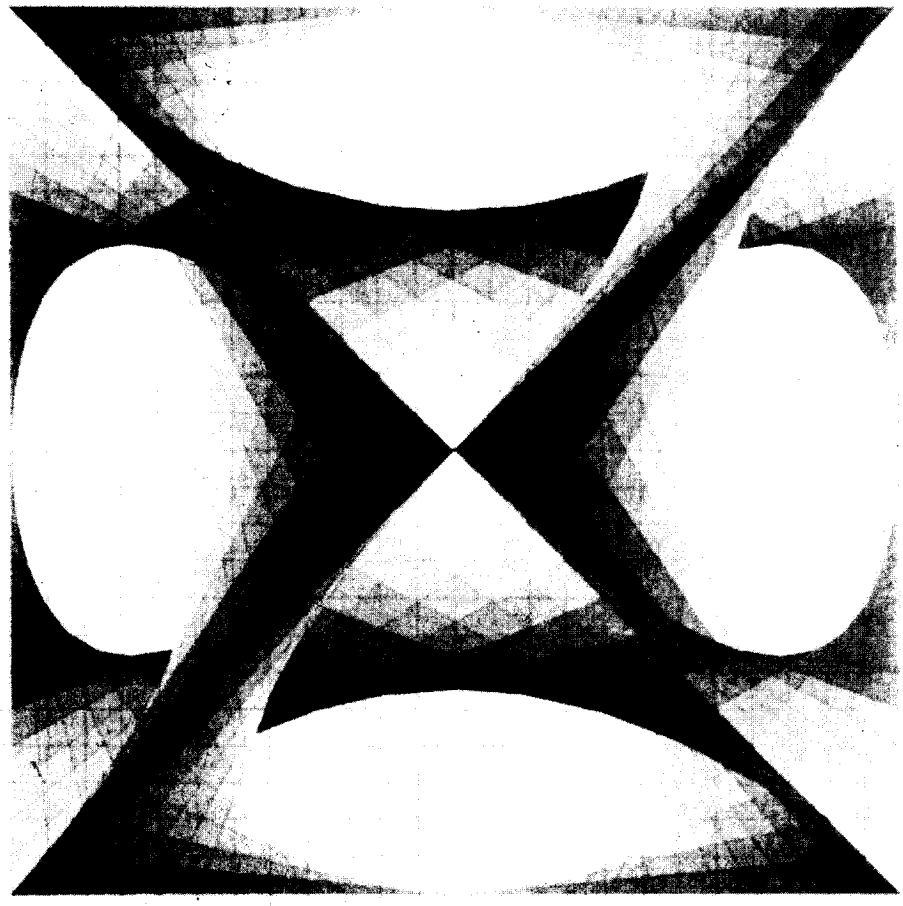
# Symmetry: Culture and Science

Wasan

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Volume 8, Number 1, 1997



*Symmetry: Culture and Science*
*Vol. 8, No. 1, 74-112, 1997*

*SYMMETROSPECTIVE*

# GOLDEN SECTION(ISM): FROM MATHEMATICS TO THE THEORY OF ART AND MUSICOLOGY
## PART 2

*In memoriam Ernő Lendvai (1925-1993)*

Dénes Nagy

Institute of Applied Physics
University of Tsukuba, Japan 305

*Continued from the Vol. 7, No. 4, 1996 issue.*

### 2.7 The "golden section" in the 19th century: when was it coined?

Surprisingly, the first documented terms for the extreme and mean ratio or divine proportion that refer to "gold" appeared in German mathematical works as late as the 1830s:

- 1833: *der goldene Schnitt* (golden section, in German) is used by F. Wolff in his *Lehrbuch der Geometrie* (Textbook of Geometry, in German, Berlin, p. 127).
- 1835: the same expression is mentioned by M. Ohm, the brother of the physicist G. S. Ohm, in his book *Die reine Elementar-Mathematik* (The Pure Elementary Mathematics, in German, 2nd edition, Berlin, p. 194, footnote; the 1st edition of 1826 does not have this term);
- 1839: *sectio aurea* (the Latin version of the same term) is given as an alternative to the *sectio divina* by J. F. Kroll in his book *Grundriss der Mathematik* (An Outline of Mathematics, in German, Eisleben, p. 189).

While most people believe that the term "golden section" is ancient or medieval, some historians of science and linguists pointed out that the term is relatively new and they credited M. Ohm's book as the first known usage in a printed work. The latter was pointed out in a detailed terminological study by J. Tropfke, a German high school director and the initiator of a reform in mathematical education, who authored a seven-volume series of books on the history of elementary mathematics (cf., *Geschichte der Elementar-Mathematik*, 2nd edition, Vol. 4, Berlin, 1923, pp. 186-187). Some decades later the Belgian-American G. Sarton, one of the 'fathers' of history of science as a new discipline, made a query "When did the term 'golden section' or its equivalent in other languages originate?" , but this did not produce any additional result (*Isis*, 42 (1951), p. 47), similar to some more recent studies. The view that the first known usage is due to M. Ohm has got a new support by the famous *German Dictionary* that was initiated by the Grimm brothers, the authors of collections of folk tales and philological works, and completed by generations of scholars in a period of more than one-hundred years (cf., "der goldene Schnitt" , *Deutsches Wörterbuch*, in 16 vols., by J. and W. Grimm, Vol. 4.1.5, Leipzig, 1958, columns 759-760; Reprint edition in 33 vols., Vol. 8, München, 1984, columns 759-760). The German usage of the term "golden section" in 1835 as the first one is also accepted by the largest English and Italian dictionaries (cf., "Golden section" , *Oxford English Dictionary*, in 12 + 5 vols., *Supplement*, Vol. 1, 1972, p. 1257, or 2nd edition in 20 vols., Vol. 6, 1989, p. 656; and "Sezione aurea" , *Grande dizionario della lingua italiana*, Vol. 18, 1996, p. 830). Thus, it is interesting that we have an earlier example. Although F. Wolff's book (1833) is not much older than M. Ohm's work (1835), there are some important consequences. It is not necessary to study M. Ohm's life and work in order to get more information on the roots of the term "golden section" , but we may refocus our survey. Indeed, M. Ohm did not use this term in the first edition of his book (1826) and he mentioned it in the 2nd edition just in a footnote (1835). The usage by F. Wolff is not only earlier, but also more direct: he introduces this term in the main text as an alternative to the extreme and mean ratio (1833):

> *Man nennt diese Construction* [sic] *auch den goldenen Schnitt.*
> (This construction is also called the golden section).

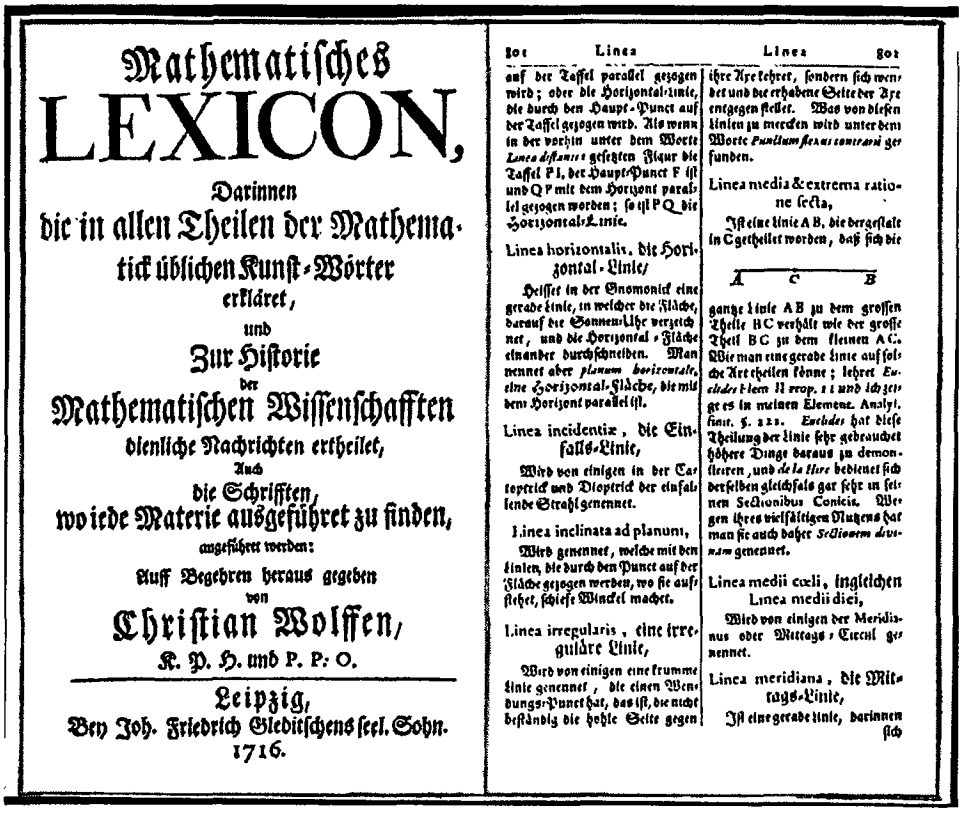
We may find many examples for the Latin *sectio aurea* in the German mathematical literature of the mid-19th century: beyond the cited example of 1839, it is also available, among others, in the title of a German mathematical-educational paper written by J. Helmes in 1844 (*Archiv der Mathematik und Physik*, 4 (1844), pp. 14-22). The latter is the earliest example for the Latinized term in the *Oxford English Dictionary*, although, as we

have seen, J. F. Kroll used it some years earlier. Incidentally, the possible reason of using the Latin *sectio aurea* in German contexts was very simple: the authors demonstrated that this mathematical concept (not the term!) is ancient. A German scholar, J. Bochenek, became so enthusiastic with such philological tricks that in 1885 he also created an 'original' Greek version for the same expression: *khrusê tomê*, i.e., the golden section. The first known book that includes the term "golden section" in its title is a 16-page mathematical brochure by A. Wiegand: *Der allgemeine goldene Schnitt und sein Zusammenhang mit der harmonischen Theilung* (The Common Golden Section and its Connection with the Harmonic Division, in German, Halle, 1849).

Note that F. Wolff does not claim that the term "golden section" is his own, but he refers to it as an existing expression. Perhaps his usage is still not the very first one, but there are precursors. On the other hand, there are some indirect pieces of evidence that this term was not available much earlier. The most convincing one is the lack of this term in Ch. Wolff's article on the extreme and mean ratio in his *Mathematisches Lexicon* (Leipzig, 1716, Fig. 3).

