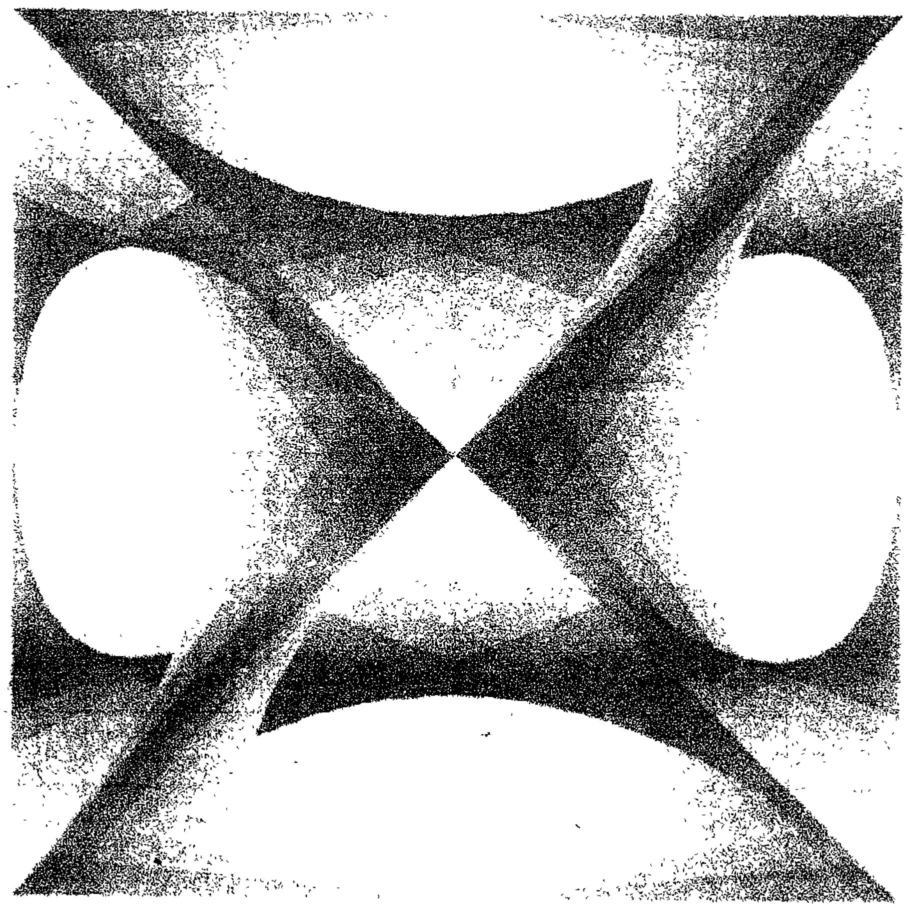
Symmetry: Culture and Science

Symmetry and Information

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Volume 8, Number 2, 1997



Symmetry: Culture and Science
Vol. 8, No. 2, 157-175, 1997

SYMMETRY: CULTURE AND SCIENCE

INFORMATION PROCESSING AND SYMMETRY-BREAKING IN MEMORY EVOLUTIVE SYSTEMS

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Abstract: *The aim of this paper is to evaluate the role of symmetry and symmetry-breaking processes on the complex information processing developed by hierarchical evolutionary natural systems, such as biological, neural, social or cultural systems. The study is conducted in the frame of the Memory Evolutive Systems, which give a mathematical model of these systems. The dynamics of a MES is modulated by the competition between a net of internal regulation centers, which act apart but encode overlapping strategies which have to be equilibrated. The main characteristics of these systems, at the root of their complexity and adaptability, is a symmetry-breaking in the passage from a higher (or macro) level to a lower (or micro) level: several disparate sub-systems with different comportments at the micro level can be undistinguishable at the higher macro level because of a similar macro behavior (Multiplicity Principle). It is responsible for the development of a dialectics between heterogeneous regulation centers, and for the emergence in time of more and more complex objects. An application to neural systems vindicates an emergentist dynamical reduction of mental states to physical states.*

1. INTRODUCTION

For the Webster, symmetry is defined as "Proper proportion in the arrangement of parts; correct balance between the two halves of an outline; graceful proportioning; the beauty of harmonious arrangement of parts". The mathematical notion of symmetry is more precise: It supposes given a set of objects on which acts a group G of transformations, and two objects of the set are said to be symmetrical (for G) if there exists a transformation in G transforming one into the other. In particular spatial symmetries around a point, or a line, or a plane, are ubiquitous in Art, and are also taken into account for taxonomic classifications.

In complex evolutionary natural systems, such as biological, neural, social or cultural systems, symmetry and symmetry-breaking occur in a multitude of different ways, at a structural, functional and temporal level. They modulate the evolution of the system and allow for a complex information processing, not relying on a convention between an emittor and a receptor, but distributed over a variety of overlapping and possibly conflicting internal regulations. Classical physical models, tailored for a specific process in a well-delimited environment, are not flexible enough to deal with these processes, though chaos theory allows for some symmetry-breaking.

The aim of this paper is to show how these processes can be studied in the frame of the Memory Evolutive Systems (or MES, introduced in preceding papers, e.g., Ehresmann and Vanbremeersch 1987, 1991, 1996), which give a mathematical model for self-organized hierarchical evolutionary systems in which the global dynamics follows from a balance between competing regulations. Each regulation depends on a particular internal organ, modeled as a sub-system of the system called a Center of Regulation (CR). The components, or actors, of a CR have the same complexity level, and cooperatively develop a stepwise more or less cyclic process with its own time-scale. In this process, the CR acts first as a filter and decoder of informations to form its landscape, which represents its internal model of the system; then it selects a strategy on this landscape to give the best answer at its level, and finally it encodes commands to effectors to realize this strategy; its choice is helped by a differential recourse to a central memory in which past experiences are stored.

We prove that one of the main characteristics of such complex systems, which give them their flexibility and adaptability, is the existence of 'multifold' objects, that is an object of a higher level which has several possible lower level decompositions in sub-systems with disparate comportment at this level, though they cannot be distinguished at the higher level (Multiplicity, or Degeneracy, Principle). It entails a symmetry-breaking in the passage from a higher to a lower level, and is responsible for the emergence in time of objects with increasing complexity order.

