysis of students' attitudes. The statistical significance of observed changes was computed using a t-test with $n = 12$. Results for the survey from the second workshop (with $n = 6$) were similarly tabulated and analyzed.

4 RESULTS

4.1 Criteria for Understanding

Mathematics education reformers argue that change in mathematics education needs to affect not only the setting in which learning takes places, but also the goals toward which learning is directed. Traditional approaches to education do not place first priority on students understanding mathematical ideas (Brandt 1994, Perkins and Blythe 1994). Tests stress coverage of the "content" of the curriculum, or limited skills acquisition. In contrast, theorists emphasize that "understanding" requires that students develop the ability to use ideas in appropriate contexts, to apply ideas to new situations, to explain ideas, and to extend ideas by finding new examples (Gardner 1991, Gardner 1993, Sierpinska 1994). The goal of the Escher's World project was to help students develop this kind of understanding of mathematics through studio activities.

4.2 Students Learn about Symmetry

4.2.1 Use of Symmetry in Designs

Students were able to use the concept of symmetry to create original designs. During the workshops all of the students (12/12) were able to make designs using mirror symmetry, and 83% of the students (10/12) were able to make designs using rotational symmetry.

4.2.2 Application of Symmetry to Analysis of Images

Students developed their ability to apply the concept of symmetry to the analysis of images. Before the workshop, students made analytical references to symmetry an average of 0.5 times while looking at 4 images in structured interviews. After the workshop, mean analytical references to symmetry rose to 4.3 references over 4 images (see Figure 2, mean change +3.8, p<0.01).

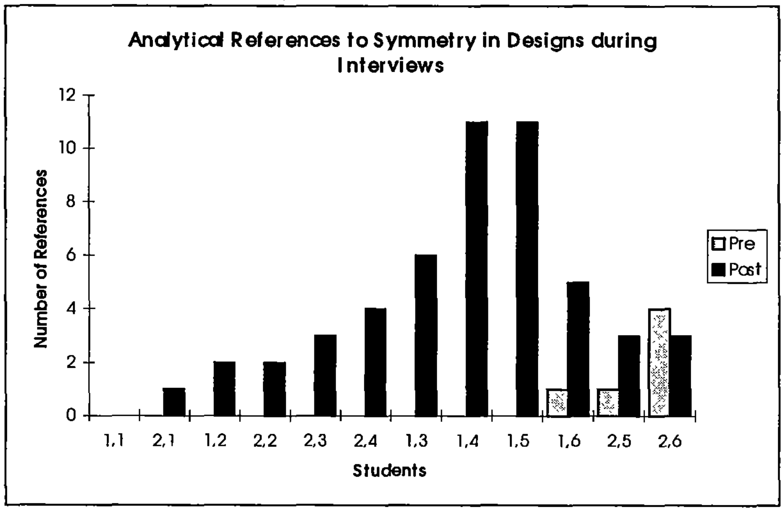


Figure 2· Students learned to use symmetry to analyze images. Students 1.1–1.6 attended the first workshop; students 2.1–2.6 attended the second workshop. Students have been ordered for clarity of presentation.

The small change in the total number of analytical comments (mean change +1.4 references; $p = 0.36$) suggests that students were not becoming more analytical overall; rather, as students began to use the concept of symmetry as a tool for analysis, they replaced some other form of analysis. As shown in Figure 3, change in analytical references to color (mean change -0.8 references; $p = 0.16$) were not statistically significant. Analytical references to composition were too small in pre-interviews (1% overall) to account for the rise in analytical references to symmetry. This suggests that students replaced an analysis in terms of the elements of form with a more mathematical analysis in terms of the principal of symmetry. This hypothesis is supported by the change in percentage of analytical comments made about forms and change in percentage of analytical comments made about symmetry, which have a coefficient of correlation of 0.76 across all students.