Wolff also wrote a textbook on mathematics which discussed the topic in a very broad context, including the problems of architecture and engineering: *Anfangsgründe aller mathematischen Wissenschaften* (Elements of All Mathematical Sciences, in German). This very influential textbook had seven editions between 1710 and 1757. Although the book is in German, he did not use any German expressions for the extreme and mean ratio beyond. Specifically, the "Register" of the 7th edition lists two related terms and both of them are Latin: *media et extrema ratione secare* and *sectio divina*).

In the 1830s the term "golden section" was not yet widely used in the German mathematical literature. Thus, the German translation of M. Chasles's French book on the history of geometry, which was very popular in that time, uses Pacioli's *proportio divina* and the German version of the extreme and mean ratio, without referring to the new German term (*Geschichte der Geometrie*, Halle, 1839, pp. 598 and 637; translated by L. A. Sohncke, whose son, L. Sohncke, later became a pioneer of the study of symmetry groups in crystallography). Last, but not least, the term "golden section" was not available in the artistic-aesthetical circles according to the very detailed survey by A. Zeising. In his book of 1854, which we will discuss in Chapter 3.1, he claims that this term is used by mathematicians and he does not know the reason of its name.



Mathematisches
LEXICON,
Darinnen
die in allen Theilen der Mathematick üblichen Kunst-Wörter erkläret,
und
Zur Historie der Mathematischen Wissenschafften dienliche Nachrichten ertheilet,
Auch
die Schrifften, wo iede Materie ausgeführet zu finden, angeführet werden:
Auff Begehren heraus gegeben von
Christian Wolffen,
K. P. H. und P. P. O.
Leipzig,
Bey Joh. Friedrich Gleditschens seel. Sohn.
1716.

go Linea Linea go

auf der Tafel parallel gezogen wird; oder die Horizontal-Linie, die durch den Haupt-Punct auf der Tafel gezogen wird. Als wenn in der vorhin unter dem Worte *Linea distantiæ* gesetzten Figur die Tafel PL, der Haupt-Punct F ist und QP mit dem Horizont parallel gezogen worden; so ist PQ die Horizontal-Linie.

Linea horizontalis, die Horizontal-Linie,

Heisset in der Gnomonick eine gerade Linie, in welcher die Fläche, darauf die Sonnen-Uhr verzeichnet, und die Horizontal-Fläche einander durchschneiden. Man nennet aber *planum horizontale*, eine Horizontal-Fläche, die mit dem Horizont parallel ist.

Linea incidentiæ, die Einfalls-Linie,

Wird von einigen in der Catoptrick und Dioptrick der einfallende Strahl genennet.

Linea inclinata ad planum,

Wird genennet, welche mit den Linien, die durch den Punct auf der Fläche gezogen werden, wo sie aufstehet, schiefe Winckel machet.

Linea irregularis, eine irreguläre Linie,

Wird von einigen eine krumme Linie genennet, die einen Wendungs-Punct hat, das ist, die nicht beständig die hohle Seite gegen ihre Axe kehret, sondern sich wendet und die erhabene Seite der Axe entgegen stellet. Was von diesen Linien zu mercken wird unter dem Worte *Punctum flexus contrarii* gefunden.

Linea media & extrema ratione secta,

Ist eine linie AB, die dergestalt in C getheilet worden, daß sich die

A C B

ganze Linie AB zu dem grossen Theile BC verhält wie der grosse Theil BC zu dem kleinen AC. Wie man eine gerade linie auf solche Art theilen könne; lehret *Euclides* I. 11 prop. 2 und ich zeige es in meinen Elementis Analyt. finit. §. 222. *Euclides* hat diese Theilung der Linie sehr gebraucht höhere Dinge daraus zu demonstrieren, und *de la Hire* bedienet sich derselben gleichfals gar sehr in seinen Sectionibus Conicis. Wegen ihres vielfältigen Nutzens hat man sie auch daher *Sectionem divinam* genennet.

Linea medii cœli, ingleichen Linea medii diei,

Wird von einigen der Meridianus oder Mittags-Circul genennet.

Linea meridiana, die Mittags-Linie,

Ist eine gerade Linie, darinnen sich

**Figure 3**: The page with the article "Linea media & extrema ratione secta" (Line cut [according to] the mean and extreme ratio) in Christian Wolff's *Mathematisches Lexicon* (Leipzig, 1716). Since this is the first mathematical dictionary in German, Wolff is eager to discuss terminological questions and to popularize German expressions. Indeed, most keywords are given in both Latin and German. However, the heading of this article and its text do not refer to any German term; just the Latin *media et extrema ratio* and *sectio divina* are used. The preference of the latter term could have been an influence from Kepler (Wolff had close links to M. G. Hansch who inherited Kepler's manuscripts and started to publish them). Although Wolff deals with questions of art and aesthetics in some other articles (see, e.g., *Eurythmia / die Wohlgereimheit*, column 609, *Symmetria / die Symmetrie*, columns 1342-1343), in the case of the extreme and mean ratio he does not give such remarks. All of his references are mathematical works (Euclid, de la Hire, and his own monograph). Note that Christian Wolff (1679-1754), who became a professor in 1706 following the recommendation by Leibniz, authored a large number of books in German and Latin on various topics, including philosophy, law, mathematics, science, and even architecture and civil engineering.

Since all of the known early usages of the expression "golden section" point into the direction of mathematical education, we may accept that the new term spread from that field: either teachers of mathematics coined the term or they adopted it from an obscure source and then popularized it. The fact that the same mathematical concept had various Greek, Latin, and Italian names, which had further German versions, was very difficult for

teachers of mathematics. They obviously needed a unified term, an impressive one that helps the inspiration of students. Their first trial was the term *stetige Teilung* or *Proportion* (continuous division or proportion), which was popularized by J. H. Lorenz's German translation of Euclid (1781 and 1798), but it could have had some inspiration from W. Schmid's *stäte Proportz* (1539), Kepler's *proportio continua* (1608). On the other hand, this term is not very imaginative and even misleading since any proportion $a/b = c/d$ can be considered as continuous. Even if we require that the first ratio $a/b$ should be 'continued' by $b$, we get $a/b = b/c$, i.e., the geometric mean. In fact, Kepler speaks about the "divine proportion" as one among the "continuous proportions" . For the given purpose the new adjective "golden" is an optimal choice since it has some association with all of the earlier names (extreme and mean ratio, divine proportion, continuous proportion). In fact, gold is

- a noble metal with many *extreme* properties,
- a typical symbol of *divinity*, and
- a material associated with *continuity* since it does not corrode or tarnish and even its value remains relatively stable.

We may even say that the phrase *goldener Schnitt* is a German version of Kepler's *sectio divina* (literally *göttlicher Schnitt)* where the religious connotation is eliminated: *Gott*, i.e., god, is substituted by the phonetically similar *Gold.* Note that the German language gives a special freedom to use the adjective "*gold*" for the purpose of referring to positive properties, as it is illustrated by a large number of family names, including Goldbach, Goldberger, Goldfinger, Goldhaber, Goldmann, Goldmark, Goldschmied, Goldstein, and by various objects in the cited Grimm brothers' fairy tales, from golden eggs to golden leaves. (Incidentally, pro-golden-section researchers did not miss these tales: their analysis suggest that among the total 585 characters in 125 tales the ratio of the negative ones to the positive ones is close to the golden section, cf., J. Benjafeld and C. Davis, "The golden section and structure of connotation", *Journal of Aesthetic and Art Criticism*, 36 (1978), 423-427.) Even the idea of connecting *Gold* and *Schnitt* appeared in a different sense: the gilt edge of books. Although the two cannot be confused, still there is a tendency to make a small orthographic difference between the mathematical term and the expression related to books: der *goldene Schnitt* versus *der Goldschnitt*. Last, but not least there was an essential factor that could have encouraged the usage of the adjective "*golden*" in the case of the extreme and mean ratio. This is the existence of some well-established terms with similar structure, especially the following two ones:

- golden rule (Latin *regula aurea* [1468], German *die goldene Regel* [1483]), i.e., the 'rule of three', the method of solving the equation of the proportion $a/b = c/x$ for $x$ (cf., Tropfke's cited book and the Grimm brothers' *Deutsches Wörterbuch*), or, in a broader

sense, any important guiding principle (e.g., the Biblical statement "so whatever you wish that men would do to you, do so to them" , Matthew 7:12),
-golden mean (Latin *aurea mediocritas* [Horace, 1st c. B.C.], German *die goldene Mitte*), i.e., the way between extremes; note that D'Arcy Thompson used this expression as the English equivalent of the *sectio aurea* in the first edition of his book *On Growth and Form* (Cambridge, 1917).

The fact that the extreme and mean ratio is a gemstone, or even a gold-nugget, of mathematics, as Kepler claimed, is a further justification of the new terminology.

According to the *Oxford English Dictionary*, the English term "golden section" first appeared in the article "*Aesthetics*" of the *Encyclopaedia Britannica* (9th edition, Vol. 1, 1875, p. 220), specifically in a reference to the German author Zeising and his theory. We will return to the latter very soon (Chap. 3.1). Although we cannot fully exclude the possibility that an equivalent of the "golden section" is still available in much earlier texts, we may clearly state that before the 19th century such an expression did not become a conventional one in the mainstream of the literature. The fact that even the seemingly classical expression *sectio aurea* was not a well-established ancient or medieval term, but it spread as the Latin version of a 19th-century German expression (sic!), is quite surprising. Indeed, the opposite view, together with an additional belief that ancient artists and then Renaissance 'masters' in Pacioli's circles applied the golden section intensively, became so widespread that they became an eternal part of the 'mathematical folklore'. Thus, the legend that the Greek sculptor Pheidias was involved in the use of the golden section was lifted to a historic fact by using the Greek letter "*phi*" , the initial of his name, for the modern mathematical notation of the golden number. The alternative version is "*tau*" from the Greek word *tomê* (section). As we have seen, the latter expression was used by the mathematician Proklos, but its meaning is not definitely the golden section. The usage of these two Greek letters does not harmonize with the fact that the Greeks had not even a simple expression for the golden section. What can we do if the facts are so much mixed with the legends? Of course, we should not ignore some 'golden' aspects of this topic. On the other hand, we should consider the golden section without the 'golden legends'...