It has also a strong influence on the dynamics of the MES which is modulated by the cooperation and/or competition between the various CRs, specially on the equilibration process between their strategies to attain a global coherence, possibly with fractures for some CRs. It leads to a dialectics between CRs with disparate complexity level and time-scales, and may cause evolutionary symmetry-breakings in the form of de/resynchronisations of the CRs.

Though developed in a different frame, these results have strong convergences with the "Physics of becoming" proposed by Matsuno (1989, 1994). They can also be compared with the features by which Farre (1994) characterizes evolutionary systems, or with the mechanisms singled out in the C8 taxonomy of Chandler (1991; cf. also Chandler, Ehresmann and Vanbremeersch 1995).

2. GLOBAL DESCRIPTION OF AN EVOLUTIVE SYSTEM

We first give an 'external' description of an evolutionary system.

2.1. Systemist Models

To model a system consists in organizing the informations that an observer (the modeler) can obtain on the system and its dynamics. In Systems theory, the informations taken into account are related to the class of its components (individuals or parts of the system) considered as more or less stable, and to the interactions between them and their temporal variation, generally measured thanks to quantitative observables (e.g., distance, or attraction forces, or energy constraint...).

Geometrically, these data can be modelled by a graph: the nodes N of the graph represent the components of the system and the arrows represent interactions between them;

we may have none, one or several arrows from N to N', and also loops from a node to itself. Though it is often supposed that the interactions between components are symmetrical, here they are oriented, and graph is always taken in the sense of such an oriented polygraph. The elements of this graph can be labelled by some observables.

In most models, it is assumed that the organization represented by the graph remains the same during all the period on which the system is observed, and the dynamics is entirely described by the variation of the labelling observables, which can often be computed by solving some differential equations. If there is only a finite number of nodes N_i and at most one arrow between two nodes, the labelled graph can be replaced by the matrix of transitions with indices i corresponding to the nodes and in which the (i,j)-term is the label $s_{ij}(t)$ of the arrow from N_i to N_j if it exists, and is 0 otherwise. For instance in the Ising model inspired from Thermodynamics, the nodes are assimilated to spins localized on the sites of a lattice, with $s_{ii}(t)$ taking two possible values 0 or 1 ('up' or 'down'), and the interactions between two spins are measured by the energy necessary for correlating their behavior. This model has been applied in the most disparate systems, from ferromagnets to connexionist neural networks or economy problems.

2.2. Evolutive Systems: The State-Categories

However in the complex autonomous evolutionary systems which we consider, we cannot restrain the study to transformations on a well-fixed (high-dimensional) space. Indeed, these systems are 'open' in the sense that they have external exchanges, and their organization varies in time, with possible destruction or rejection of some components, and emergence of new elements, either taken from the environment or internally generated. The Evolutive Systems (or ES) introduced in (Ehresmann and Vanbremeersch 1987) give a model for such systems, based on Category Theory (cf. Mac Lane 1971), a recent domain of Mathematics. An ES describes the successive states of the system at each date of a given time-scale, and the transformations between them; the time-scale can be a sequence of reals, or an interval of the real line. (Thus it describes the closure and the conformation of the system in the sense of Chandler 1991.)

The state of the system at a date t of the time-scale is modeled by a category, called the state-category at t . A category is a graph on which there is given some 'transitivity' of interactions, generated by a law associating to two consecutive arrows, say f from N to N' and g from the same N' to N'' a well-determined arrow $f.g$ from N to N'', called their composite. This *composition law* must satisfy two axioms: the first to ensure that a path of consecutive arrows gives by composition the same unique arrow in whatever way it is 2-2 decomposed (associativity); the second to ensure that each element has an 'identity' arrow.

The state-category embodies all the informations an observer with a complete view of the system could get on it at the date t. The nodes (called *objects*) of this category represent all the components of the system as they appear at t, and the arrows (called *links*) from N to N' represent the various relations between them at this date. Some of these links are more or less invariant 'structural' links such as causal or topological relations (e.g., desmosomes between 2 contiguous cells, or canals through which informations can be transmitted from N to N', as a synapse between two neurons), whatever be the nature of these informations: spatial, temporal or energy constraints, communication exchanges or commands... Other links are more labile connections, representing a temporary interaction between two objects. The composition law, extended to paths, determines which paths represent equivalent information canals between the two extreme objects, so that it delimits the constraints on information processing.

2.3. ES: The Transition Functors

The above category models the state of the system at t. The change of states from t to a later date t' will be modeled by the *transition functor* from this category to the state-category at t'. (A functor is a map which is compatible with the graph structure and the composition law.) The transition functors must satisfy the transitivity of changes: if $t < t' < t''$, the transition functor from t to t'' is the composite of the transition functor from t to t' with that from t' to t''.

For an observer, the transition functor gives informations on the becoming of the components between the two dates. Let us explain this, because it is different from usual models. When we speak, say, of a cell in an organism, we consider that it remains 'the same' cell at different times. Here this cell will be represented not by one object, but by the sequence of all the objects in the different state-categories which represent its successive states; if we denote C_t (the object representing) its state at a time t of its life, these objects are connected by the fact that $C_{t'}$ is the image of C_t by the transition functor from t to t'. In other terms, the cell as a unit is represented not by one of its states, say C_t, but by the trajectory of C_t formed by its successive states. Naturally this trajectory supposes that the observer can 'transcend' the present and compare successive states to recognize symmetrical occurences of an object in different state-categories (which might not be possible internally). Depending on the context and when no confusion might arise, the terms component or object may refer either to a particular state or to the whole trajectory.

Now the cell is destructed at some date s. To represent the possible disappearance of elements or links in time, we suppose that the state-categories possess an *objet 0 on* which are mapped all the components having been suppressed (destructed or rejected out of the system). Thus the death of the cell at s will be represented by the fact that its state C_s becomes 0 while its preceding states were not 0.

The *emergence* of new components (coming from the environment, or internally generated) is also revealed by the transition functors. Indeed, there may exist objects in the state-category at t' which are not the images of any object by the transition functor from t to t'; such objects have emerged at some date between t and t'.

The data of the time-scale, of the state-categories at its different dates, and of the transition functors between them is called an *Evolutive system*.

2.4. Ponderations on a Category

The above representation of a system is geometrical but not quantitative. In concrete applications, quantifications are introduced by labelling the links of the state-categories by real numbers (or possibly by vectors), called their weights (e.g. strength of the link, its capacity as an information canal, its propagation delay...). Then we say that the category is pondered if there is given a law to deduce the weight of a path from the weights of its factors (most often, the weight of the path is the sum, or the product, of those of its factors). The weights may be invariant of the structure, or may be function of time. By analogy with the case of a canal activated when an information is transmitted, we say that a link is *activated* at a time t if its weight increases at t, and quiescent if it does not vary.