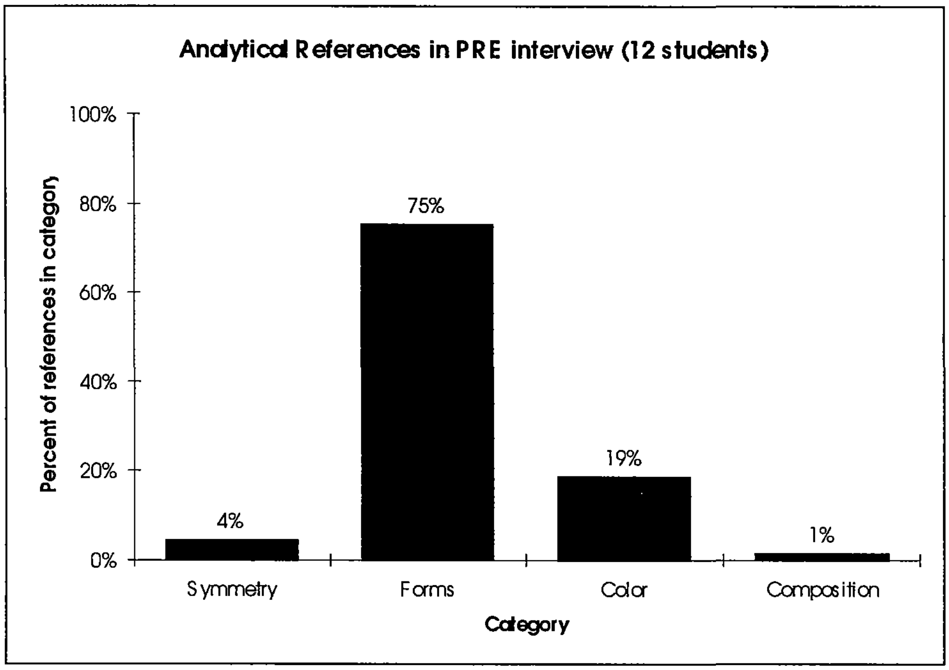
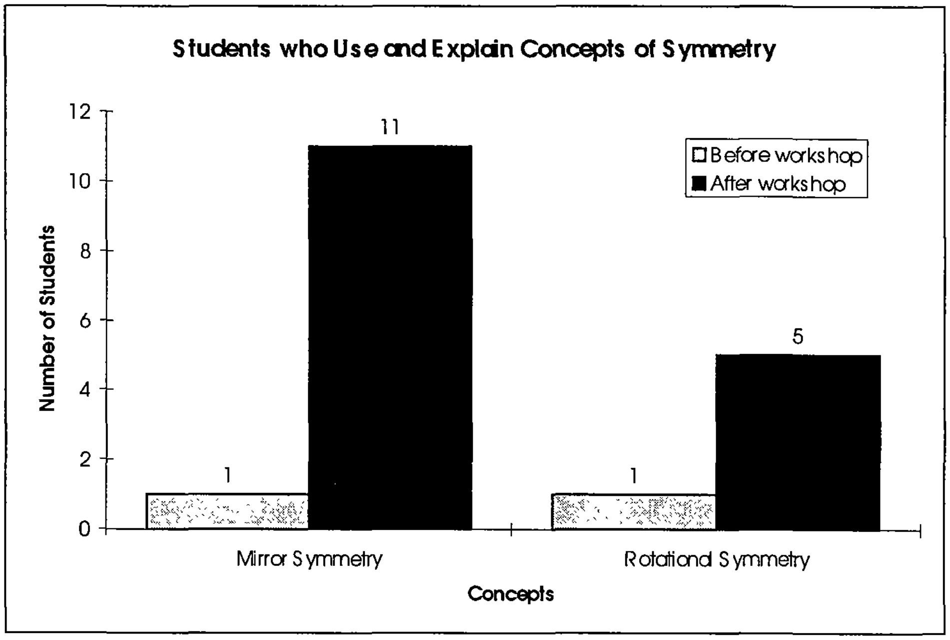


Figure 3: Rise in analytical references to symmetry was related to a drop in analytical references to forms. Graph shows aggregate data for 12 students in 2 workshops. Change in symmetry references is +30%; change in references to forms is –29%.

It should be noted here that students also showed an increase in analytical references to composition, which will be discussed in a later paper about arts learning in Escher's World.

4.2.3 Use and Explanation of the Concept of Symmetry

The number of students who could use and explain formal concepts of symmetry rose dramatically over the course of the workshop (see Figure 4). Some student explanations of symmetry were fairly general descriptions even after completing the workshop, but others are quite specific. For example, one student said: "If you drew a line down the middle—the line of symmetry—the two halves would be identical. They would be exactly the same: mirror images of each other."



4.2.4 Finding New Examples of Symmetry

After the workshop, students started to see symmetry in the world around them: 75% of the students (9/12) reported thinking about symmetry beyond the context of the workshop in post interviews or follow-up interviews. Students reported seeing symmetry in drawings, chairs, wallpaper, rugs, video games, flowers, and clothing.