# 3 GOLDEN SECTIONISM IN ART THEORY

## 3.1 Zeising's "basic law of morphology" and the age of universal principles

From the mid 19th century there was a special interest in finding some universal principles and theories in various fields of art and science. This was very strong in the German speaking countries where the 'golden sectionism' was started. Probably the socio-political situation, the struggle for uniting the German states, was also a component, at a general cultural level, of looking for unifying principles. Starting in the 1830s, there was a growing interest in the golden section in the German states, which was also marked by the presence of various factors that are usually necessary for a 'golden age' of an idea:
- new results: mathematical theorems associated with the golden section and Fibonacci numbers, modern interpretation of old results in textbooks,
- new interdisciplinary connections: the discovery of the importance of the same concepts in botany, specifically in the case of the arrangement of leaves (phyllotaxis, cf., Chap. 1 and Fig. 1), later the aesthetical dimensions of the topic,
- good terminology: the expression goldener Schnitt solved the problem of terminological confusions and spread very quickly,
- 'management' of the topic: first teachers of mathematics and some botanists gave an enthusiastic group of supporters, later they were joined by some authors who popularized the topic (together with some overstatements!) in the broadest context of art and science.

The first comprehensive book that discusses the golden section as a basic law penetrating nature and art, and where we may speak about an -ism as a distinctive doctrine, was Adolf (Adolph) Zeizsing's book of 1854. The author of this monograph was one of the very few polymaths of his age: he published aesthetical and mathematical works alike, wrote poems, novels, and translated Xenophon's work from Greek into German. His book referred to has an unusually long title: *Neue Lehre von den Proportionen des menschlichen Körpers, aus einem bisher unerkannt gebliebenen, die ganze Natur und Kunst durchdringenden morphologischen Grundgesetze entwickelt und mit einer vollständigen historischen Uebersicht der bisherigen Systeme begleitet* (A New Theory of the Proportions of the Human Body, Developed from a Hitherto Unrecognized Basic Law of Morphology Penetrating the Whole Nature and Art and Accompanied by a Complete Historical Survey of the Earlier Systems, in German, Leipzig, 1854, xxii + 457 pp.). There are many striking things in this title. The author speaks about a new theory with an unrecognized law and he makes clear that this statement is based on a detailed survey of the earlier systems. He does not use the expression "golden section" in the title, but emphasizes the aspect of human proportions. Zeising introduces a whole system based on Fibonacci numbers (Fig. 4).

Indeed, it was based on some interesting observations: later various statistical data confirmed that in the case of European humans the navel divides the height according to a surprisingly good approximation of the golden section.

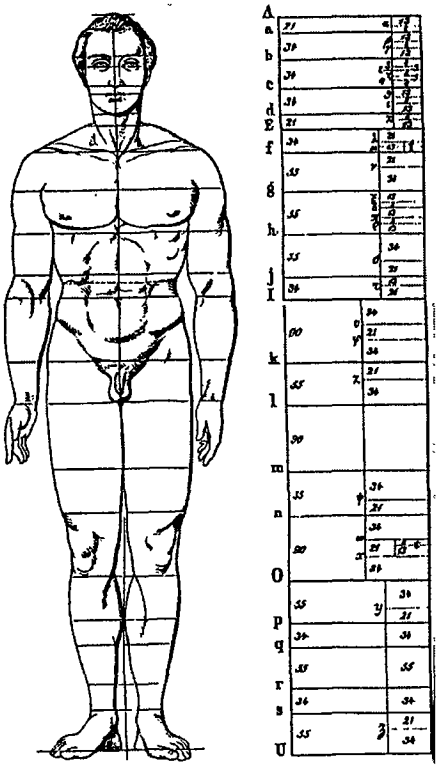


**Figure 4**: Zeising's system of human proportion from his book of 1854 (Figures 49 and 50, pp. 214-215). Although some of the division lines are seemingly subjective, he gives an anatomical definition of each of them (in German and in Latin). Note that 34 + 21 + 34 should be 89, not 90 (cf., the line segments between *I* and *K*, *n* and *O*, and perhaps *l* and *m*). After this correction, we have Fibonacci numbers elsewhere. Incidentally, this "mistake" was made by purpose. Zeising slightly modified the Fibonacci sequence to reach 1,000 for easy calculations (the first row is the Fibonacci sequence, the second is modified one):

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

(the same) 90, 145, 236, 381, 618, 1,000

Zeising's book emphasizes the aspects of anthropomorphism in art and science. It would be interesting to study Zeising's possible influence in this context, including Darwinism in biology and Positivism in philosophy of science, although he was not directly associated with these. We also should see the possible links between this aesthetical anthropomorphism and the dangers of racism.

On the other hand, Zeising tries to extend the importance of the golden section and Fibonacci numbers excessively to all details of the human body and then, as a following

step, to all fields of nature and art from crystals to animals, from sculptures to buildings. Although one may feel an obsession here, the actual contents of the book is less dogmatic than the title suggests. Zeising's book clearly deserves some credit for a rich set of data: a 130-page historic survey of aesthetical works from the Pythagoreans to Hegel (surprisingly Pacioli is missing!), very many interesting references and quotations in Greek, Latin, Italian, French, English, and German, informative tables of proportional systems, and last but not least 177 wood-cut illustrations. The concept of the golden section is introduced only on pages 159-163. This presentation strongly differs from many later books where the 'extraordinary properties' of the golden section, or another 'universal' proportional system, are declared on the first pages and then this proportion dominates all of the rest of the text. Indeed, Zeising does not see elsewhere and exclusively the perfect golden section. For him the golden section is a general tendency that appears with relatively good approximations. He even includes a full chapter "On the modifications of the lawful proportions by sex, age, nationality, and individuality" (pp. 296-320).

One may say that Zeising's mathematical aesthetics shows some analogy with the later approach in mathematical crystallography:
- first, as a general law, the symmetry groups of the geometrically ideal structures are discussed,
- then the necessary modifications are considered for describing real crystals.

There is no doubt that nature is so complex that the first mathematical models may target just the "ideal" cases. However, this approach is dangerous in the case of human bodies: it is easy to develop the related statements into the direction of racism as ideal human proportions versus bad ones. Unfortunately, Zeising also made such steps. He lived in an age when travellers went to distant regions and carefully recorded various data. The suddenly discovered "variety" of humans initiated many papers that compared the anthropological data of different groups of people. Zeising was inspired to use his golden section theory and to explain the differences in such terms. These works, of course not only by Zeising, made a 'hotbed' of prejudice and even a 'scientific basis' of the later *Übermensch* theory in Germany. This is such a problem that the community of scholars never should forget: seemingly naive theories may become extremely dangerous in the hand of the 'wrong people'. We must have a unity among scholars that prevents such developments!

Zeising, one year after the publication of his book on the golden section, embedded his theory into a general aesthetical context in a second monograph *Aesthetische Forschungen*

(Aesthetical Researches, in German, Frankfurt am Main, 1855, xv + 568 pp.). Here he dealt with the golden section just on a few pages in the framework of a much broader topic. Specifically, he considered regularity (symmetry), proportionality, and expression (character) as the three basic concepts that make harmony among the formal elements. The golden section is introduced in the sub-chapter on proportionality. However, this book is not just a work on formal elements. Zeising also studied various other fields of aesthetics. Thus, he devoted long chapters to the manifestations of the comic, tragic, sublime, charm, and humor, and also discussed the relationship among different fields of art. We emphasize this fact because a large number of authors claim that the book *Aesthetische Forschungen* is a second monograph by Zeising on the golden section. Perhaps they never have seen this book and did not believe that Zeising was able to switch to different topics so quickly. On the other hand, Zeising, an extremely prolific writer, also continued to publish on the golden section. Interestingly, he himself never used the expression "golden section" in the titles of his books, with the exception of the posthumously printed brochure *Der goldne* [sic] *Schnitt* (The Golden Section, in German, Halle, 1884, 28 pp. and 1 table). Surprisingly, this last work does not deal with aesthetical questions at all, but focuses on mathematical problems. S. Günther, a leading historian of mathematics, published a paper "*Adolph Zeising als Mathematiker*" (Adolph Zeising as a mathematician, in German) and introduced him as a founder of mathematical aesthetics (*Zeitschrift für Mathematik und Physik, Historisch-literarische Abtheilung*, 21 (1876), pp. 157-165). Perhaps Zeising's success was also associated with the fact that many other scholars had a similar interest in bridging art and science. A notable example is J. Metzger's book *Gesetze der Pflanzen- und Mineralienbildung angewendet auf altdeutschen Baustyl* (Laws of Plants- and Minerals-Formation Applied for Old-German Building-Style, in German, Stuttgart, 1835, viii + 24 pp. and 8 tables). This book discusses various aspects of symmetry and spiral forms, from crystals to leaf arrangement, but without touching on the golden section and the Fibonacci numbers. While Metzger remained at the level of interesting analogies between nature and architecture, Zeising went beyond this: he provided a special focus and built a theory around this. Obviously, it was very much appealing for the general public. Many people became delighted, together with Zeising, that they may understand the beauty of a large number of art works by a single theory. Some artists, usually not the great masters, were also enthusiastic by the fact that Zeising offered them some 'help' from outside. The acceptance of Zeising's theory by a relatively large number of people is not surprising: his fine scholarly work in various details concealed the basic troubles.

