REFLECTIONS ON ROTATIONS

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Publications with relevance to symmetry: Language of Shape and
Movement, Tel Aviv: Tel Aviv University (1983); Symmetry and Notation:
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Exhibitions with relevance to symmetry: (1969) New Tendencies, Zagreb,
Yugoslavia (participation); (1971) Art and Science, Tel Aviv Museum
(participation); (1972) Festival de Due Mondi, Spoleto, Italy (in
conjunction with performances of Noa Eshkol’s Chamber Dance Group);
(1973) Graphic Art and Movement Notation, Museum Haaretz Science and
Technology Pavilion, Ramat Aviv, Israel; (1992) ‘Variations on a
Pomegranate’ - video installation and paper cuts, Kalisher Five Gallery, Tel Aviv (in conjunction with
‘Movement Notes’ exhibition); (1993) ‘Emergences’, ZOA House, Tel Aviv (in conjunction with performances
of Noa Eshkol’s Chamber Dance Group).

Abstract: A sequence of 16 pictures employing dynamically conceived shape is used to
demonstrate how symmetry operations can constitute one of the connective forces in a
work of visual art that is systemic and periodic. The aim of the study was to generate a
set of pictures comprehensible by virtue of the appeal to transformations that have been
present in art for thousands of years. The sequence was composed in EW movement
notation, the principles of which are briefly sketched. The composition of the sequence
is explained, using illustrations together with their symbolic expression in movement
notation, and also typographical representations of aspects of its overall structure. The
visual realization of the structure comprises the full sequence of 16 pictures.
INTRODUCTION

The study presented here is a part of work begun over 30 years ago, all of it based upon the use of Eshkol—Wachman (EW) movement notation as a tool of composition in visual art, including kinematic forms such as videotapes. Since EW movement notation entails the perception of movement as shape, it is the natural vehicle for this work, in which shapes are perceived as the paths of movements. Indeed, the notation was the catalyst in the conception of this model for the generation of shape. It provides a firm basis for the organisation of the material, because the conception of visual forms as movement traces allows their precise definition in the notation. EW notation is ideal as the support for a serial or periodic approach to composition which rests upon quantities and order. By its use, it is possible to control and pursue complex and extensive variations of shape, within a reasonably concise symbolic scheme. Furthermore, anyone who is literate in the notation is able to follow in depth the formal operations it describes.

The aim of the present study was, however, to generate a sequence of pictures that would be as far as possible comprehensible independently of any knowledge on the part of the viewer, of the ideas underlying movement notation. This leads to the question: upon what could such comprehension be based, if the work of composition is in fact based upon those ideas? The solution is here assumed to lie in the intuitive and probably innate perception of transformations such as symmetries, which have been present in all art for thousands of years (Weyl, 1952; Avital, 1996; Avital, under review). These verbally nameable operations would be reflected in any thoroughgoing notation used in the domain of visual art, including them as an integral part of a single compositional act. Where EW is the notation that serves this end, the quantification of these operations will also certainly be possible (Harries, 1986).

In this study, operations on the basic set and on its successive transformations are all intended to be intuitively identifiable and verbally nameable, and also quantifiably expressible in EW notation. Movement notation provides for the quantified approach essential to any well defined composition. At the same time, the presence of quantitative differences can be detected even without knowledge of the precise values involved. For the sake of simplicity, the principles through which this study is organised were confined to two types of transformation of the motif series.

The employment of EW movement notation in visual art, is itself inevitably based upon some intuitive transformations: recursion, rotation, scaling, and positive/negative sense of movement; and it necessarily involves taking into account other operations such as selection of trace type, colour modification, layering.
In this black and white sequence, the basic motifs are obtained by splitting up a single motif taken from an earlier sequence in colour ("Emergences", 13 paper cuts, first exhibited in Tel Aviv, 1992). The composition of that sequence of pictures was based entirely upon a serial and periodic approach. In the present case, a similar serial and periodic approach manifests an overall systemic structure, in which the ordered reduction of a complex pattern to a basic set of motifs is followed by a cumulative redevelopment of that set through successive transformations and recombinations. These metamorphoses will be described in the text, both verbally and in terms of movement notation.

The use of EW movement notation in visual art has been described extensively elsewhere (Harries, 1969, 1975, 1983); for the purposes of the present article, a brief outline of this application is provided in the next section.

**EW movement notation**

A subset of Eshkol-Wachman (EW) movement notation serves abstract composition of visual images in several ways: as a record, for communication, for systematizing, and above all for formulating ideas in symbols that make it possible to grasp structure through a scheme of manageable size. Any bounded area can be regarded as the path of the movement of a line, called in descriptive geometry a 'generatrix'. The line sweeps out a trace, the shape of which is determined by the way the generating line moves. This idea is useful so long as it is possible to define exactly how the line does move. This is achieved when the movement is expressed in the symbols of EW.