4.3 Students Learn to Solve Mathematics Problems Visually

4.3.1 Use of Visual Repesentations Shows Mathematical Understanding

The workshop did not deal with mathematics word problems, or explicitly with the use of visual representation as a tool for solving traditional mathematics problems. After the workshop, however, students used visual representations as a successful problem solving strategy. Only 33% of the students (4/12) used visual representations to solve word problems before the workshop, while 75% of students did after the workshop (see Figure 5, $p < 0.06$).

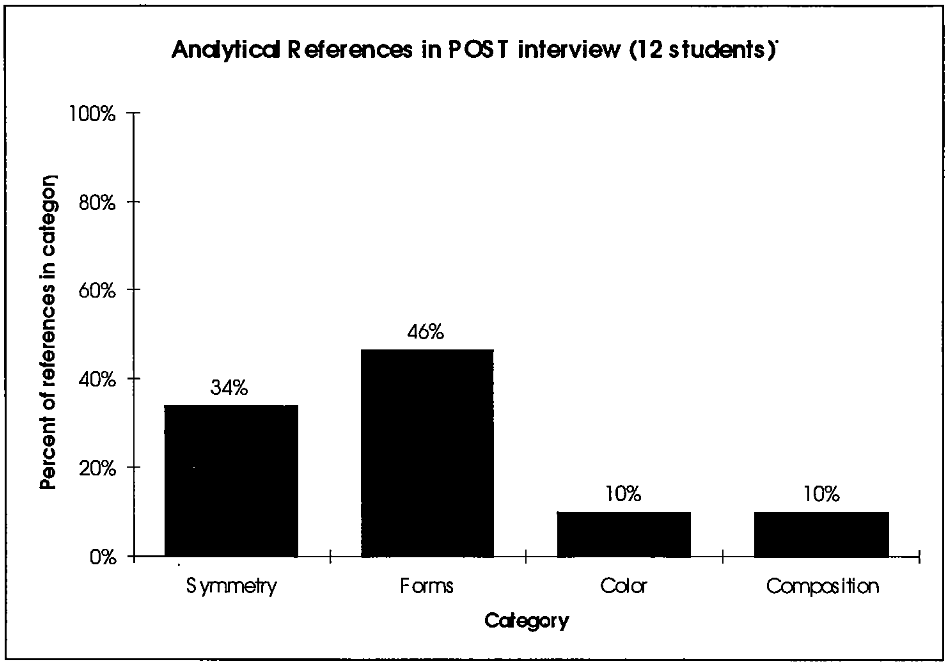
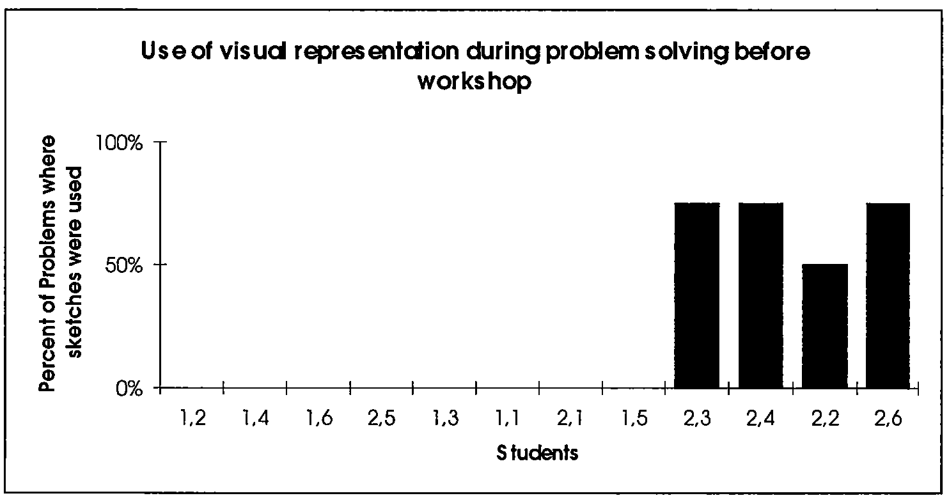


Figure 4: Students learned to use and explain symmetry through design. Graph shows data for 12 students in 2 workshops