### 3.2 Fechner's restriction and new direction; the spread of golden sectionism in Germany and Russia (with an outlook to other works in Hungary, the United States, and France)

In the 1860s and 1870s the golden section, together with some criticism and change of direction, reached an even broader field of aesthetics via the experimental-psychological works by G. T. Fechner, one of the most influential, but also controversial psychologists. Perhaps it is useful to introduce him very briefly, since his biography would give an insight into his unusually broad interest. Originally Fechner graduated as a medical doctor, but became a physicist at the University of Leipzig. In the same time, he also published satirical writings and spiritual works, as a reaction against growing materialism. Indeed, this period is marked not only by his noted experimental-physical work on the Galvanic battery (1831), but also by his booklets on the comparative anatomy of angels (1825) and on life after death (1835). Some years later he became seriously neurotic and resigned from his position as a professor of physics. After a long illness, Fechner suddenly recovered and started a new carrier. He published books on the soul life of plants (1848) and the matters of heaven (1851), which are not obvious topics for a scientist again, and concluded that mind and matter should be studied together. This interest led him to the hypothesis that there is a logarithmic relationship between sensation ($S$) and stimulus ($R$)

$$S = k \log R$$

where $k$ is constant (Weber-Fechner law). Fechner went on and shaped the new field of psychophysics in a period of ten years, which culminated in his first monograph on this subject (1860). Then, suddenly he turned to a new topic and laid down the basis of experimental aesthetics. Perhaps Zeising's theory on the golden section and its overstatements played a decisive role at this turning point. An interesting document of this period is a rarely cited paper by Fechner, which was published in an obscure journal: "*Ueber die Frage des goldenen Schnittes*" (About the question of the golden section, in German, *Archiv für die Zeichnenden Künste*, 11 (1865), pp. 100-112). Fechner, as an experimental scientists, enthusiastically measured the dimensions of very many pictures and produced statistical data that disproved the universality of Zeising's theory. His talent as a satirical writer also sounds here: are our spoons and forks are based on the golden section? Even his interest in life and death, as well as in angels, has a say: let us study the proportions of crosses on the graves. Zeising theory again failed. On the other hand, Fechner also realized some importance of the golden section. He outlined a new direction of research: he recommended to study much less complex objects than master pieces of

great artists and initiated tests where the subjects should choose from a given set of crosses our rectangles. Fechner's main conclusion is that Zeising's theory on the golden section should be restricted. In this way it may remain interesting, a really important discovery. He also explained his view on a new experimental aesthetics that should be built from "below" , instead of projecting an abstract theory to art works from the "top" . These were not just empty words, but Fechner's first step towards a new field of study that dominated the next decade of his life and sidelined the psychophysical studies for a long period. Fechner measured the frames of about 20,000 paintings and conducted many tests in two different ways:

- method of choice - the subjects were asked to choose from a given set of simple figures, for example rectangles,
- method of construction - the subjects were asked to construct figures.

He summarized these results in the frameworks of his two monographs on experimental aesthetics (*Zur experimentelle Aesthetik*, Leipzig, 1871; *Vorschule der Aesthetik*, Leipzig, 1876). In the most widely cited test, for example, he investigated the preference in the case of ten randomly arranged rectangles with the same area, where the ratios of their sides varied from 1/1 (square) to 2/5. He found that about 35% of both men and women chose the golden rectangle (actually the ratio of the two sides was 21/34 = 0.617...) and a further 44% of men and 34% of women preferred the 'neighboring' two rectangles (2/3 = 0.666... and 13/23 = 0.565...). In the case of the test for rejection, nobody choose the golden rectangle, while many people 'disliked' the square and the longest rectangle.

Perhaps Fechner's studies became essential that the golden section theory survived. His pragmatic move of restricting the importance of the golden section made the whole approach more reliable, while a large number of 'super-theories' that focused on other 'universal' proportions disappeared. Fechner's works made clear that the golden section is not the key to everything, but still a widely preferred proportion. While the debates about the golden section were often simplified to a yes/no question, Fechner demonstrated that it is a statistical problem. His experiments were very well designed and provided a great challenge for later generations of psychologists. Interestingly, many criticism against Fechner's experiments are based on the lack of knowing his original works. In the same time, two enthusiastic groups of people continued to popularize the topic of the golden section:

- teachers of mathematics, who remained mostly at their field and used just a few additional examples from art and nature,
- believers of a universal theory, who analyzed even more object of nature and art.

The first group made a lot of service to the topic. Indeed, the golden section leads to many interesting problems in mathematics, from geometrical constructions to the theory of polyhedra, from irrational numbers to the theory of approximation by Fibonacci numbers, and it is useful to beautify the process of education. Parallel, the second group produced many interesting measurements of objects, which were relevant for further studies, but also made a lot of harm by shaping a rigid golden sectionism. These authors believed that a growing number of examples make the theory more universal. They missed Fechner's question: what about a spoon that has a different proportion? Anyway, until the end of the 19th century more than a dozen books or booklets appeared about the golden section in German, which referred to the term *goldener Schnitt* in their titles. These works covers many fields, not only mathematics and fine art, but also botany, medical science, typography, and arts and crafts. Despite the overstatements in some of these books, they also contributed to a positive development: many people understood that we should have more bridges between different fields of art and science. This form of interdisciplinary thinking is beneficial for both education and research.

The topic of the golden section also reached Russia in 1876 when a small book was published on Zeising's method with some additional remarks by Yu. F. Vipper: *Zolotoe delenie, kak osnovnoi morfologicheskii zakon v prirode i iskusstve (Otkrytie prof. Tseizinga)* [Golden Division, as a Basic Morphological Law in Nature and Art (The Discovery of Prof. Zeising), in Russian, Moskva, 1876, 24 pp.]. Note that this book is signed by Yu. F. V., without giving the full name of the author. (Yu. F. Vipper was the father of the noted historian R. Yu. Vipper and the grandfather of the art historian B. R. Vipper.) The Russian culture was also receptive to universal principles or laws that would provide a comprehensive picture about a whole field. In the 1860s Mendeleev's periodic table of the chemical elements and Gadolin's exhaustive list of crystal classes (32 point groups), which opened the way to Fedorov's classification of all the possible ideal crystal-structures (230 space groups) in 1890, present good examples for this. The cited crystallographical works also show a strong commitment to geometrical-morphological approaches. This interest could have made some room for golden sectionism from a scholarly point of view. We may also consider some socio-political factors. In a vast country where many nations are living together, golden sectionism could have had a similar importance at a general cultural level as in Germany during the unification of that country. Later, at the beginning of the Soviet period, golden sectionism had an acceptance and even some ideological support as

- a bridge between old traditions and new forms of geometric art,
- a rational basis of aesthetics for new art theories,

- a reflection of anthropomorphism in art, science, and technology.

The golden section also had a vulgar importance: it is associated with two basic symbols of the Soviet ideology, the five-pointed star (pentagram) and the idea of spiral development. Since artists were overwhelmed by orders for works where these symbols should be well 'reflected', it is not surprising that the golden section often appeared in theoretical works on art and design. An interesting example of golden sectionism in architecture is G. D. Grimm's book *Proportsionalnost' v arkhitekture* (Proportionality in Architecture, in Russian, Leningrad, 1935, 147 pp.). Since this book was the result of a 40-year study, it is an important document of both Russian and Soviet interest in the topic from the 1890s until the 1930s. From the late 1910s constructivism gave a new emphasis to geometric methods in art. The Soviet State initially supported this movement, but later rejected its 'desanthropomorphism'. Social Realism 'required' the representation of workers, peasants, and soldiers, therefore human figures had a strong emphasis. Here gave the golden section some 'rescue' to geometric artists, since they could have claimed that they represent anthropomorphic 'objects' by utilizing human proportions. Indeed, the golden section gained some importance in industrial design, which is represented by many theoretical works. Although the Russian golden sectionism led to various dogmatic works, the scientific attitude towards the topic also inspired some remarkable results. We will discuss some of them in the next chapters.

Let us go back to the mid-19th century. Zeising's theory was not the only attempt to explain the proportions of large number of objects in compact form. It is true that most of them were quickly rejected or did not generate too much waves. One of the few exceptions was the proportional system of the Hungarian archaeologist and art historian Imre (Emerich) Henszlmann. The international recognition of his approach is demonstrated by the fact that he gave a talk about his system at the Royal Institute of British Architects in 1852 (two years before Zeising's book was published!), the French Emperor Napoleon III supported the publication of his book *Théorie des proportions* (Paris, 1860), and the revised 1867 edition of Gwilt's *Encyclopaedia of Architecture* quotes his statements (Reprint edition, New York, 1982, p. 975). Henszlmann, unlike Zeising, suggested not just one preferred ratio, but a whole sequence. Starting with the ratios of the "triangle of the cube" , which is shaped by an edge (1), a face-diagonal ($\sqrt{2}$), and a spatial diagonal ($\sqrt{3}$), he developed a "harmonic system" of ratios using increasing and decreasing triangles. Although this system had many interesting features, it was too complicated and gradually became overshadowed by the golden section. On the other hand, Henszlmann contributed to interesting discussions on proportional systems, including the criticism of golden

sectionism. For example, while the Hungarian architect and educator Myskovszky analyzed a Gothic church using the golden section, Henszlmann summarized his negative view and presented his different approach (these two works appeared together in the Hungarian journal *Arhaeologiai Közlemények*, 11 (1877), pp. 101-111). In a following paper we will return to Henszlmann's theory and some Hungarian works that pointed out the limits of both Zeising's and Henszlmann's approaches.

We should also note a tendency in the history of art: while even more people - usually specialists of theory of art and aesthetics and rarely practicing artists - became involved in proportional theories, in many cases the golden section was discussed in a broader context. For example, the American J. Hambidge's term "dynamic symmetry" , introduced in the 1910s, refers not only to the (golden) "section" , but to any kind of ratios where square roots of integers are used. He considered various rectangles with such ratios of the sides: "root-two rectangle" , "root-three rectangle" , etc. He used these rectangles and their diagonals intensively in his analyses of classical art works and at his lectures on composition. In the 1920s, the Albanian-Rumanian M. Ghyka, living in France, became a 'champion' of (over)using the golden section in proportional analysis of both old and new art works. He presented interesting, but often wayward proportional analyses where the regulating lines connect irrelevant points. Both Hambidge and Ghyka were prolific writers. Their books and papers, despite some immediate criticism, gained great popularity among many people, from American archaeologists to French artists, because they filled a gap between the classical art and the new geometric methods of design, between irrational mysticism and rational calculations, and, at a more general level, between art and science. Even the architect Le Corbusier's proportional system, the *modulor*, was strongly influenced by Ghyka's works on the golden section. In the 1930s Ghyka's and Hambidge's books were translated from French and English, respectively, into Russian, and in the 1940s and 1950s Ghyka's books also appeared in English. Did Hambidge make original discoveries? The architect M. Borissavliévitch, who worked in France and published a remarkable book on *The Golden Number* (French ed., Paris, 1952; English trans., London, 1958), gave a negative answer for this question. Specifically, he claims (English edition, pp. 22-23) that Hambidge did not discover anything new, but his system was influenced by Henszlmann's work of 1860. Note, however, that I could not establish yet a direct link between them: Henszlmann's French book of 1860 is not available in any American public library according to the National Union Catalog and the brief summary of his methods in Gwilt's *Encyclopaedia of Architecture* is not enough to understand it. Of course we may suspect many other links, not definitely via Henszlmann's French book, but also via the architects who attended his lecture at the Royal Institute of British Architects. Returning to

M. Borissavliévitch's book, I recommend to pay more attention to it. He presents many interesting facts on the golden section, but also points out the "sterility of philosophical aesthetics" and the "error in the use of mathematical theories" in the aesthetics of architecture.