The primary use of EW notation is in the context of human movement (Eshkol and Wachman, 1958). Movements are treated as the paths produced by the limbs, which are regarded, for the purpose of analysis, as chains of articulated axes.

Using EW, it is possible to define the movement of any line, and thus to describe shapes in terms of movements of articulated generating lines, in relation to a system of reference encompassing two or three dimensions of space, plus time. The instructions of the notation are general in that they apply to any medium, but specific in the way they work for each - for instance: pencil and paper, computer graphics as in the present study, or (when the moving lines are the axes of limbs) human performers.

Seen in this way, a still picture is not only a motionless object but also one stage in a formative process, and the potential point of departure for subsequently emerging form. EW notation preserves the continuity of the static and dynamic, and of the two-
three-dimensional. In three-dimensional space, if a single generating line moves about one of its ends, which remains at a fixed position, the line may sweep out a curved surface or a plane. (In the case of a solid limb, rotation about its own axis is also significant.) In two-dimensional space, a circular shape results. A more complex shape is obtained if a second generator is articulated with the moving end of the first, and simultaneously moves about their common 'joint'. Modifications of the circular path also result from any changes of length of the generator. Chains of any number of such generators may be formed, moving about the points of linkage.

In every movement of articulated generating links, these are characterised in terms of EW as 'heavy' or 'light', i.e., carrying or carried by a neighbouring link. (A generator may simultaneously carry one neighbour and be carried by another.) When a generator moves, it carries with it all other links that are further away from the origin, thereby changing their positions; the origin of the heaviest link is analogous to the base of support in the case of a living organism.

When independent movements of the light links occur at the same time as they are carried by a heavy link, the change of position of each link is the result of the simultaneous movement of the carried link together with the movements of the heavier links. The movement of each generator is written as though in relation to an immobile carrying link; but in fact the path of this movement will be modified because its heavy neighbour moves as well; see Figure 1. The figure shows simultaneous movements of generators, both as successions of positions and as the shapes that they sweep out. In (a) the carried link moves at twice the rate of the carrying link; in (b), it moves at half the
rate of the carrying link. The varied synchronisation of the movements of two or more articulated generators is the source of the apparently endless wealth of shape that can be obtained and composed using this system of representation.

The notation of each of the shapes in the figure indicates, to the left of the frame, the relative lengths of the generating links. Above the frame, the value of the unit of movement is given (one unit = 10 degrees). The plane in which the movements take place is given in parentheses; following these, arrows indicate the sense of the movement (clockwise or counterclockwise), and numbers specify the amount (in units) of the movement of each generator. If the length of a generator were to change, this too would be indicated in the appropriate horizontal space. Specifications of colour can be added in an additional space, parallel with the movement score itself.

Work designed to be displayed in time is conveniently provided for in EW, where the measured flow of time is represented by the columns of the basic grid, upon which the synchronised patterning of movements of the generating links is written and easily perceived. The information conveyed in these scores is implemented as abstract moving computer graphic images, or as videotapes. The quantitative nature of EW makes it ideal for computer input. The software I have developed provides for the entry of data in an EW score on screen; when this score is completed, the visual process it represents is displayed in movement on the screen, and can be recorded on videotape (Harries, 1981, 1983).

All of the work of visual composition is written in the same notational system and does not require new parameters or new modes of symbolisation for different projects. The generality of the notation is more than sufficient and it can equally well encompass the domain of three-dimensional structure. Furthermore, shapes and processes are defined with as much accuracy as can be matched in the chosen medium; this allows both for a maximum of control and for great subtlety of variation.

The picture sequence: 'Reflections on Rotations'

This picture sequence is intended to be in principle comprehensible without special knowledge of the means used in its composition. The use of movement notation in generating such a picture sequence involves processes that can be learned, and any EW-literate person could understand the structure by studying its score. It is, however, doubtful whether even such a person would be able to follow the structure in its entirety without studying the score, if there were no visually explicit integrating principle. Therefore, while this study is firmly anchored in every detail to definitions and
procedures of variation formulated in EW notation - which guarantees full control of the compositional procedures - at the same time, all compositional choices were made in cognizance of the nature of the transformations that result from those procedures, and from the way in which they are bonded together by their ordering in relation to one another. To understand the meaning of the structure is to understand the place of a motif (in any one of its transformed states) within a picture; the place of a picture within a group of pictures; and the place of a group within the whole. (This is the essentially hierarchical principle which Avital [under review] has pointed out as being at the foundation of all viable art.) If the bonding principle is one that is universally understood in visual terms without the help of verbal or other symbolic explanation, it is reasonable to hope that both an observer who is not EW-literate and an observer who is EW-literate but had no access to the score, would be able through the visually apprehended structural procedures that emerge in the composition, to perceive the meaning of the whole in visual terms. It may be that even such a simple composition as the present is not immediately obvious; instant comprehension is, however, not the aim.