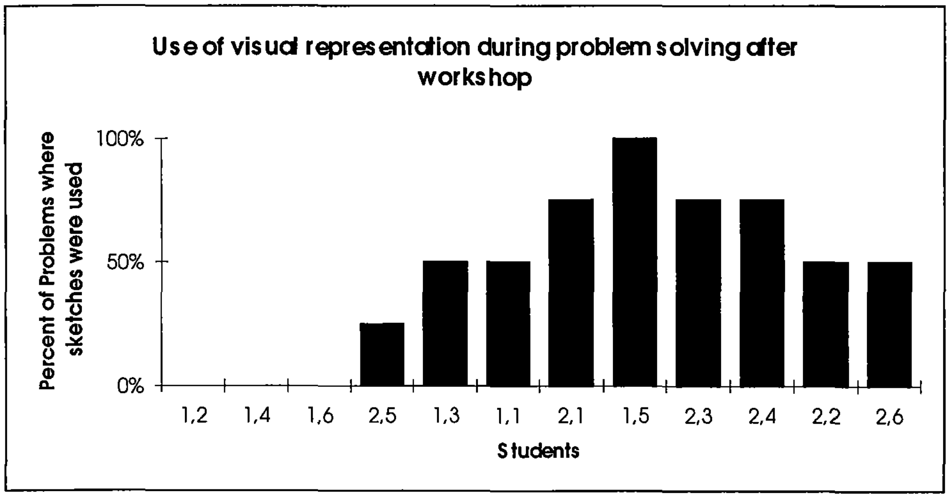


Figure 5: Students learned to use visual representations during problem solving. Students 1.1–1.6 attended the first workshop; students 2.1–2.6 attended the second workshop. Students have been ordered for clarity of presentation.

For example, in Figure 6, the student did not use a visual representation to solve the problem: "One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 3 miles due east. After Julie turns right again and walks for another 2 miles, she decides to go home. How far does she have to go to get back to her home?" After the workshop, the same student working on a similar problem used a visual representation of the problem situation.

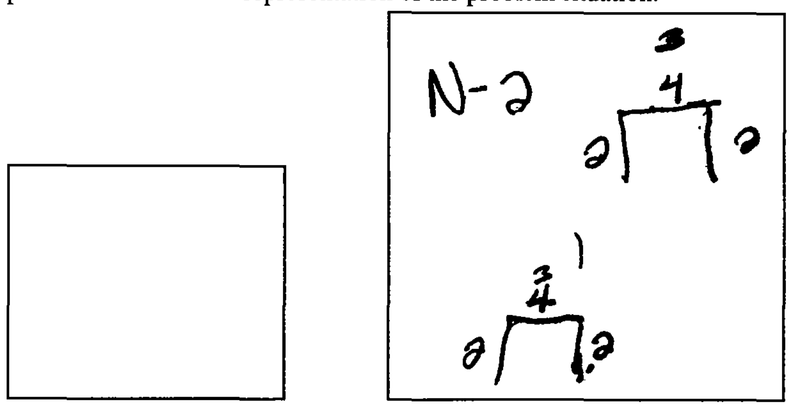


Figure 6: One student's notes while solving a problem during interviews. In the pre-interview (left image), the student did not use a visual representation. While solving a similar problem during her post-interview (right image), the student represented the problem visually andproduced a correct solution.

4.3.2 Visual Representations as a Successful Problem Solving Strategy

Use of visual representations for word problems after the workshop was correlated with success in problem solving during post interview problems (see Figure 7; $r = 0.83$). Some students solved problems without using visual representations, and some students used representations but failed to solve problems; however, no student solved more problems overall than the total number of problems they attempted using visual representations. This is reflected in the absence of data points above and to the left of the dotted line in Figure 7.