### 3.3 Golden sectionism in Japan: partly as a Western influence, partly as a topic in traditional mathematics

We can trace some influence of golden sectionism in Japan as early as 1928 when T. Miyashita, a pioneer of design education, published his *Soushoku kousei no kenkyuu* (Studies of Decorative Composition, in Japanese, Tokyo, 442 pp.). The author, after visiting Europe and America, gave a detailed survey of the works of various schools of design from the Bauhaus to the Russian Constructivists and also discussed Zeising's theory on the golden section and Hambidge's dynamic symmetry. In that time the Japanese society was very much open to adopt Western ideas. On the other hand, the golden section did not become a central concept in Japanese design, but remained one of the possible proportional systems. Interestingly, Miyashita missed an interesting point: the very concept of the golden section was not at all new in Japan. It was available, although without a specific name, much earlier in *wasan* (from *wa-* Japanese, *san* mathematics), the traditional Japanese mathematics that flourished from the late 17th century. The members of *wasan* schools produced many manuscripts and block-printed books, moreover they also 'published' their geometric results on beautifully engraved wooden tablets in shrines and temples. They frequently dealt with pentagons, pentagrams, as well as circle packings in these figures, and the related calculations led them to the value of the golden number. Originally, I believed that they did not went beyond this, but a recent observation has changed my view. I surveyed various *wasan* books where regular pentagons and their diagonals, which intersect each other according the golden section, were discussed. I was surprised to see that A. Aida, the founder of a new school in *wasan*, denoted the smaller part of the golden section by the Sino-Japanese character *ten*, heaven (*Sanpou tenseihou shinan*, Vol. 3, 1810, p. 19). This choice could be just a coincidence or a simple reflection of the fact that *tensei* or *tenzan*, which begins with the same character *ten*, refers to a Chinese-Japanese method for solving algebraic equations with an unknown 'celestial element'. Still, it is tempting to say that Aida considered the golden section as the "heavenly proportion" in some sense. An even more striking example for the Japanese interest in this topic is the book by K. Yamagami entitled *Ikei (Igyou) doujutsu* (Different Problems with the Same Technique, 1843; the first word in the original title has two

readings and meanings, "different form" and "strange or fantastic appearance" ). The printed version of this work was revised by D. Komatsu and, as a preface points out, it is recommended for "everyday use" . The book presents 32 problems without further details and the reader should 'discover' that all of the solutions are related to the same idea, the golden section. Interestingly, just two problems are based on the regular pentagon, while the remaining 30 ones are associated with interesting circle packings. In most cases the golden section appears in the solution as a 'fantastic' surprise, despite the fact that we 'know' the answer from the previous problems. Yamagami's rich collection of problems still remains unique in the mathematical literature, moreover we may make an even stronger statement. If we do not consider Pacioli's book, where only the first part deals with the "divine proportion" in the mathematical sense, Yamagami's work of 1843 is the first ever book on the golden section. Indeed, it is slightly earlier than those German works that we discussed in the case of Western golden sectionism: Wiegand's mathematical booklet of 1848 and Zeising comprehensive survey of 1854. Although we must admit that Yamagami's book deals with the golden section in a hidden form, without naming it or discussing its properties, the importance of this concept will be evident for those readers who take up the challenge and solve some of the problems.

What was the reason behind this specific interest? The answer for this question is not easy, but perhaps the cultural context gives some hint. *Wasan* was not a fully "academic field" , but a set of knowledge, partly mathematics, partly art or technique, that had some links to everyday problems. In the 17th century the formation of *wasan* was strongly influenced by Chinese mathematics, but this statement is less valid in the case geometry. The close packing of circles in various figures became a specific Japanese field of study. It was practised widely not only in big cities, but also in small villages until the late 19th century when the Ministry of Education required to replace *wasan* by "Western mathematics" in the official system of education. (Interestingly, the problems of circle packings became popular in Western mathematics from the 1950s and now this topic is 'reintroduced' into Japan via the translation of Fejes Tóth's book.) Let us turn to the possible sources of the Japanese interest in the topic of the golden section long before the adaptation of Western mathematics and the related term (*ougon bunkatsu*, golden section). The regular pentagon plays an important part in Japanese culture: not only as a flower motif and design element, but also as the traditional form of flat ties (knots) of cloth-belts in storage and of paper-strips with love letters. The seal of the medieval geomancer Abe Seimei (mid-Heian period, 10th-11th cc.) was, similar to the symbol of the Pythagoreans, the pentagram. It was used as a talisman in yin-yang geomancy and still survives as a symbol on swaddling clothes. On the other hand, we should add that the golden section cannot be considered as

the basic form of human proportions among Japanese people (sorry, Zeising!) and the typical unit in design - from calligraphy to architecture - is the square, not a rectangle with "dynamic symmetry" (sorry, Hambidge!). Note that the double-square has a special importance in architecture: it is the shape of the tatami mat (its exact dimensions slightly varies from region to region), which is the module-element of the floors of Japanese-style rooms. The dimensions of rooms and the capacity of heaters are frequently given in tatami units even today. Incidentally, one may feel that here we have a link to the golden section. If we consider the diagonal of a double-square and subtract from this the length of the side of a square, we may produce a construction for the golden section. However, the latter steps are not typical in Japanese design. (Still, this construction could have inspired Mukaigawa to formulate his discussed theory on Leonardo.) I believe that in the case of Yamagami's book three traditions came together:

- an interest in the golden section via the pentagram,
- the study of circle packing problems, including the arrangement of circles in larger circles, squares, rectangles, ellipses, and other figures,
- the idea of presenting different problems with the same solution, which was initiated by Fujita.

Although there are many geometrical figures in Yamagami's book that may have artistic importance, there is just one that can be directly linked to arts and crafts. It is a pattern that decorates the surface of a fan, which is a relatively frequent topic in *wasan* books. On the other hand, Yamagami's work did not become so influential as the cited German works: it did not attract a large number of 'followers' of golden sectionism. Still, this book is an interesting document, not just a 'fantastic' or 'grotesque' journey. It demonstrates in an 'extreme form' that many mathematicians (*wasanka*) had some interest in the golden section and also hints a link to the problems of design.

Here we may mention another interesting example: the Japanese flag with the rising sun at the center. It uses just one motif, the "sun disc" (*hinomaru*), which divides the flag into two parts: a red circle is surrounded by a white field. The ratio of the vertical side to the horizontal one is 2/3, while the ratio of the diameter of the circle to the vertical side is 3/5. As an interesting coincidence, this old Japanese symbol was adopted as the flag that should mark their ships in the same year of 1854 when Zeising's book on the golden section was released. The described ratios were declared in 1870, very close to the time when Fechner published his first exprimental-aesthetical monograph (1871). Interestingly, in early 1870 the ratio of the sides was defined as 7/10, but it was modified to 2/3 in the same year. Perhaps pro-golden-sectionists would claim that this is a systematic use of the Fibonacci

numbers (1, 2, 3, 5, ...) for the approximation of the golden section (2/3, 3/5, ...), but we have no evidence for this. Probably the switch from 7/10 to 2/3 indicates that the main reason was, beyond some general aesthetical considerations, a preference for ratios of small integers. Incidentally one of the most comprehensive survey-books on Western golden sectionism is a Japanese work written by the art critic M. Yanagi (Yanagui): *Ogon bunkatsu: Piramiddo kara Ru Korubyuje made* (Golden Division: From the Pyramids to Le Corbusier, Tokyo, 1965 and 1977, 254 pp.). Another book by him is a trial to extend this topic even into Japanese art, despite the fact that the golden section has no traditional basis there: *Ogon bunkatsu: Nihon no hirei, Hôryûji kara ukiyoe made* (Golden Division: Japanese Proportions, From the Hôryûji [temple] to Ukiyoe [style of woodblock prints], Tokyo, 1977, 151 pp.).

### 3.4 The methodology of detecting the use of the golden section in art

Golden sectionism initiated some interesting contributions and later inspired some artists, from French painters to Russian constructivists, although the actual use of the golden section was much rarer than some of the statements suggest. On the other hand, golden sectionism also led to many wayward analyses of art works and objects of nature, and helped the birth of other 'universal theories'. The works of these 'proportion hunters' can be characterized by two types of mistakes:

- Forced use of regulating lines based on the golden section, which have no importance from the point of view of the original composition.

- Claiming the presence of the golden section by measuring some data and then using approximate calculations (this is an important methodological problem because, let us say, 0.645 can be the approximate manifestation of both the golden section, i.e., 0.618..., and the simple ratio 2/3 = 0.666...).

Indeed, we cannot state that an artist clearly used the golden section without some further evidence. The latter could be, e.g.,

- a direct or indirect statement by the artist (notebooks, letters, interviews may include such references),
- sketches where the geometric construction of the golden section can be identified (just one example: if the artist identified a regular pentagon in the composition and uses the points of intersection of its diagonals, we clearly have a commitment to the golden section),

- systematic approximation to the golden section by Fibonacci-type numbers (expecting a large number of cases that are clearly identified by the artist).

Of course we cannot exclude the possibility of the unconscious usage of the golden section, but this is a very problematic issue. In the literature related to the golden section there are frequent claims that Fechner's experimental-psychological tests 'proved' the aesthetical importance of the golden section. However, we should not ignore the fact that the findings by Fechner, which were discussed earlier (Chap. 3.2), are more delicate: he dealt with simple figures and presented statistical data on preferences. Remember that 35% chose the golden rectangle from a set of ten, which is a relatively big number, but definitely not a 'proof' of an absolute preference. In the psychological literature, many similar experiments were described since Fechner's studies: some of these indicated a preference for the golden section, while other ones did not show any significant trend or presented different preferences (e.g., Haines and Davies in 1904; Schiffman in 1969; Zusne in 1970; Davis and Jahnke in 1991; cf., the detailed survey of both 'sides' by C. D. Green in the journal *Perception*, 24 (1995), pp. 937-968). May we refer here to an important problem: most of the experimental-psychological researchers present to the subjects various significantly different proportions, while we would suggest rather such experiments where 3/5 = 0.6, 5/8 = 0.625, etc. are also included, together with the 'real' golden section (0.618...). Although we may expect further psychological studies on the golden section, including experiments based on both choice and construction, we hardly can expect any definitive 'proof' beyond some preference around (*sic*!) the golden section. On the other hand, testing different groups of people with various cultural and ethnic backgrounds, and even extending the experiments for animals, may provide some interesting data not only for psychologists, but also for artists and designers. For example, Chinese and Japanese people, who have a great tradition of using squares and simple rational proportions from calligraphy to architecture, could have less preference for the golden section. Some psychological tests also hint the same thing (e.g., Berlyne in 1970; Nakajima and Ohta in 1989). Indeed, in the case of industrial design both the different preferences and different anthropometric data should be considered. Note that the architect Le Corbusier's proportional system, the *modulor*, is associated with the data of Western culture, which is not definitely the best for, let us say, Asian people. Thus the golden section is still an interesting challenge for comparative studies.