A pondered category is often constructed from an initial labelled graph G as follows. We first extend the graph by adding all the paths of G, such a path being weighted by the sum of the labels of its factors. We so obtain the *category of paths* if we define the composite of two successive paths by their concatenation. In this category there may exist several arrows from N to N' with the same weight, i.e., which transmit an activation from N to N' in exactly the same way. This is redundant on a functional perspective. So we will identify two paths with the same extremities and the same weight. After this identification, we obtain a new category in which the objects are still the nodes of G, but in which an arrow from N to N' is a class of paths with the same label. Many structural (or 'geometrical') properties of the system will be deduced from a study of this category, forgetting how it has been constructed and the weights of the arrows, these properties depending only on which paths have the same composite, hence play symmetrical roles to transfer informations. Let us remark that this identification of paths is an essential step for it completely modifies the internal organization of the category. (In particular it allows for the existence of colimits for non trivial patterns, which is very important in the sequel, but is not the case in a category of paths as proved in Ehresmann, forthcoming.)

For instance the *category of neurons* is obtained by such a construction from the graph with nodes the neurons and arrows the synapses (from the pre-synaptic neuron to the post-synaptic one), the label representing the force of the synapse, related to its manner to transmit the influx from one neuron to the other.

3. DIFFERENT MEANINGS OF INFORMATION IN AN ES

In an ES, informations will be processed along the links of the state-categories. However the word 'information' must be given a precise definition in this context.

3.1. Classical Information Theory

The usual meaning of information is "communicated knowledge or news", and so it supposes some intention in the process. In mathematical theories of information, of which the theory of Shannon is the best known, we have an emittor which encodes a message, a canal which transmits the coded message, and finally a receptor decoding the resulting signal. The message can correspond to the transfer of some material object (as a string of letters, or a quantity of motion), or to a modification of the state of the receptor, e.g., by a wave or a vibration. Its information content depends on a statistical evaluation of the probability of each signal used. The aim of the theory is to study the number of bits of information which will be faithfully transmitted in the process, without any regard for the meaning of this information. This pre-supposes some convention between emittor and receptor determining the class of possible messages. Without such a convention, any message would appear as pure hazard, and it is proved (Benzecri 1995) that the practical establishment of an efficient code between actors through action is a complex process.

This theory can be interpreted in a subjective perspective: what is the information gain for the receptor, measured by the difference between its incertitude with respect to the message before the reception, and its incertitude after reception. More objectively, i.e., for an external observer knowing the codes, the information content of the message is its specificity, that is the difference between the variety of all possible messages and the sub-variety of those messages which can lead to the received signal.

3.2. Information through Action

But the notion of information processing which is at the root of the dynamics of complex ES such as biological, social or cultural systems is much more general, in that it resorts of an 'action language', in which the mirror symmetry of receptor and emittor through their common convention is destructed. While usual theories are concerned with the correct material transmission of a message between two actors, here the importance of messages, be they endogeneous (commands, constraints, spontaneous oscillatory processes) or caused by external perturbations, comes from the interactions they generate or in which they participate and the feedbacks they trigger. The same message can be decoded by several actors in a different way; for instance, a color center and a shape center in the brain 'decode' different informations from the same visual target. Conversely, several messages can be decoded as the same signal, e.g., different blue objects are not separated by a color center.

In our categorical model, information transfers are effected through the links between objects. At a date t, an object N can receive informations only through the links to N which are activated at t ; these links form what we call the *reception field* of N . It may send informations to other objects by activating its links toward them, which form its *operation field.* (These two fields are exchanged by the mirror-symmetry which replaces a category by its opposite category obtained by inverting the direction of each link; but in natural systems this formal operation has no meaning.)

In any case the transmission of a message depends on the structure of the link, on its weight (to measure the amplitude or the frequency of the signals), and on its propagation delay which determines the transmission delay. But the activation of a link is not sufficient to speak of information.

3.3. Information Transfer in an ES

The activation of a link from N to N' represents an information transfer only in the following cases:

— If N' receives the signal (a letter can be lost, or written in an unknown language!), decodes it and takes it into account in its subsequent action. The response can be immediate, or delayed, or opposed to the message, or even a non-response (the differentiation of a cell consists in omitting to express some genes). The signal may be non-intentional, such as the traces left by a prey which reveal its presence to a predator, or some instability in a cell, as a change in an oscillatory process, which is transmitted to contiguous cells. For a cell, the signal can also result from the diffusion of a product (say, an hormone) secreted in a far off cell and which diffuses through the circulation or the conjonctive tissue, after a more or less long delay.

— If the emittor N sends a message (constraint, command...) with the specific objective to modify the action of N', and later on receives some feed-back of this action. It is specially the case in human societies, where the sending and reception of the message can be intentional. But natural selection has led to the development of organisms able to send instinctive signals which are decoded as messages by other organisms, possibly after modification during their transfer from the source to the receiver (e.g., emission of pheromones by a female insect to attract the male).

— If there has been established (intentionally or not) some convention between N and N' for a real transfer of informations between them. In this case, we can speak of a communication exchange. But this 'communication' can be an artefact caused by an external observer, who detects that a particular signal determines a correlation between some state of the emittor and some state of the receptor.

4. HIERARCHICAL SYSTEMS

In the definition of a category, all the objects seem to play a symmetrical role. But in a complex natural system, an observer can distinguish all a hierarchy of types: individuals, groups of cooperating individuals able to perform a task that they could not do separately, groups of groups, and so on. For instance, in an organism: atoms, molecules, cells, tissues. How can we distinguish such a hierarchical structure in the purely categorical framework?

4.1. Patterns of Linked Objects

Any cooperation between components of a system requires a possibility of exchange of informations between them. We have said that such exchanges are done through the links of the system. Thus a group of cooperating objects will be represented by a pattern in the state-category. A *pattern* consists of a family of objects N_i , and some distinguished links between these objects. A *collective link* from the pattern to an object N' is a family of individual links f_i from N_i to N', correlated by the distinguished links of the pattern.