The concern was, then, to present a serially formed picture sequence, designed to be directly - i.e., visually - understood as a single systemic structure, by virtue of its being bound together by familiar and intuitively comprehensible transformations. The transformations chosen were threefold rotational symmetry, and reflective symmetry. These transformations were to be unambiguously specified in EW Movement Notation, no less explicitly than were the details of the motifs and series out of which the work was formed. The whole sequence is shown in Figure 7.

In the following exposition, no special previous knowledge is assumed on the part of the reader. It includes illustrations in which the same idea is conveyed symbolically in movement notation, and also visually by the shapes generated in accordance with the notation. Some simple typographical schemes are also used to indicate aspects of general structure without going into detail.
Movements of a linkage of four articulated axes are specified (Figure 2), and the trajectory of the distal end defines a curve, \( G \).

The curve \( G \) is used as the visual generating link in a movement sequence; i.e., its movements are assumed to generate shapes. The curve \( G \) is substituted for the 'lightest' (upper) link in the three-movement sequence which gave rise to the curve itself. (Its axis for this purpose is the line joining the two ends of the curve.) This is illustrated in Figure 3; the states of this new curved link are shown as they appear at the beginning and end of each movement.

The other three links are not visual, but contribute to the form of the shape generated, by carrying the (independently moving) curved generating link. This sequence is repeated a further three times, from the states reached. Figure 4 shows the notation of the movements, and position lines selected from the path they sweep out.
The four elements \( a, b, c, d \), are generated as the four consecutive parts of this extended sequence. They are recombined (with a single shift of the sequence) to form four motifs, \( A, B, C, D \), as shown in Figure 5.

\[
\begin{array}{cccc}
\text{a} & \text{b} & \text{c} & \text{d} \\
\text{d} & \text{a} & \text{b} & \text{c} \\
\hline
\text{Motif A} & \text{Motif B} & \text{Motif C} & \text{Motif D}
\end{array}
\]

**Figure 5**

These form a group of four pictures: \( A : B : C : D \) which constitutes the central group in the whole series. Each motif contains one element in common with each of its neighbours, emphasizing sequential continuity between the four. Each motif is made more readily identifiable by being displayed as a different type of trace: fully swept trace, trajectory, and selected position lines from the swept path (in two different weights of line).

In the following schematic representation of the structure of the entire series, the groups are separated by vertical bars, and the pictures within the groups are separated by colons:


The groups to the left and right of the central group contain increasing accumulations of the motifs, so that their complexity increases with their distance from the centre. In the groups to the left, the pictures at successive levels (groups) are connected by relations of reflective symmetry \((\sigma)\). In terms of EW movement notation, this means that the movements of the links that form each motif start from positions bilaterally symmetrical to those in the appearance of the same motif in the group next nearest to the central group; and the links move in the opposite sense to the movements that produced the same motif in the group next nearest to the central group. The groups on the right are connected by relations of threefold rotational symmetry \((C_3)\), expressible in EW notation as reiterations of the motif at a series of positions separated by intervals of \(120^\circ\). In all appearances of the motifs in their various transformations, the initial positions of the 'heaviest' links are permutations of the positions \((0), (3)\) and \((6)\) where \(1 = 40^\circ\).
A diagram of the structure

Since it is intended that the structure be perceived in the picture sequence itself, to describe it in words and diagrams may appear to be merely paradoxical and superfluous. But in fact their use need no more interfere with a proper appreciation of the composition than consulting a map detracts from the direct viewing of the actual terrain. A map represents only certain aspects of the actuality, but this very selectivity may facilitate an overall understanding.

In the typographical scheme given above, only the additive aspect was represented. The next 'map' is a schematic representation of the interrelation of motifs, transforms, combinations, and levels of hierarchic order. In Figure 6, the hierarchic order is shown integrated with the composed sequence.

In this scheme, A, B, C and D again represent the motifs. The boxes indicate pictures, each labelled with a number vertically beneath it. All pictures shown on the same level on the page belong to the same level of order. Arrows indicate the direction of ascent to a new level of order, and transformation of the motif (by reflective symmetry or by rotation). All states of the motif from lower levels appear together with each of its transformations. If the pictures are viewed in order from 1 to 16 (or indeed the contrary - from 16 to 1), the changes consist firstly of subtractions of transformed motifs from level to level, and continuity (similarity) is revealed through the gradual isolation of the
original, undeveloped motifs. This process comes to completion in the central group (7 to 10). From that stage onward, the changes consist of *additions* of transformed manifestations of the motifs, while continuity is further maintained through the persistence of the transforms that have already appeared at lower levels.