A further problem is that even the seemingly straightforward documents on the deliberate application of the golden section could be misleading. Specifically, golden sectionism became so popular in some periods that artists 'backdated' the use of this proportion to

works where they did not really employ it. This tendency is not definitely a falsification or wrongdoing to mislead the art historians, but a psychologically understandable reinterpretation of the facts. When an artist, freshly infected by golden sectionism, checks his earlier works, may believe that, indeed, the golden section was there intuitively or the original regulating lines were just a little wrong. Something similar could have occurred in the case of the architect Le Corbusier who, after reading Ghyka's book of 1927, 'revised' the sketches of some of his earlier works and modified the original regulating lines according to the golden section. At least, this possibility is demonstrated by Herz-Fischler in two papers with convincing arguments ("The early relationship of Le Corbusier to the 'golden number', *Environment and Planning*, B6 (1979), pp. 95-103; "Le Corbusier's 'regulating lines' for the Villa at Garches (1927) and other early works, *Journal of the Society of Architectural Historians*, 43 (1984), pp. 53-59). Another possible problem is the metaphorical use of the golden section in the sense of good proportions. Most people would accept as an obvious fact that the "Section d'Or" (Golden Section) group of painters, who were associated with cubism and in 1912 made a big exhibition in Paris, used the golden section. In a following paper we will see that this is not the case. Thus, even in the straightforward references to the golden section require critical analysis. These are further challenges to art historians.

# 4 THE GOLDEN SECTION IN MUSICOLOGY

## 4.1 From Zeising's book to Liszt's interest in the golden section, with some open questions (1850s); the later developments (Bösenberg's book of 1911)

Golden sectionism also reached the field of music, first in the German speaking countries and then, on a larger scale, in Russia. Indeed, Zeising's comprehensive book on the golden section, which was discussed earlier (*Neue Lehre von den Proportionen*, Leipzig, 1854, xxii + 457 pp.), includes a chapter entitled "The importance of the proportional law in the field of music as the foundation of harmony" (pp. 414-444). This is primarily a musicological-mathematical survey of how to produce consonances by dividing a string according to given ratios. This topic has a long tradition back to the Pythagoreans. Zeising also referred to the works of Kepler who had a special interest not only in the harmony of the universe, but also in musicological questions (p. 424). Interestingly, he did not emphasize Kepler's contribution to the topic of golden section and Fibonacci numbers. Zeising made a special effort to unite the problem of musical harmonies with the scientific results of acoustics. Obviously this topic has theoretical importance, but perhaps it is less

important for the composer who uses the traditional scales. However, there is a related problem that is useful in practice: the consideration of intervals with 1, 2, 3, 5, 8, ... semi-tone steps. We should add immediately that only 4, 6, and 7 are not Fibonacci numbers among the first eight integers. Thus, we may need musical reasons in order to speak about harmonies based on Fibonacci numbers or the approximation of the golden number. Zeising considered this problem at the level of theory of harmony, without discussing concrete musical pieces or presenting examples of notes. Finally, he referred to proportions considering time-intervals, but just in the very last sentence of this 30-page musicological chapter (p. 444). It is true, however, that he touched a similar problem at the beginning of the next chapter where he discussed poetical works and dramas (pp. 444-446). One year later, Zeising returned to musicological questions in his cited second monograph on aesthetics (*Aesthetische Forschungen*, Frankfurt am Main, 1855, xv + 568 pp.). The chapter "On the sounds" (Von den Tönen, pp. 235-248) slightly extends the topics covered earlier, including the problem of proportions in time (pp. 242-244). Zeising's surveys were impressive enough to attract some attention among composers and musicologists. Since his father was a famous violinist, his name was familiar in musical circles.

The Hungarian composer, piano virtuoso, and conductor Ferenc (Franz) Liszt, one of the leading figures of musical life in his age, had some links to Zeising. The two men were almost the same age and shared a lot of common interests. Liszt was very much excited by Zeising's work on the golden section, as it is illustrated by his letter of May 6, 1859 to Princess Marie Sayn-Wittgenstein, the daughter of his partner in life, Princess Carolyne Sayn-Wittgenstein:

> I am delighted [...] that you have also been seeing Zeissing [Zeising] again. If there was any way at all to incorporate the latter with the "German Music" , it would be a very nice gain. He formerly knew Brendel, and perhaps he might decide occasionally to honor the columns of the "Neue Zeitschrift" with his fine style. I am very much counting on you to indoctrinate him, as I am also counting on you to explain the secrets of the "Goldene Schnitt" [golden section, in German] to Fainéant ["Idler" , in French]: a method I would really like to apply in my own compositions. Mme. Zeising gave me the impression of being a very well-bred woman - have you seen her again?

Note that the "Idler" is Liszt himself: he frequently signed his letters to Marie by this nickname. The original French draft of the cited letter is at the Manuscript Department,

Houghton Library, Harvard University (No. 47 in Volume 1 of Liszt letters to Marie Sayn-Wittgenstein, Accession number *89M-62; I am indebted to Ms. Betty Falsey for her kind help.) According to my knowledge the original text of this letter has not been published, just its English translation, which appeared in the collection *The Letters of Franz Liszt to Marie zu Sayn-Wittgenstein*, translated and edited by H. E. Hugo (Cambridge, Mass., 1953, Letter No. 45, see the cited section on p. 106). Unfortunately, Hugo replaced the German reference to the golden section by the English expression "Golden Model" , which made the statement very obscure. We slightly revised this translation and reintroduced Liszt's original term for the golden section.

Hugo, in his comments to this letter, emphasizes Liszt's preoccupation with aesthetic theories and his interest in justifying the school of program music (pp. 102-103). He demonstrates this tendency by Liszt's earlier essay "Berlioz und seine *Harold-symphonie*" (Berlioz and his Harold-Symphony, in German, 1855). In this literary work, Liszt explains, among other things, the importance of formal elements in structural schemes and presents his view that the genius is a mediator between art and nature. Although Hugo points out these interesting facts from Liszt's essay, he does not follow the direct reference to the golden section and its possible meaning as a formal element in art and nature. Instead of this, Hugo turns our attention to Hegel, "Zeissing's master" , which is a simple misunderstanding. Hegel died during the Berlin cholera epidemic in 1831 when Zeising was just a 21-year old student in Halle. It is true that earlier Zeising spent a short period at the University of Berlin and he could have visited some lectures by Hegel, but he cannot be considered as a "pupil of Hegel" (p. 314). (Incidentally, Zeising presented a 130-page survey of aesthetical theories in his cited book of 1854, and he discussed Hegel in just a few paragraphs, pp. 118 and 127-130.) Perhaps Hugo is misled by the fact that Liszt frequently referred to Hegel's aesthetics in his essay on Berlioz. However, we should not ignore the fact that the actual letter of 1859 is not about Hegel.

I believe that the reference to Zeising and the golden section needs more attention. Indeed, Liszt wrote about him in two further letters to Princess Carolyne Sayn-Wittgenstein. Following the quoted letter to Marie (May 6, 1859), Liszt confirmed his original recommendations to Carolyne, too, and emphasized that the "cultivation of Zeising" is his intention (May 10, 1859). He also praised that Carolyne organized a meeting with Zeising. Liszt gave further details in connection with Zeising and made recommendations how to treat this "great man" . A few days later, in another letter, Liszt again mentioned Zeising (May 15, 1859). The original French drafts of these two letters are published in the collection *Franz Liszt's Briefe* (Franz Liszt's Letters, comments in German, letters in

original languages, edited by La Mara [pseudonym of Marie Lipsius], Vol. 4, Leipzig, 1899, see pp. 478 and 483). What was the possible motivation of Liszt's interest in Zeising and the golden section? Since Liszt also saw the danger that program music could have degraded to a simple imitation of natural sounds and other phenomena, he needed more sophisticated "laws of assimilation" . His compositions were often inspired by literature, painting, and nature, but he had less interest in pictorial realism than Berlioz and some other composers. If we consider the fact that Zeising interpreted the golden section as a law of morphology penetrating the whole nature and art, this proportion could have been an additional component for Liszt's own version of program music. Note that Zeising and Liszt shared a common interest in Goethe, who was not only a giant a literature, but also contributed to the study of biological morphology. For example, Zeising started his 1854 book by quoting Goethe (and Humboldt), while Liszt published a booklet in order to establish a Goethe Foundation.

Unfortunately, Liszt's extensive correspondence with Princess Marie, which could have given further details of his interest in the golden section, was interrupted by family problems. Shortly after Liszt's quoted letter was written (May 1859), Marie married to Prince Constantine von Hohenlohe-Schillungsfürst (October 1859) whose family was displeased about Carolyne's relationship with Liszt (Carolyne did not get permission from the church to divorce from Marie's father and to marry Liszt). This conflict between the two families led to the sad fact that Carolyne refused to see her daughter Marie and did not allow Liszt to correspond with her for a longer period. At least, there is no surviving letter between Liszt and Marie from 1860 until 1869, although there are conjectures that such letters may have existed (cf., Hugo's cited book, p. 111). In addition to this, Liszt had a very turbulent period in his life between 1858 and 1860, which also marked the end of his very creative period of twelve years in the German city of Weimar. Specifically, he resigned from his full-time post as a conductor, following a demonstration against him (December 1858). Liszt also had various personal problems, including the death of his son (December 1859). The accumulation of problems led to a breakdown and the 49-year old composer even prepared his last will (September 1860). Finally, Princess Carolyne, and later Liszt, moved to Rome, where he became more involved in religious music. Obviously, this period was not useful for keeping close connections with Zeising in Munich (München) and starting new experiments with the golden section. Did he carry out such a composition? The enthusiasm of the cited letter (May 1859) gives a clear evidence that Liszt had such an intention, despite all of the difficulties in his life. On the other hand, he had a soaring character: he could pick up an idea with great enthusiasm and drop it suddenly (as it happened in the case of using lantern slides and a wind machine illustrating

his music). However, his curiosity about the golden section, either temporary or constant, is remarkable since Liszt was a very influential person with a large number of pupils and admirers and he kept close connections with many composers. We will later discuss views that the golden section played an important role in the works of Debussy and Bartók. Thus, it is interesting to note that Liszt was visited by the young Debussy in Rome and he also initiated the foundation of the Academy of Music in Budapest where Bartók was later educated and became a professor of piano. Moreover, Debussy composed a piece entitled *Rhapsodie in the Style of Liszt*, which is unfortunately lost, and Bartók authored two papers on Liszt (1911, 1936).