Such a collective link models a common action (emission of a message, constraint, command,...) performed by the pattern as a whole, and which could not be realized by the objects of the pattern acting separately. The cooperation can be temporary, as in a group of people who decide to unite to realize a particular task. But if it lasts for a long period, their association can take an identity of its own, and become institutionalized in the system by the formation of a higher order object N (their concatenation in the sense of Chandler 1991), which 'binds together' the pattern and such that the links of N to any N' correspond to the collective links of the pattern.

4.2. Binding of a Pattern into its Colimit

Categorically, this binding object will be modeled by the colimit (or cohesive binding) of the pattern. The colimit (cf. Mac Lane 1971) of a pattern is an object N such that there is a canonical collective link (c_i) from the pattern to N, and each collective link (f_i) from the pattern to any N' binds into a unique link f from N to N' (in formula: $f_i = c_i.f$ for each i). The colimit does not necessarily exist, but if it exists it is unique (up to an isomorphism). For instance, a molecule is the colimit of the pattern formed by its atoms with the distinguished chemical bonds which determine its spatial configuration. Roughly, in the colimit the degrees of liberty of the objects are freezed to ensure a better cooperation along their distinguished links.

The situation can also be seen 'upside-down': an object N which is the colimit of a pattern can be considered as a complex object admitting the pattern as an internal organization into more elementary components, and the pattern will be called a decomposition of N. But while the pattern univocally determines its colimit (if it exists), the inverse is not true because the same object may have several (non-equivalent) decompositions, so that the map associating to a complex object its decompositions is one-to-many. We'll come back on this important property later on. For instance, the same amino acid can be obtained from different codons (degeneracy of the genetic code).

4.3. Limit of a Pattern

Symmetrically, the pattern can act collectively to decode some messages which could not be decoded by its objects separately. We define a common message from an object M to a pattern as a family of individual links from M to the different objects of the pattern, which are correlated by the distinguished links of the pattern. And an object representing the pattern in its capacity of acting as a common decoder will be represented by the limit L of the pattern (defined in the same way as the colimit, but by inverting the arrows).

Though the same pattern can operate collectively as an emittor of messages, or as a receptor, there is a symmetry-breaking in the way its distinguished links operate in both cases, and it is revealed by the fact that the higher objects which represent it in these two roles, namely its colimit and its limit, are different. It follows that several patterns which play symmetrical roles as emittors (they have the same colimit) have different (non-isomorphic) limits, thus play non-symmetrical roles as receptors.

4.4. Hierarchical Systems

An ES is called a Hierarchical ES (or HES, cf. Ehresmann and Vanbremeersch 1987) if each state-category is hierarchical in the following sense: the objects are divided into a finite number of complexity levels, numbered 0, 1, ..., m , so that each object N of a level $n+1$ is the colimit of at least one pattern P of linked objects of the lower level n .

Then N has also a complex organization as an iterated colimit of lower levels. Indeed, each object of P is itself the colimit of a pattern of linked objects of the level $n-1$, so that the links from N to any object can be deduced by an iterative process from the collective links of these lower level patterns; we say N is a 2-*iterated colimit*, or admits a 2-*ramification* based on the level $n-1$. And so on down the ladder, to the 0-level.

However the hierarchy is not purely ascending. The different levels are intertwined, with interlevel and intralevel interactions from lower to higher levels, and vice-versa. An object can receive informations simultaneously from objects of any level, and in response send messages to any level, as long as the energy constraints are satisfied.

For instance, an enterprise has such a hierarchical organization with several levels: The objects of level 0 represent its employees. More complex objects represent departments, from small producing units affected to a specific task, to higher directorial levels. The links between the members of a department represent the canals by which they exchange informations and collaborate to achieve a common goal. The hierarchy is intertwined, because higher levels can send orders to lower levels, but at the same time they can depend on some work done by the lower levels. The construction of a machine is stopped if some material necessary for its construction is not produced in sufficient quantity by a lower-level unit.

5. SYMMETRY-BREAKING BY PASSAGE TO A LOWER LEVEL

We have said that a complex object has several decompositions in more elementary objects. We are going to precise this situation and indicate some of its important consequences.

5.1. Simple Links Binding a Cluster

In a HES, there are some links between complex objects which are deducible from a lower level. Indeed, let N be the colimit of a pattern P and N' the colimit of a pattern P'. A *cluster* from P to P' is a family of individual links between the components of P and P', correlated by the distinguished links of the two patterns, so that this cluster binds into a link from N to N', called a *(P, P')-simple link.* These simple links only transmit informations mediated through individual members of the patterns. A composite of simple links binding adjacent clusters is still a simple link.

In particular, two decompositions P and Q of N are said to be *equivalent* if the identity of N is a (P, Q)-simple link, so that P and Q are connected by a cluster. If moreover Q is a sub-pattern of P, we say that Q is a *representative sub-pattern* of P. Roughly it means that all the collective actions of P are entirely determined by those of Q, the elements of P not in Q being 'constrained' by the comportment of those in Q. An example is given by the Representatives of a nation (whence the terminology).

The notion of a representative sub-pattern is used to define the stability span of an object (cf. Ehresmann and Vanbremeersch 1987).

5.2. Stability Span of a Complex Object

We have seen (cf. 2.3.) how, in an ES, we can keep track in time of a component by its trajectory. Now if the component is a complex object N of level $n+1$, its internal organization of level n at t may be different from its later organization; for instance the 'same' cell perdures though its different components are renewed in time. If we know a decomposition P of N at t, how can we pass from P to a decomposition P' of (the state of) N at a later date t' so that we may recognize directly at the level n that P' is still a decomposition of the 'same' object?

For this, the change in P must be smooth enough, and this 'smoothness' is measured by defining the *span of the decomposition* at t : It is the longest period during which there exists a representative sub-pattern of P whose successive states remain a decomposition of the successive states of N . The *stability span* of N at t is then defined as the lowest upper bound of the spans of its different decompositions at t. This stability span is related to the renewal and degradation rates of the components of N, and N preserves its complex identity as long as this stability span does not become too small. For an organism, the decreasing of the stability span denotes its aging.

The fact that the stability span of N is less than its lifetime emphasizes that, on the long term, there is an evolutionary symmetry-breaking in the passage from a level to a lower level, since the stability of an object during its evolution at its level is not reflected in a stability of its organization of the lower level.

5.3. Multifold Objects

Complexity can also cause another type of symmetry-breaking in the passage to a lower level, more 'structural' since it is observable at a given date as follows.