A 'reading' of the outer groups of pictures depends more upon visual differentiation than is the case in the more central groups, which rather invoke the retention of the images in visual memory when viewing individual pictures as interrelated members of a group.

The full picture sequence is shown in Figure 7 (1-16).
Notating the transformations

A less schematic representation than that given in Figure 6, would include the definition of the orientation and type of transformation involved at each level of the sequence for each motif. This is tantamount to their symbolic and quantitative expression in terms of the notation actually employed in composing the sequence - i.e., EW movement notation.
The motifs $A B C D$ have been defined as simultaneous appearances of pairs of the elements $a b c d$. For example, motif $B$ is the combination of elements $a$ and $b$, the first and second parts of the source sequence shown and notated in Figure 4. The motif appears (alone) in picture no. 7 in a specific orientation, as notated in Figure 8.

![Figure 8]

The transformation of $B$ in picture no. 11, can be seen as the recreation of the elements from scratch, in a new orientation, as notated in Figure 9.

![Figure 9]

This is equivalent to the rotation of the whole motif $B$ through 120 degrees about its origin. An equivalent of further rotation of the motif by the same amount as in picture no. 14 would be notated as shown in Figure 10.
These three (original and two variants) would reveal threefold rotational symmetry.

If the medium had been one in which the display changes in actual time - as in a videotape - this mode of notation would be the only way of expressing the movements to be displayed. However, that is not the present case. The original shapes of the motifs have already been fully defined, and these definitions have served as the basis for the 'macros' A B C D. In this structured set of immobile pictures interrelated through symmetry operations, it will therefore be more appropriate to notate them in a way that reflects the symmetry type, while also (since we are dealing with real specific visual forms) indicating what forms are involved, and how they are orientated. This requirement is fulfilled by treating the paths (shapes) of the motifs as invariant forms, and defining the movements of the total shape, which produce the transformation. For example, in the case of motif B the three states of the motif can be adequately indicated as shown in Figure 11.

$$(r) = (\theta)$$

Figure 11

The positional series in Figure 12 expresses the whole sequence of rotational symmetries, by indicating the positions of the appearances of the motifs; and also which of the modified motifs 'survive' in each picture. (Note that in the end all of them survive - in the sense that they all reappear in each group.)
This constitutes a new - third - level of operation: movements of the invariant forms - entire motifs that have already been defined as 'frozen' or 'fossilized' traceforms - paths of movement generated by the movements of a curve, the shape of which was itself previously defined as a path of movement.

Similarly, in Figure 13, the transformation of motifs by reflective symmetry can be expressed as the rotation through 180 degrees ('flipping'), of the invariant forms (motifs) about specified axes lying in the picture plane.
Here again there is explicit indication of which transformations survive in each picture.

We have now explained how it is possible to give in EW notation a fully detailed account of a structured sequence of 16 pictures that includes the definition of generating links, the generation of elements and motifs, and the full deployment of successively transformed combinations of those motifs, including a representation of the detailed operation of the two symmetry types employed in this sequence.

The last picture (16) manifests the total rotational symmetry towards which the second half of the sequence tends throughout its intermediate stages. This tendency is the mode of articulation that connects the parts of the sequence that include pictures 7-16. In the reflective transformations, there is complete symmetry between pairs of motifs at every stage. The successive elimination of these symmetries down to the isolated asymmetrical motifs in pictures 7-10, is the mode of articulation - or rather, of separation - of the parts of the sequence that include pictures 1 to 10. Furthermore, the overall connecting principle which gives the sequence its unified structure is that of reduction followed by accumulation. (In its accumulative aspect, the sequence has symmetry about the central group.)

Besides movement notation, we have for the purpose of this explanatory article also used typographical representations of the overall structure in explaining the successive grouping and recombination of the motifs, and in a 'map' of the evolution of the motifs and their recombination in transformations, resulting in increasing complexity as the distance from the middle group of pictures increases. These schemes are comprehensible with a minimum of verbal explanation. While they are not detailed notations, they make sense because their frame of reference is the unambiguous detailed specification fully formulated in terms of concepts of EW notation. Different selections from these schematic representations would yield descriptions of varying generality - some so simple that they could be given in words and commonly accepted symbols such as $\sigma$ and $C_3$, as was done in the penultimate paragraph of the section entitled 'The picture sequence...'

Ultimately, only the pictures themselves can give a proper visual realization of the structure, for no description or symbolic representation can fully convey the landscapes of visual experience.
REFERENCES
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