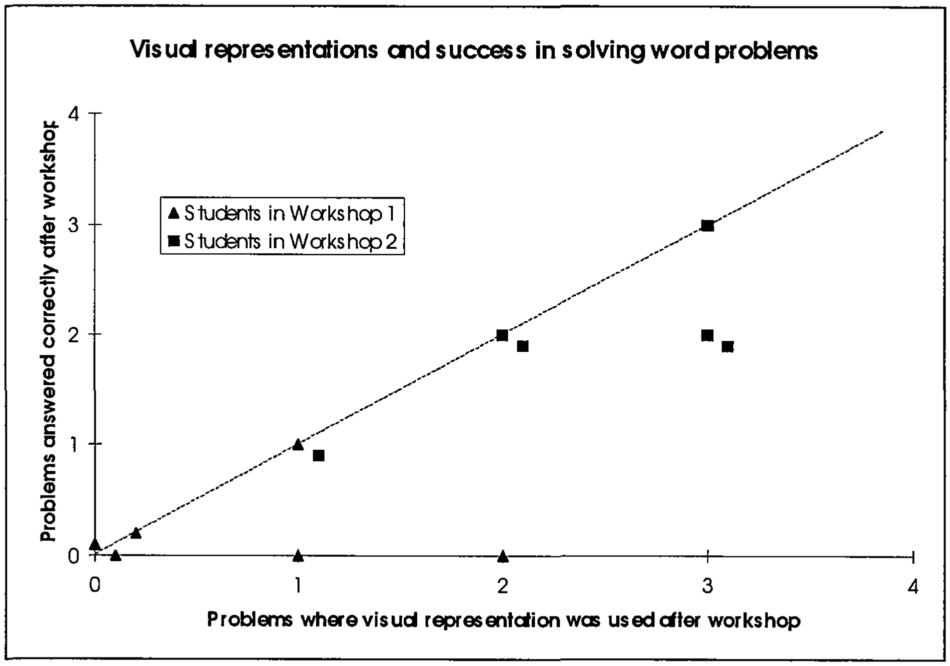


Figure 7: Visual representations helped students solve problems ($r = 0.83$)

4.4 Students Like Mathematics More

In post-interviews and follow-up interviews, 67% of students overall (8/12) reported feeling more positive about mathematics as a result of the workshop. This reported change was supported by data from a written survey. The survey was given to six of the twelve students who participated in the workshops. In the survey, students responded to 4 prompts about mathematics:

> "I like math class/I don't like math class."

"I like doing math problems/I don't like doing math problems."

"I like thinking about math/I don't like thinking about math."

"I understand math/I don't understand math."

Students marked a scale from 5 (most positive) to 1 (least positive). As shown in Figure 8, total rating for the 4 mathematics questions went up for two-thirds of students who were given the written survey (4/6). No student's total rating went down from pre- to post-interview survey. Change for the "I like math class/I don't like math class" prompt (mean +0.67, $p < 0.01$) was particularly striking.

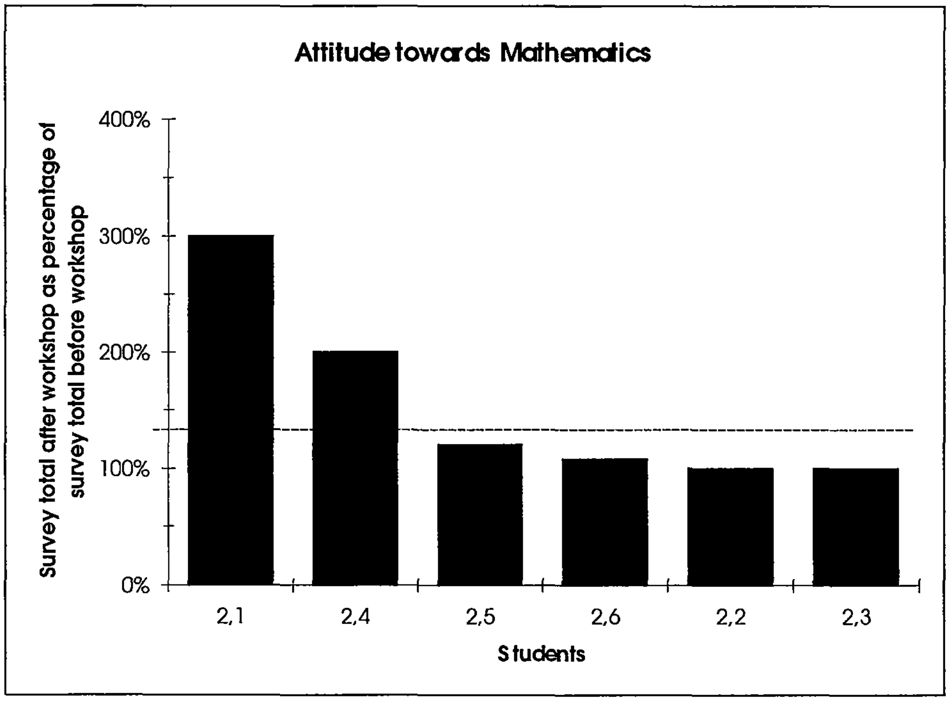


Figure 8: Students felt more positive about mathematics after the workshop. Graph shows data from survey conducted for the second workshop only. Students have been ordered for clarity of presentation.