A complex object generally admits several equivalent decompositions. However there may exist objects N of level n+1 at t admitting also non-equivalent (i.e., non connected by a cluster) decompositions of level n in the state-category at t. Such an object is called a *multifold object* (Ehresmann and Vanbremeersch 1995).

If two patterns represent equivalent decompositions of N, this can be recognized directly at the level n of these decompositions by the existence of a cluster of links between their components. But for non-equivalent decompositions there is no possibility to recognize at their level n that they act as symmetrical emittors; this symmetry emerges at the higher level where only their collective comportment is taken into account. Thus there is a symmetry-breaking in the passage from the 'macrolevel' n+1 (where the patterns are binded into the same object) to the 'microlevel' n where the patterns are not connected (in particular they do not act symmetrically as receptors). For instance 2 different codons of the same amino acid seem unrelated at the atomic level, though they give rise to the same molecule at the molecular level. Or the two possible images in an ambiguous figure gain their 'symmetry' only when they are interpreted in relation to the complete figure, not when apprehended separately.

By analogy with the entropy of a gaze in Thermodynamics, we define the n-entropy of a multifold object of level n+1 as the number of (classes of) its non-equivalent decompositions of level n, thought of as its different possible 'microscopic' states. It is a measure of its structural (or 'horizontal') complexity.

6. MULTIPLICITY PRINCIPLE AND COMPLEXIFICATION

6.1. Multiplicity Principle

We say that a HES satisfies the *Multiplicity Principle* (or MP) if there exist multifold objects of level n+1, and if, conversely, an object of level n can be a component of several objects of level n+1 .

As shown above, the first property entails a *symmetry-breaking in the passage to a lower level*, that might be thought of as a degeneracy (as we say that the genetic code is degenerated). In fact the MP has initially been called the *degeneracy principle*, following a terminology introduced by Edelman (1989) in the case of neural systems. The MP allows for much flexibility in the comportment of the system, and that makes it one of the main characteristics of complexity for evolutionary systems.

For instance, it is related to the well-known though not easily understood scaling and universality properties, which entail that disparate complex systems behave similarly near their critical points though their microscopic comportment is very different. Indeed, near a critical point, all these systems are represented at the macrolevel by a same multifold object.

6.2. Complex Links

In a HES with no multifold objects (so that the MP is not satisfied), a composite of simple links is still simple. But this result does not extend to the HES which satisfy the Multiplicity Principle. Indeed, in such a system, there may exist non-simple links obtained by composing a path of two (or more) simple links which bind non-adjacent clusters; they are called *complex links* (Ehresmann and Vanbremeersch 1995).

More precisely, the composite of a simple link f from N to a multifold object M of level n+1, with a simple link g from M to N' must exist. However, since M is multifold, it admits non-equivalent decompositions of level n , so that f may bind a cluster from a decomposition P of N to a decomposition Q of M, while g binds a cluster from a non-equivalent decomposition Q' of M to a decomposition P' of N'. In this case, the composite of f and g is generally a complex link. For instance, the link from the group of authors of a Journal to the group of its subscribers is a complex link, mediated by the Journal as such, considered as a multifold object representing both its editorial staff and its administration.

Though a complex link of level n+1 is not simple, it can partially been handled at the lower level n, through the links of the clusters that its factors, say f and g, bind, except for the 'switch' between the two non-equivalent decompositions of the intermediate object M; this switch can be recognized as such only at the level n+1, where a symmetry between the two non-equivalent decompositions is generated.

A composite of complex links is generally a complex link (though it might exceptionally be simple).

6.3. Complexification

The change of states in biological systems consists in exchanges with the environment, formation of new objects by association of components, and decomposition of some complex components. For instance for a cell: endocytosis and exocytosis, synthesis of new proteins, decomposition of some products. To model these operations in an ES, we define the complexification of a category with respect to a strategy. In the ES we consider in the sequel, we suppose that the transition functors are obtained by iteration of this complexification process with respect to adequate strategies on the state-categories.

A *strategy* on a category consists in the data of a set of new elements 'to absorb', a set of components 'to destruct', a set of patterns without a colimit 'to bind' and a set of colimits 'to decompose'. Then we construct a functor embedding the initial category into a new category, called its *complexification*, in which the objectives of the strategy are fulfilled in the most economical way, both from an algorithmic point of view and with respect to the energy cost.

The complexification can be constructed explicitly (cf. Ehresmann and Vanbremeersch 1987). In particular in this new category the components to destruct become the 0 object, and there is added a new complex object for each pattern to be binded; this object, which becomes the colimit of the pattern in the complexification, can be thought of as the pattern in itself, taken as an integrated unit. In concrete examples, its formation (e.g., synthesis of a protein) entails the strengthening of the distinguished links of the pattern. The links between these new objects are both simple links binding together a cluster of links between the patterns they bind, and complex links obtained by composing simple links.

6.4. Emergentist Reductionism

In a hierarchical evolutive system, an object N of level n+1 is (by definition) the colimit of at least one pattern P of level n , but also a 2-iterated colimit based on the level n–1, and so on down the scales. In some cases, we can 'skip' the level n , and directly present N as the colimit of a 'large enough' pattern of level n . An important result (Ehresmann and Vanbremeersch 1995) is that such a presentation is possible if all the links of P are simple, but not if some of the distinguished links of P are complex.

Thus the level of an object is not a faithful mirror of its *dynamic* (or 'vertical') complexity. To measure this complexity, we define the *order* of N as the smallest k such that N is the (non-iterated) colimit of a pattern of level k–1 . From the results recalled above, it follows that the existence of complex links (and a fortiori the MP) is a requisite for the existence of objects of order more than 2.

In particular, the objects which emerge in a complexification can be of the same order as the pattern they bind (*horizontal complexification*) or of a higher order (*vertical complexification*). For instance, the synthesis of a protein remains at the molecular level, but Evolution has led to the emergence of more and more complex organisms.

An iteration of horizontal complexifications can be replaced by a unique complexification with respect to a suitable 'large' strategy which subsumes all the successive strategies. But in HES satisfying the MP, we may have an iteration of vertical complexifications which cannot be replaced by a unique complexification, hence which necessitates a 'dynamical' construction in several steps. And it leads to the emergence of objects of strictly increasing orders. This result (Ehresmann and Vanbremeersch 1995) vindicates an emergentist reductionism in the sense of Bunge (1979).