It would be essential to locate further documents on Liszt's interest in the golden section. Since he liked to dedicate his compositions not only to sponsors, but also to friends, colleagues, and relatives, we checked the records of his more than 700 works. Although we did not find any piece dedicated to Zeising, there are some compositions that were written for Princess Marie. We recommend studying the autograph manuscripts of these and some other works of the corresponding period, since these may include further clues. Of course it is highly possible that Liszt did not make 'mathematical calculations' for his compositions, but first understood the theory of the golden section, then applied it more or less intuitively. We do not know the main focus of Liszt's interest in the usage of the approximate golden section (ratios of Fibonacci numbers). We may suspect both of the earlier discussed topics:

- intervals measured in semi-tone steps (theory of harmony) or
- time-intervals (temporal organization of musical pieces).

It is well-known that Liszt had a special interest in the adaptation of Hungarian melodies where a pentatonic (five-tone) scale with such semi-tone steps is typical. In the 1920s the Russian musicologist Sabaneyev analyzed, among others, the temporal structure of 100 pieces by Liszt and claimed that 87 of them include the golden section; we will return to this work in the next chapter. On the other hand, we admit that neither Zeising's chapter on musicological questions (which fills less than seven per cent of the pages of his book of 1854), nor Liszt's cited letter can be interpreted as a breakthrough in musical golden sectionism.

While Fechner's visual-psychological study of the golden section attracted some attention in both the German- and English-speaking countries (cf., the papers by Witmer, 1894; Pierce, 1894; Angier, 1903), the similar musicological-psychological questions did not gain a momentum. This tendency was continued even after the publication of the book by

F. Bösenberg *Harmoniegefühl und Goldener Schnitt; Die Analyse der Klangfarbe: Zwei musikpsychologische Abhandlungen* (Harmony-Sensation and the Golden Section; The Analysis of Tone-Color [Timbre]: Two Music-Psychological Treatises, in German, Leipzig, 1911, x + 109 pp.; see Part 1, pp. 1-45, and the conclusions, pp. 92-104). Although the author of this book was strongly influenced by both Zeising's and Fechner's proportional studies, he ended up at a "new golden section" : log2/log3 = 0.630..., which approximates the golden number 0.618... (actually he referred to the inverse cases: log3/log2 = 1.585... and 1.618...; pp. ix and 36-44). Bösenberg's starting point was the psychology of musical intervals, in particular the fifth (from C to G). The latter is associated with the ratio 2/3 if we consider a vibrating string. Then he considered log2/log3 referring to the logarithmic relationship between sensation and stimulus (Weber-Fechner law, see Chap. 3.2). Interestingly, Fechner did not linked directly his law in psychophysics with his studies on the golden section in visual-aesthetical problems. Now Bösenberg made this move. He also shaped a rectangle where the ratio of the sides is equal to log2/log3 in order to compare it with the real golden rectangle. He claimed that one hardly can detect any difference in aesthetical preference (pp. 38-39). We should note two important facts: Bösenberg, similar to Zeising, (1) emphasized the aspects of harmony and not the questions of psychology of time and (2) did not change the general view that the actual golden section is primarily a visual problem.

### 4.2 Russian musicological works on the golden section: Vipper, Rozenov, Sabaneyev (1870s-1920s); some new developments in semiotics, medical science, computing (and a psychological question)

Interestingly, the Russian case was different: Vipper's 24-page booklet of 1876, which discusses Zeising's theory, devoted almost half its length to musicological questions (pp. 14-24). This work, with well selected illustrations, is even handier for interested artists than the original monograph. In the case of proportions in nature and plastic art, the Russian author simply adapted the text and the figures from Zeising's book. However, in the chapter about music he added an original part on the golden section of time-intervals (p. 24) and extended the text by two new figures (Figs. 19 and 21), including an example of notes. After such a start in Russia, it is not surprising that the golden section also became a topic of musicological discussions in a wider scale than Zeising's book initiated in its original form. Indeed, É. K. Rozenov, a colorful figure of Russian artistic life as a musicologist, art critic, pianist, educator, and organizer of events, gave various lectures on the golden section in music and poetry in Moscow and St. Petersburg, which led to some

articles and to the booklet *O primenenii zakona "zolotogo deleniya" k muzyke: Esteticheskoe issledovanie* (On the Application of the Law of the "Golden Division" to Music: An Aesthetical Study, in Russian, S.-Peterburg, 1904, 19 pp.; Parallel edition as journal article, *Izvestiya Sankt-Peterburgskogo obshchestva muzykal'nykh sobranii*, 1904, June-August issue, pp. 1-19). This publication represents an important shift away from the application of the golden section, or rather Fibonacci numbers, in the theory of harmony to the analysis of time-intervals. Rozenov first studied seven poetical works, five by Lermontov, one by Schiller, and one by A. K. Tolstoi, then analyzed in detail a piece by each of the composers Bach, Beethoven, Chopin, and Wagner. In the 1920s Rozenov published further works about the golden section in music and poetry, while in 1930 L. Mazel' came out with a musical-educational article about the golden section.

Another important step, which involved the analysis of a large number of works and providing extensive statistical data, was done by another important figure of the Russian artistic life, the musicologist and composer L. L. Sabaneyev (Sabaneev, Sabaneiev). He presented his comprehensive study in a two-part article in the journal *Iskusstvo* (Art, in Russian): "Étyudy Shopena v osveshchenii zakona zolotogo secheniya" (The études of Chopin in the light of the law of the golden section, in Russian, *Iskusstvo*, 1925, No. 2, pp. 132-145; 1927, Nos. 2-3, pp. 32-56). First he analyzed 1,770 works by 42 composers and claimed that 75% of these include the golden section (with an error not greater than 2%), then he dealt with Chopin's études in detail. This large percentage led Sabaneyev to the conjecture that composers may use the golden section intuitively. Note that the publication of the first part of the article almost coincided with the release of Eisenstein's earlier cited movie *The Battleship Potemkin* (1925), where the golden section is used as an organizing principle in time. This period was very exciting from the point of view of avant-garde movements and lively artistic discussions in the new Soviet State. However, the first signs of the limits of freedom appeared very soon. Sabaneyev's studies were also interrupted by his immigration to the West in the mid 1920s. He was already abroad in 1927 when the second part of his article on the golden section appeared. Perhaps this fact also contributed to the unfortunate situation that Sabaneyev's cited work remained in the shadow of this transitional period: the end of his years in Russia and the start of a new life partly in France, partly in England, Germany, and the U.S.A. where he spent longer periods. Indeed, Sabaneyev's work on the golden section is almost unknown outside Russia, while his colleagues at home had good reasons not to cite an author who left the country. (Incidentally, we are aware of a few references to Sabaneyev's corresponding paper by Russian authors, but these were mostly in non-musicological publications, i.e., in such fields where his name was less prominent and thus did not cause 'problems'.) On the other

hand, Sabaneyev soon became an internationally recognized musicologist in the West following his German and English books, including the monograph *Modern Russian Composers* (New York, 1927 and 1975) that became a classic.

This is, however, not the end of musicological golden sectionism in Russia. The interdisciplinary study of the golden section and symmetry inspired partly new approaches in musicology (Marutaev), partly comprehensive surveys where musical questions are also discussed (Korobko and Korobko, also see Petukhov, Sonin, Soroko, and others). In 1982 a collection of Rozenov's musicological papers were reprinted, including one of his works on the golden section (*Stat'i o muzyke: Izbrannoe*, [Articles on Music: Selected Works, in Russian], Moskva, see pp. 119-157 and 245). The composer M. A. Marutaev - one of the contributors of the *First Interdisciplinary Symmetry Congress and Exhibition*, Budapest, 1989 - more recently co-authored with two architects a book: I. Sh. Shevelev, M. A. Marutaev, and I. P. Shmelev, *Zolotoe sechenie: Tri vzglyada na prirody garmonii* (The Golden Section: Three Views on the Nature of Harmony, in Russian, Moskva, 1990, 342 pp.). In parallel, the Russian school of semiotics 'rediscovered' Eisenstein's studies on the golden section, who, beyond applying this proportion for time-intervals in his movies, also recorded many interesting details in his notebooks. This approach is interpreted as an important part of the beginnings of the formal analysis of art works and semiotics in Russia (V. V. Ivanov). Although the golden section did not become a central concept for this movement, some leading figures used it occasionally:

- the golden section as an exact method in the theory of art (Fechner's survey of 1876 was translated and commented in a collection of papers, editors Lotman and Petrov in 1972),

- approximate golden section (e.g., 3/5, 5/8) in the structural arrangement of poetical works (Tsereteli in 1973, Jakobson and Lübbe-Grothues in 1981),

- the aesthetics of the golden section in the context of the lateralization of the brain (Lotman and Nikolaenko in 1983).

The importance of the golden section in time was also considered in the field of medical science, which is marked by two monographical works (Tsetkov in 1993; Subbota in 1994). One of the challenging topics here is the possible role of the golden section and the Fibonacci numbers in the organization of cardiac cycles and the minimization of energy expenditure for the functioning of the heart (Tsetkov). Last, but not least, we should turn to applied mathematics and computing science. It was demonstrated that the optimal method of searching the maximum or minimum value of an unknown function (with some

conditions) is based on an algorithm related to the golden section. Specifically, we should approach the desired value by dividing the considered intervals according the golden section. This algorithm was described by an American mathematician (Kiefer in 1953), but the topic has a new Renaissance in the work of Russian and Ukranian scholars (see, e.g., Stakhov's book *Kody zolotoi proportsii*, Codes of the Golden Proportion, in Russian, Moskva, 1984). I warmly recommend to study this topic also in the framework of experimental aesthetics. Both the eye motion in space and the perception in time can be linked to a dynamic process of "algorithmic aesthetics" where the subject searches for a point of climax (maximum). Psychologists developed various theories in order to explain some preference for the golden section, but they missed this theory of optimal search, which has an exact mathematical basis, not just subjective considerations. Do similar optimum principles play a role in nature, including human perception?