5 DISCUSSION

5.1 Background Work

5.1.1 Constructionism

The design of the Escher's World workshops was based on the constructionist theory of learning developed by Seymour Papert at the MIT Media Laboratory (Papert 1991a, Papert 1993). There have been a number of attempts since the turn of the century to introduce "learning by doing" into American education (Dewey 1938, Prawat 1995). The learning theory of "constructivism," developed by Jean Piaget and applied to mathematics education in the United States by Les Steffe, Jere Confrey, Paul Cobb, Ernst von Glasersfeld, and others, has been used as a psychological basis for a variety of education reforms in the United States beginning in the 1960s. Constructivism asserts that students must create knowledge for themselves out of their own experiences (von Glasersfeld 1995, Phillips 1995). These two ideas came together in the theory of constructionism, which suggests that building things is a particularly rich context for building understanding. The theory of constructionism has been supported by investigations into the way students learn through the design and construction of real objects and virtual microworlds (Kafai and Harel 1991, Resnick 1991, Resnick and Martin 1991, Resnick and Ocko 1991). By demonstrating that effective mathematics learning takes place in the context of a design studio, Escher's World shows that the visual arts are another potentially effective environment for constructing mathematical understanding by constructing physical and virtual objects.

5.1.2 Visual Art and Mathematics Learning

Escher's World shows that the mathematical concept of symmetry can be explored and learned in an art studio environment. This basic result supports the ideas of numerous theorists who have suggested that learning in traditional academic disciplines can be enhanced or even transformed by the arts (Read 1943, McFee 1961, Arnheim 1969, Field 1970, Silver 1978). In a recent study, for example, Willett showed that mathematics learning was more effective in the context of arts-based lessons than with standard mathematics pedagogy at the elementary school level (Willett 1992). Arthur Loeb's visual mathematics curriculum (Loeb 1993) has not been studied formally, but substantial anecdotal evidence supports his approach to the study of symmetry through a design studio as an effective learning environment for undergraduate students.

5.2 Significant Results of Escher's World

The results reported in this paper extend earlier research in two important dimensions.

5.2.1 A New Mode for Problem Solving

Willet's research established that elementary students can learn mathematics content effectively in an art studio setting. The results of Escher's World support this same conclusion for high-school students, but also show that after exploring mathematical ideas in an art studio setting, students gain access to an additional mode for thinking about mathematics problems. In the Escher's World workshops, learning mathematics in the context of visual arts helped students learn to use visual thinking as part of their mathematics problem solving.

Although a number of researchers argue that visual thinking is related to successful mathematical thinking (Piemonte 1982, Hershkowitz and Markovits 1992), recent work by Campbell et al. suggests that students' visualization ability is not necessarily a factor in their success at solving problems (Campbell et al, 1995). Data from the Escher's World study shows that students' use of visual thinking during problem solving was correlated with their success in solving word problems after the workshop. This suggests that the workshop activities helped students make a more effective connection between visual thinking skills and mathematical problem solving.

5.2.2 Change in Affect

Data from Escher's World also suggests that students' learning of content and skills in an art studio environment is connected to a positive change in attitude toward mathematics. Willett's study of elementary students did not address students' attitude toward mathematics. In another study of the effect of arts activities and mathematics with fourth grade students, Forseth found that students' attitude towards mathematics improved, but that there was no significant improvement in students test scores compared to a control group (Forseth 1976). Results from Escher's World suggest that under the proper circumstances, positive change in students' attitude towards mathematics can be achieved in combination with meaningful changes in the way students approach mathematics problems.

5.3 Limitations of the Study

While the students in the two workshops discussed in this paper showed significant development in their mathematics knowledge, skills, and attitude, it is important to remember that Escher's World represents a very brief intervention. It was necessarily limited in the number of mathematical topics that students could investigate, and in the depth to which students could explore any one topic. It is not clear that the large positive changes seen in this brief but intensive intervention would continue at the same rate over a longer intervention. This fact, combined with the small sample size of the experiment, suggests that some caution should be used in making sweeping claims based on these data.

6 CONCLUSION

The data from Escher's World reported in this paper suggest that a studio setting is productive context for learning mathematics. This result supports other research findings regarding connections between mathematics and the visual arts. However, data from Escher's World goes beyond previous work by suggesting that while learning mathematics in visual arts environment, students not only learn specific mathematical concepts, they also develop the ability to use visual thinking as an effective tool for problem solving.

The limited size and scope of workshop results reported in this paper, combined with the apparent success of the "mathematics studio" as a learning environment suggest that further work should be done to develop the concept of studio mathematics. In particular, it would be useful to have a better understanding of the mechanism by which students develop new modes of mathematical thinking in a studio context. This will be the subject of a future paper on the Escher's World project.