7. MES AND THEIR CRS

7.1. Memory Evolutive Systems

We are going to study how the dynamics of self-organized autonomous systems, such as living systems, is modulated by internal overlapping regulations. In the global description of a HES given up to now, we have seen that there are symmetry-breakings between the levels, so that an internal actor cannot transcend them for having a complete view of the system. It explains that the regulation must be distributed between several cooperative and competitive internal centers of various levels; each one operates at its own time-scale, with some access to a Memory, in which informations on the preceding experiences met by the system are organized for a better adaptation when the same circumstances occur anew.

This situation is modeled by the notion of a *Memory Evolutive System* (or MES, Ehresmann and Vanbremeersch 1991). It is a hierarchical evolutive system, generally satisfying the Multiplicity Principle, in which the evolution is partially regulated by a net of internal organs of regulation, called *Centers of Regulation,* or CR. Each CR is a subsystem of the system whose objects, called *actors*, have a definite complexity level, and cooperatively direct a stepwise dynamic process at a specific discrete time-scale; the CR acts by itself, in the best possible way for it, depending on the informations it can internally decode and encode. But the realization of its strategies is subjected to coherence constraints coming from the confrontation with the strategies of other competitive CRs and of external perturbations. We also suppose that the system contains and develops a sub-system which represents a central hierarchical *Memory* to which the various CRs have a differental access, both for storing and retrieval.

Let us first describe one step of a CR. Its length is determined by the time-scale of the CR, and constitutes the *actual present* of the CR. The step is divided in several more or less overlapping phases.

7.2. The Decoding Phase

In the first phase, the CR acts as an internal observer to form its actual landscape, which represents the informations, coming from receptors of external processes or from various internal components, that the CR can decode during its actual present. It is constructed as follows.

The informations received by a specific actor A are modelled by the links (or aspects) in its reception field which are activated during the actual present of the CR and propagate some signal (e.g., transfer of energy) to A. For instance in a neural system, it could be the arrival of a photon on a cone A of the retine, or the ear vibration generated by a spoken word. In any case, A can only extract some characteristics of an object, because of structural constraints (a photon interacts only with specific molecules), or energy constraints (a neuron must receive a sufficient input to fire), or temporal constraints (during the latency delay of a neuron it cannot receive any information).

But the actors of the CR must respond as a group and not individually. So there is an exchange of informations between them through their distinguished links in the CR; for instance if the CR is a tissue, through direct contact between its cells. Two aspects which are correlated by a zig-zag of distinguished links between the actors give the 'same' information to the CR; we say they are in the same *perspective*. The *landscape* of the CR is the category in which the objects are the perspectives for the CR, and the links come from links in the system compatible with two perspectives. The landscape is an *internal model* of the system for the CR; the possible error in this representation can be measured only *externally*, by the *distortion functor* from the landscape to the system.

7.3. The Decoding Phase

In the second phase of the step, the actors analyze the informations extracted from this landscape and select a common strategy to adapt to the situation. Each actor corresponds to a degree of liberty, but the fact that the actors must act cooperatively introduces constraints transmitted through their communication links. The choice of the strategy is facilitated by a recourse to that part of the Memory which the actors can attain, and in which preceding strategies and their results may have been memorized.

The third phase consists in encoding commands to effectors to realize the selected strategy. The anticipated landscape for the next step should be the complexification of the actual landscape with respect to the chosen strategy. Its adequation will be *internally* evaluated at the next step, by comparing it with the next effective landscape (through the *comparison functor*). If some errors are so detected, one of the objectives of the next strategy will be to correct them. Another objective of this strategy will be to memorize the preceding strategy and its result for the CR (possibly under the form of iterated colimits) to allow for better choices in similar situations later on.

7.4. Information Processing by the CR

To sum up: the CR acts first as a collective decoder to form its landscape. But the signals it so collects from various parts become informations for it only in so far as it uses them to select a strategy. After that, the CR emits commands to realize this strategy, and evaluates the result not directly on the fidelity of the transmission to the effectors, but indirectly in its next landscape, by comparison with its anticipated feedback (whence a loop between the commands sent by the CR and their later effects for it). In some way, we could say that the CR handles the informations in a solipsist manner, to obtain the best return of its internal measurement process (in the sense of Matsuno, 1989). Naturally it does not always succeed, for its actions may be counteracted by the other CRs, up to a premature interruption of its step, if a strategy cannot be found, or if, once selected, it cannot be realized. Then we say that there is a *fracture* for the CR.

On a quantitative (energy) level, the reception of informations in the landscape corresponds to an increase of the weight of a perspective (computed as the mean of the weights of the aspects which compose it), and thus transfers a gain in energy to the actors. This energy is dissipated through the choice and realization of a strategy, so that the energy of the CR is at its lowest at the end of the step. For lower CRs, this mechanism has been described by Schneider (1991) under the name of a "molecular machine", of which an example is the union of EcoRI and DNA leading to a specific cutting of the DNA.

8. EQUILIBRATION BETWEEN STRATEGIES AND MEMORY

8.1. The Interplay among Strategies

Each CR operates its decoding/encoding process on its own landscape, which is only its formal and more or less partial and deformed model of the system. But the effective realization of its strategy involves the system itself. Thus, at a given date, the actual strategies of the different CRs in their respective landscapes are repercuted to the system via the distortion functors. All their commands enter in competition, with more or less conflict, so that an equilibration process is necessary to produce a coherent global strategy which will be effectively realized by the system. This equilibration process is called the *interplay among strategies.* As a result, some commands of the CRs may be rejected or opposed, possibly leading to fractures for the corresponding CRs or even to a blocage of their action.

The interplay among strategies is not directed. It is a free competition, in which each command (link to an effector) of the different strategies intervenes with a specific weight. It also profits from the symmetry-breaking in the passage from a higher to a lower level (entailed by the MP, cf. 6.1.). Indeed, if a command concerns a multifold object, its activation can be done through any of its non-equivalent decompositions, which may have different weights. This allows for a flexibility in the response, depending on the context. For instance if we have to recognize the classical ambiguous figure with a young and an old woman, and if this occurs in the course of a story which speaks of fairies, it will be interpreted as the young woman. Another example is the recurrence of a same word when we write quickly or are tired.

8.2. Development of the Memory

It develops from an initial innate kernel, to retain the results of past strategies for a better adaptation when the same inputs occur anew. This Memory acts. as a detector of symmetry between consecutive inputs or strategies. Among the commands of the strategies of each CR figures the storage of the preceding informations decoded in its landscape, of the commands of the strategy it has chosen and of its result for the CR. If it is accepted after the interplay among strategies, the subsequent complexification adds these items to the existing Memory, under the form of (iterated) colimits and complex links which become images of the corresponding items, and are accessible for a CR under some perspective.