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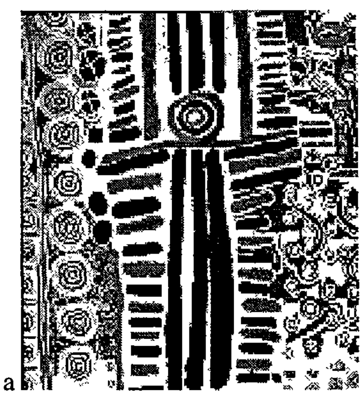
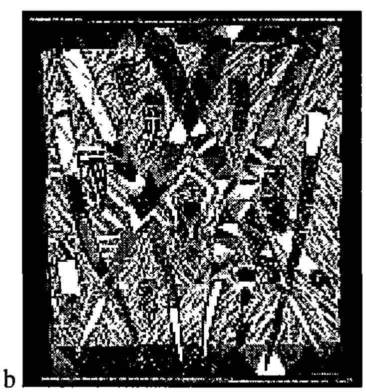
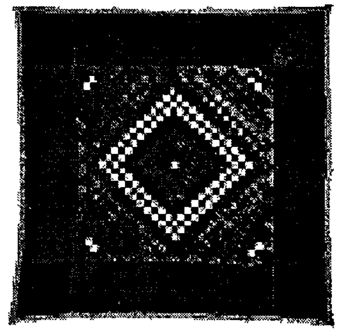
8 APPENDIX

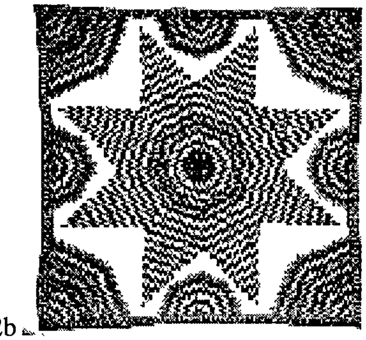
8.1 List of images used in interviews

Students were shown four images in both pre and post interviews. One image was chosen from each of the sets 1-4 below. Note that set 3 contains only one image; all students were shown the same image in both pre and post interviews for this set. All images were resized to 5 inch width and reproduced in color.

1a 

1b 


2a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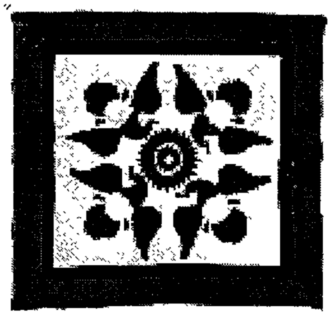
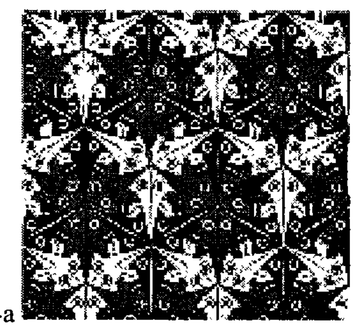
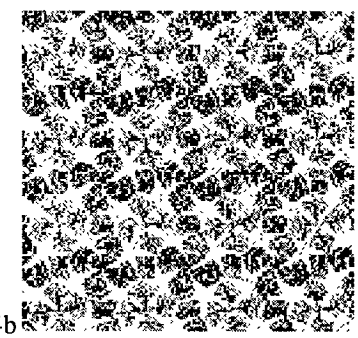

3a


4a


4b

8.2 List of problems used in interviews

During pre and post interviews, students from the first workshop were given two out of four problems 1.1-1.4 below. Students from the second workshop were given 4 problems, one each from sets 2.1–2.4 below. Students were given the problems one at a time, each typed on a separate sheet of paper. Students had unlimited time to work on each problem and were provided with a pad of lined paper and a pen or pencil.

Problem 1.1

Ms. Jones has to 25 pencils and 10 pens to give to her students. She gives each student the same number of pencils. She gives each student the same number of pens. At most how many students does Ms. Jones have?

Problem 1.2

Bob and Tanecka each have a 12-inch pizza pie. Bob cuts his pizza into 8 pieces. Tanecka cuts her pizza into 6 pieces. If you put the pizza's one on top of the other, at most how many cuts in Bob's pizza would be in the same place a cut in Tanecka's pizza?

Problem 1.3

Sally and Juan each have a 14-foot ladder. Sally's ladder has 21 rungs on it. Juan's ladder has 15 rungs on it. If you put the two ladders side by side, how many rungs would be in the same place?

Problem 1.4

A group of students has 12 apples and 15 oranges. They share the apples and oranges so that each students has the same number of whole apples and whole oranges as every other student has. At most how many students could there be in the group?

Problem 2.1a

One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 3 miles due east. After Julie turns right again and walks for another 2 miles, she decides to go home. How far does she have to go to get back to her home?

Problem 2.1b

One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 4 miles due east. Julie then realizes that she dropped her watch 1 mile back. She turns around and walks until she reaches her watch. After she picks up her watch, Julie turns right again and walks for another 2 miles due south. Now Julie wants to go home. How far does she have to go to get back to her home?

Problem 2.1c

A leaf falls and lands 5 yards east of the tree it was on. A boy picks up the leaf, and walks 10 yards north. The boy sees a swing set on his left, 15 yards to the west. He runs to the swings, but drops the leaf 10 yards before he reaches the swings. How far is the leaf from its tree after the boy drops it?

Problem 2.2a

One border of Theo's backyard is 20 yards long. Theo wants to put up a fence along this border. If he puts up one fencepost every 5 yards, how many posts does Theo need?

Problem 2.2b

One border of Theo's backyard is 20 yards long. Theo wants to put up a fence along this border. If he puts up one fencepost every 5 yards, how many posts does Theo need? Problem Number 2 Theo wants to put up fences along two borders of his backyard. One border is 15 yards long, the other is 20 yards. If he puts up one fencepost every 5 yards, how many posts does Theo need?

Problem 2.2b

Yolanda has a pipe that is 8 feet long. She needs to cut the whole pipe into pieces that are 2 feet long. How many cuts does she need to make?

Problem 2.3a

A snail is stuck on the inside wall of a well, 5 feet down from the top of the well. It moves 3 feet up the wall every day. But every night, the snail slips 1 foot down the wall. After how many days will the snail reach the top of the well?

Problem 2.3b

A snail is stuck on the inside wall of a well, 5 feet down from the top of the well. It moves 2 feet up the wall every day. But every night, the snail slips 1 foot down the wall. After how many days will the snail reach the top of the well?

Problem 2.3c

Yanni goes to his favorite restaurant and orders an 12 ounce soda. Every time he finishes drinking 4 ounces of his soda, a waiter pours 1 more ounce of soda into his cup. How many times will the waiter pour soda into Yanni's cup before Yanni completely empties his cup?

Problem 2.4a

Leo asks Luanda if she will lend him money for a hamburger. Luanda says, "But you borrowed money for a hamburger from me yesterday! And you still owe me 1 dollar for the soda you bought last week!" So Leo says, "Well, okay – lend me money again today, and I'll owe you 4 dollars all together." How much does a hamburger cost?

Problem 2.4b

Leo asks Luanda if she will lend him money for a hamburger. Luanda says, "But you borrowed money for a hamburger from me yesterday! And you still owe me 80 cents for the soda you bought last week!" So Leo says, "Well, okay – lend me money again today, and I'll owe you 4 dollars and 30 cents all together." How much does a hamburger cost?

Problem 2.4c

Two years ago, Janelle was four times as old as Sangita. If Janelle is twenty years old now, how old is Sangita now?

8.3 List of survey questions used in interviews

Students were given the following "sample question":

For each set of statements, circle the number that represents how you feel.
Example:
I like talking on the phone. 5 4 3 2 1 I don't like talking on the phone.

If you really like talking on the phone, circle 5. If you like it most of the time, circle 4. If you like it half of the time and don't like it the other half, circle 3. If you don't like it most of the time, circle 2. If you really don't like talking on the phone, circle 1.

The survey consisted of 15 questions, all in the same format as the example:

1. I like school./I don't like school.
2. I like math class./I don't like math class.
3. I like doing math problems./I don't like doing math problems.
4. I like thinking about math./I don't like thinking about math.
5. I understand math./I don't understand math.
6. I like to make art./I don't like to make art.
7. I like looking at art./I don't like looking at art.
8. I like thinking about art./I don't like thinking about art.
9. I understand art./I don't understand art.
10. I like computers./I don't like computers.
11. I understand computers./I don't understand computers.
12. I learn a lot from listening to the teacher./I don't learn a lot from listening to the teacher.
13. I learn a lot from working on a computer./I don't learn a lot from working on a computer.
14. I learn a lot from working with other students./I don't learn a lot from working with other students.
15. I learn a lot from working by myself./I don't learn a lot from working by myself.