The recognition of an item is induced by the symmetry between this item and its image memorized as (iterated) colimit. Indeed, if the 'same' situation recurs later on in the CR landscape, its image in the Memory is activated, thus allowing its recognition by the CR, and the retrieval from the Memory of a suitable strategy. The recognition and the retrieval of an item can proceed through any of the different (non-equivalent) decompositions or ramifications of its image, thus affording some flexibility to the process. A symmetry-breaking occurs when the strategy who had succeeded before does no more succeed, and then the CR will encode a modification of the Memory at the following step. The memorized strategies and their results form a sub-system of the Memory which is called the *Procedural Memory.*

8.3. Measure of the Information Content

In systems with only a small number of well-adapted comportments and in a stable environment, there is generally few conflicts, the strategy interplay does not cause fractures, and the CR strategies are realized, if not immediately at least with only a small delay. In this case, the Memory remains stable and it allows to quantify the information content decoded by a CR.

Indeed, the informations that a CR could recognize during one of its steps come from the perspectives issued from the Memory in its landscape, and their probability is function of the weight of the perspective. The information content effectively decoded by the CR is measured by a comparison between the number of these perspectives, pondered by their probability, and the number of those perspectives which are activated during the step. The information content encoded by the CR strategy is defined symmetrically, if we consider the perspectives coming from available strategies in the Procedural Memory.

But in changing environments, and for systems with a great number of CRs and a large choice of strategies represented by multifold objects, the strategy interplay can take a great importance and thoroughly modify the comportment of the CRs, thus leading to new strategies and necessitating changes in the Memory.

For instance, in an Ising model, the correlations between the comportment of two spins should decrease exponentially with their distance. However it has been theorized long ago (Gaunt and Domb 1970, and recently verified experimentally by Back et al 1995) that, for a specific 'critical' temperature, the correlations extend to spins far apart, and the rapid exponential decay is replaced by a long-range power-law decay, with a critical exponent which is the same for all an invariance class of systems. In our model it is explained by the fact that, when the number of spins increase, there are more complex links along which they can be correlated. Among the applications of this result: correlations between DNA base-pairs, or lung inflation, even city growth or economics.

9. DIALECTICS BETWEEN HETEROGENEOUS CRS

Here we will distinguish different causes of fractures, and study their implications for the evolution of the system.

9.1. Structural Temporal Constraints of a CR

Each step of a CR may have a different length, in particular this length might be very short if there is a fracture. A step of a CR which is completed without a fracture is called a normal step. We define the *period of* the CR at a given date as the mean length of its preceding normal steps since a fracture. Generally the period of a CR increases with its complexity level.

Lower CRs with few available strategies (e.g., the promotor of a gene in a cell), have a more or less cyclic comportment, with periodical recurrence of the same situations and strategies; and the period remains almost constant. But for more complex CRs, the period may vary, as long as it remains subjected to *some structural temporal constraints* which must be respected for a step to be completed in due time, and avoid fractures.

These constraints are formulated in the form of inequalities (and not equalities): the period of the CR at a given date must be less than the mean time-lags of its actual decoding and encoding processes (measured through the propagation delays of the corresponding links), and more than the stability spans of the objects which send perspectives in its landscape. They leave some flexibility in the operation of the CR and, as long as they are satisfied, its period can be maintained. Though they concern the temporal dimension, they have implications on the energy requirements, because propagation delays and stability spans ultimately depend on the availability of energy resources.

If these constraints cannot be realized during a long enough time, there will be a lasting fracture, or *dyschrony*. Its repair may impose some temporal symmetry-breaking in the form of a change of period, leading to a *de-resynchronisation* of the CR with respect to the other CRs. In particular, we have proposed (Ehresmann and Vanbremeersch 1993) a *Theory of Aging* for an organism, based on such a cascade of de-resynchronisations for CRs at increasing levels; this theory seems to unify the various physiological theories.

9.2. Dialectics between Heterogeneous CRs

Generally there is no direct concertation between two CRs, though higher associative CRs supervise some lower level CRs and may impose strategies on them, possibly to help repair a fracture. But in any case, the comportment of a CR may affect the comportment of another one, through the interplay among strategies and the fractures which might issue from it. Let us consider this situation in the case of two heterogeneous CRs, say a lower level (or micro) CR with a short period, and a higher level (or macro) CR with a long period. Let us consider one step of the macro CR, and its corresponding unique actual landscape.

This macrostep covers many steps of the micro CR, during which it can accumulate changes which are not individually perceived in real time by the macro CR because of the propagation delays, or because each one does not affect the stability of a higher level multifold object due to the symmetry-breaking in the passage from the macrolevel to the microlevel. However their long term accumulation makes the unchanging landscape of the macro CR more and more unreliable, and may ultimately cause a fracture to this CR, for instance if they have progressively destructed all the components of some object which plays a role in its strategy.

To repair its fracture, the macro CR will have to initiate a new step with a new landscape, and a new strategy. In some cases this change of strategy will retroact sooner or later on the micro CR and impose on it a change of strategy, as a consequence of an interplay among strategies in which the new macrostrategy takes precedence over the microstrategy. This back-firing of fractures between the two CRs generates a *dialectics between heterogeneous CRs*, which modulates the dynamics of a MES.

This process explains why long-term prediction is impossible for a complex system (for instance in meteorology). Because of the symmetry-breaking between the levels, the analytic models usually proposed for physical systems can only describe the situation at a specific level, say at the level of a macro CR. In other terms such a model represents not the system as such, but a macro landscape. As we have just seen, the approximation afforded by this landscape, hence by the model, becomes more and more unreliable in time, up to a fracture, at which time the model must be completely modified to accommodate the change of landscape (cf. Rosen, 1985).

The dialectics between CRs has also some bearing on the problem of the irreversibility of time: is the time-arrow an artefact? We cannot discuss this question here. Let us just say that, even if we could reverse the time at the microlevel, the reverting process itself would be transmitted to the macro CR with a latency, as above, so that there can be no common reversibility for the two CRs. And here we only consider two CRs, while a complex system has all a net of CRs which interfere in the interplay among strategies.

9.3. Application to Neural Systems

A neural system can be modelled by a MES based on the category of neurons (defined at the end of 2.4.). A pattern of neurons generally has no colimit in the *category of neurons*, except for some patterns corresponding to important features of the environment or to innate comportments, for instance the grasping reflex of the child. However it can acquire a colimit by a complexification process.

The formation of this colimit corresponds to the synchronization of the pattern, which becomes an assembly of synchronous neurons (in the sense of Hebb 1949, or a neuronal group for Edelman 1989); medical imagery has shown that mental processes activate such assemblies. The colimit, which can be considered as representing the assembly itself, but taken as an integrated unit, has been called a category-neuron (Ehresmann and Vanbremeersch 1991). It takes its own identity in time, and can become activated by other equivalent or non-equivalent (in the sense of 5.3.) assemblies of neurons.

As the Multiplicity Principle is satisfied, successive complexifications can lead to the emergence of a hierarchy of category-neurons of increasing order, integrating assemblies of assemblies of neurons, assemblies of assemblies of assemblies, and so on (cf. 6.4.). In particular in the development of the Memory, there emerge such category-neurons, which represent complex comportments or higher cognitive processes. Their activation consists in the 'dynamical' unfolding of one of their ramifications, down to the neuron level. For a category-neuron of order 1, representing a simple assembly of neurons, it reduces to the physical state in which this assembly is excited. But for category-neurons of order more than 2, the unfolding necessitates at least 2 steps and cannot be effected in a unique step by skipping the intermediate levels (cf. 6.4.); so it does not reduce to a pure physical state, though it may be described as a physical dynamical event, in which all the operations are well characterized. Thus we could be justified to say that mental states 'dynamically' emerge from physical states of brain but are not identical to them (*emergentist monism* in the sense of Bunge 1979).

9.4. Semantics and Consciousness

In the neural system, the net of CRs contains lower CRs corresponding to different areas of the brain decoding some specific informations, for instance a color CR, or a shape CR, supervised by higher associative CRs of increasing complexity. The actors of a CR can be neurons or, in higher CRs, category-neurons.

The CRs can act as detectors of symmetries between items in the Memory. We cannot describe here how this operation is done in a 2-step process, first leading to an 'acted' classification, which (in the case of higher animals) is internally reflected so that each class of symmetrical items becomes represented by a new category-neuron, called its *concept* (it is the limit of the pattern of actors activated by the items of the class; cf. Ehresmann and Vanbremeersch 1992).

The concepts form a sub-system of the Memory, called the *Semantical Memory*. The activation of a concept relies on a double indeterminacy: firstly choice of a particular instance of the concept, and then choice of a particular decomposition of this instance, by-passing twice the symmetry-breaking in the passage from higher to lower levels. Thus the development of this semantical memory affords a greater flexibility, e.g., in an interplay among strategies chosen under the form of concepts.

This flexibility, which makes the animal more adaptable, could be at the root of the emergence of consciousness. We have proposed to say that a CR is *conscious* if it is able, by an increase of attention after a fracture: (i) to extend its actual landscape retrospectively to past lower levels; (ii) to operate an abduction process in this extended landscape to find the probable cause of the fracture; (iii) and finally to planify a strategy for several steps ahead, through the formation of internal 'virtual' landscapes in which strategies (as concepts) can be tried without energy costs to realize them. In this sense, consciousness would amount to an internalization of Semantics and Time which allows to punctually transcend both the stream of time and the complexity symmetry-breakings, thus giving a selective advantage.

REFERENCES

Back, C. H., Wursch, C. H., Vaterlaus, A., Ramsperger, V., Maler, U. and Pesire, D. (1995) Experimental confirmation of universality for a phase transition in two dimensions, *Nature*, **378**, 597-600.

Benzecri, J.-P. (1995) Convergence des processus..., *Cahiers de l'Analyse des données*, **20** (4), 473-482

Bunge, M. (1979) *Treatise on Basic Philosophy*, Vol. 4, Dordrecht: Reidel,

Chandler, J. L. R. (1991) Complexity· A phenomenological and semantic analysis of dynamical classes of natural systems, *WESS communications*, **1**(2), 34-42.

Chandler, J. L. R., Ehresmann, A. C and Vanbremeersch J.-P (1995) Contrasting two representations of emergence of cellular dynamics, In. Farre, eds., *Proceedings Symposium on Emergence, InterSymp'95*.

Edelman, G. M. (1989) *The Remembered Past*, New York. Basic Books

Ehresmann, A C. and Vanbremeersch, J.-P. (1987) Hierarchical evolutive systems, *Bulletin of Mathematical Biology*. **49** (1), 13-50.

Ehresmann, A. C. and Vanbremeersch, J.-P. (1991) Un modèle pour des systèmes évolutifs, *Revue Internationale de Systémique*, **5** (1), 5-25.

Ehresmann, A. C. and Vanbremeersch, J.-P (1992) Outils mathématiques pour modéliser les systèmes complexes, *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, XXXIII-3, 225-236.

Ehresmann, A C. and Vanbremeersch, J -P. (1993) Rôle des contraintes structurales; In: *AFCET* 1993 , Vol. 8, Versailles, 103-112. (It also contains a list of all our preceding papers, with a guide through their maze.)

Ehresmann, A. C and Vanbremeersch, J.-P. (1995) Multiplicity principle and emergence in MES, *Journal of Systems Analysis, Modelling, Simulation*; to appear. (Condensed translation of *Emergence et Téléologie*, Amiens 1993.)

Farre, G L. (1994) Reflections on the question of emergence, In *Advances in Synergetics*, Volume I: Systems research on emergence, Windsor The International Institute for advanced studies in Systems research and Cybernetics.

Gaunt, D S. and Domb, C. (1970) *Journal of Physics* C, **3**, 1442-1461

Hebb, D. O (1949) *The Organization of Behaviour*, New York: Wiley.

Mac Lane, S. (1971) *Categories for the Working Mathematician*, Berlin: Springer.

Matsuno, K. (1989) *Protobiology: Physical basis of Biology*, Boca Raton: CRC Press.

Matsuno, K. (1994) Hidden symmetries, symmetry-breaking and emergence of complexity, In: Gruber, B., ed., *Symmetries in Science* VIII, New York Plenum.

Rosen, R. (1985) *Anticipatory Systems*, New York: Pergamon.

Schneider, T. D. (1991) Theory of molecular channels (I and II), *Journal of Theoretical Biology*, **148**, 83-137.