### 4.3 Lendvai's study of the compositions by Bartók (1881-1945): a new wave in musicology (from the 1940s)

The topic of the golden section in music had a fresh start when some musicologists and composers became involved in it as an organizing principle in time. The pioneering work of this second wave is due to the Hungarian musicologist Ernô Lendvai (1925-1993) who started his related studies in the 1940s. Independently of him, J. H. D. Webster also published a paper on this topic ("Golden-mean form in music", *Music and Letters*, 31 (1950), pp. 238-248). Lendvai was not aware of the Russian works and his first steps were very different. He analyzed the temporal structure of some compositions of Béla Bartók and during this work he 'rediscovered' the Fibonacci numbers without any information about this concept. When he gave a public lecture on this approach, an engineer pointed out to him the mathematical background of these numbers. These results became an important part of Lendvai's study of the style of Bartók, Kodály, and some other composers (Fig. 5).

Lendvai also considered the golden section and the Fibonacci numbers in another context: intervals of 1, 2, 3, 5, 8, 13, etc. semi-tone steps. He pointed out that a pentatonic (five-tone) scale, which is typical in Hungarian folk music, is based on such steps. These results were integrated into a complex system with interdisciplinary connections. Lendvai thus extended his research into the musical role of the golden section as a symbol of organic nature.

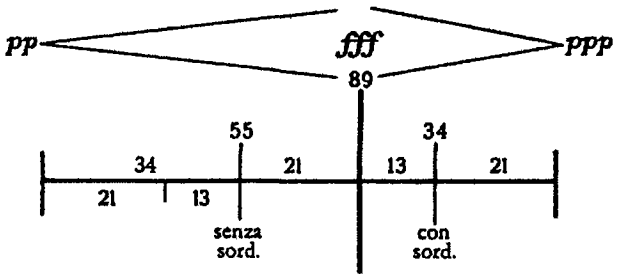



**Figure 5**: Lendvai's analysis of the temporal structure of the opening movement ('pyramid-fugue') in Bartók's *Music for Strings, Percussion and Celesta* (1936). The numbers indicate the lengths of time-intervals in bars (all of them are Fibonacci numbers), *pp* = pianissimo, *fff* = fortefortissimo, *ppp* = pianopianissimo, *con* or *senza sord. (sordino)* = played with or without mute. Lendvai completed the movement by a rest bar, similar to Bülow's Beethoven analyses. (After Lendvai's book *Bartók stílusa* [Bartók's Style, in Hungarian], Budapest, 1955, p. 144, also see his *Symmetries of Music*, Kecskemét, 1993, p. 58.)

After some publications in Hungarian, Lendvai's results reached an international audience: first via a collection of papers on Bartók's life and work in French and German (edited by B. Szabolcsi, Budapest, 1956 and 1957), then via the English paper "Duality and synthesis in the music of Béla Bartók" in *The New Hungarian Quarterly* (Budapest, 1962). The latter was reprinted in a very influential collection of essays edited by G. Kepes (*Modul, Proportion, Symmetry, Rhythm*, New York, 1966; Reprint, London, 1967; French and German translations, 1967). This book includes contributions, among others, by the mathematician Stanislaw Ulam, the psychologist Rudolf Arnheim, and the composer John Cage. Lendvai's small monograph *Béla Bartók: An Analysis of his Music* (London, 1971, xi + 115 pp.) and its Japanese version *Barutouku no sakkyoku gihou* (Bartók's Compositional Technique, Tokyo, 1976, 126 pp.) made Lendvai widely known in musicological circles. Note that he never used the term "golden section" in the titles of his monographs. He considered the golden section as a tool for a better understanding of music, not as the main focus of studies. Lendvai's related works also influenced some conductors and composers, among others Bárdos (Hungary), Hajdu (Israel), Járdányi (Hungary), Kurtág (Hungary), Ligeti (Germany), and Rickenbacher (Switzerland). Interestingly, but not surprisingly, Lendvai's approach attracted not only positive responses, but also some very negative views: this is a typical event in the history of great ideas. Lendvai's case, however, was very special because of the circumstances in Hungary. Instead of scholarly debates, he was often attacked at a political-ideological level and was forced into an isolated position. As professor of musicology and the conductor Lendvai was silenced. He continued to work almost totally alone without the possibility of regular interactions with musicologists and students. Although his books appeared in Hungary,

only a limited number of copies were printed. If we take a look at his impressive list of publications, which includes English, French, German, Hungarian, Italian, Japanese, Polish, and Russian works alike, we may immediately see that these works rarely appeared in the mainstream of musicological literature. Perhaps this strange situation also contributed to Lendvai's 'repetitive' style in his publications. He was aware that his readers have no easy access to his other publications. Thus, he preferred the repetition of some basic blocks, often with minor corrections, instead of simply referring to earlier works. We emphasize these facts in order to draw a more objective picture of Lendvai's life and work than some unfavorable reviews of his late publications did. We do not say that some of the critical remarks have no basis, but we suggest considering Lendvai's contributions, from the pioneering works to the heroic struggles, in a much broader perspective[3].

I should report here an almost unbelievable event. The journal *Leonardo* published J.-B. Condat's book review of the same work by Lendvai (*Leonardo*, 21 (1988), pp. 217-218), which is composed of plagiarized paragraphs and sentences from the most negative parts of M. Gillies's survey. Occasionally, there are minor stylistic changes in the text. Condat did not hesitate to copy even such sentences that include the phrases "in my opinion" and "what concerns me" (perhaps cheating children would make more efforts to rephrase such statements during a test!). I invite the readers to compare these two reviews and to ascertain that the 'work' by Condat (if it is his real name!) includes not a single sentence that is not available in the other one, with the exception of two French items in the references. Please be patient: Condat's review is 'composed' from 11 pieces of the other one and the jigsaw puzzle is not always easy. For example, the last two sentences are taken from two very different places: p. 293, para. 2, lines 2-4 and p. 295, para 2, lines 3-4. Incidentally, the latter is also useful for the condemnation of Condat or the person(s) who made this shameful act: "Academic honesty suffers, and doubt should be cast on his credibility in the music analytical [and the whole scholarly - my addition] world." I do not understand how this 'review' reached the journal *Leonardo* with such bombastic

[3] For example, M. Gillies's review on Lendvai's book *The Workshop of Bartók and Kodály* (Budapest, 1983, 762 pp.), which was published in the journal *Music Analysis* (Vol. 5 (1986), pp. 285-295), has some far reaching negative conclusions on Lendvai and his forty-year research on Bartók and Kodály, without clarifying that the considered work is not a new synthesis, but a collection of lecture notes written, and mostly published in a less appropriate form, in the 1970s. (Of course it would be interesting to discuss Lendvai's difficulties with the slow process of publishing in Hungary, instead of hinting that he could have had some priority; cf., review p. 295). On the other hand, Gillies has a positive view on the chapter on modality, atonality, and function, but here he immediately adds that this is not new, but based on a paper of 1977 (p. 293). We may see with some irony the reviewer's efforts to prove that Lendvai is not an academic analyst (p. 292). Perhaps the first person who would agree with this conclusion is Lendvai himself: he never considered himself as an analyst and he was outside of the `academic world'.

statements and, surprisingly, many references to musicological works in Hungarian, including the documents of the Lendvai-Somfai debate (which are also listed by Gillies). It is true that the editors also noticed some peculiarity and invited Lendvai to comment it, which was followed by Condat's reply (*Leonardo*, 21 (1988), pp. 339-340).

J.-B. Condat also edited a trilingual collection of essays on the golden section in music: *Nombre d'or et musique - Goldener Schnitt und Musik - Golden Section and Music* (Frankfurt am Main, 1988, 186 pp.). This book includes 15 papers by different authors, nine of these are reprints from a diverse set of journals (ranging from the *Journal of Music Theory* to *Mathematics Teaching*), while six were not published before, and a brief survey of eight dissertations about the golden section. Condat did not select any work by Lendvai for reprinting. In the introductory survey, he mentions just one book by Lendvai in a footnote (the same work that was reviewed in the journal *Leonardo*), claiming that it takes "several hours per page to collect the truisms, incorrectnesses, approximations, and errors" (in French, p. 11). This statement is supported by two book reviews, one by Gillies (see above) and another one by Condat himself (forthcoming in that time). As a sharp contrast to this, many of the statements in the book are based on Lendvai's work or give credit to him (see the statements on pp. 23 and 24 by R. Haase; pp. 31 and 33 by J.-P. Dambricourt; p. 45 by E. Antoni; p. 123 by D. W. Haylock; pp. 125, 130, 132, and 133 by R. Howat), including references to the denounced book (R. Howat's references). We also see a very different opinion of Lendvai in the paper by D. W. Haylock who emphasizes (p. 123) that "Lendvai has provided a fascinating analysis of Bartók's music [...], which will repay careful study" .

In January 1996, I tried to contact Jean-Bernard Condat at the post box address given in his cited works (B.P. 8005, 69351 Lyon Cédex 08, France), but my mail was returned with an official note that this address is not valid any more. Still, it would be interesting to solve this jigsaw puzzle of "academic honesty".

I also believe that some of Lendvai's important ideas remained in the shadow of his works on the golden section of time-intervals. Among others, Lendvai made original contributions to the study of modality and polymodal chromaticism. His "axis system" is a very useful tool for considering subdominant-tonic-dominant chord function. Although Lendvai's name is associated mostly with Bartók and Kodály studies, he also authored monographs on Toscanini, Verdi, and Wagner. Instead of further details, we refer to Lendvai's list of publications that was compiled for the issue "In Memoriam Ernô Lendvai" , which also includes Lendvai's last work as recovered from his computer,

scholarly papers dedicated to his memory, and recollections by his colleagues (*Symmetry: Culture and Science*, Vol. 5 (1994), No. 3).

In connection with Lendvai's approach to the golden section, there is an obvious question: is there any evidence that Bartók consciously calculated such proportions? Unfortunately, the documentation of Bartók's 'secrets' is not an easy task. While he authored many musicological works, especially about folk music, he did not mix these with the interpretation of his own compositions. Bartók was always very reserved in the case of such questions: he believed, contrary to Schoenberg, Stravinsky, and some other 20th century composers, that his music should 'speak' without any further explanation. For Lendvai, it was perhaps more exciting to find out Bartók's 'secrets' in an intellectual way than to study documents in archives. On the other hand, he had an isolated position in Hungary: he had no access to the New York Bartók Archive and some other American collections where such clues may exist. Still, he referred to various indirect evidence: Bartók's favorite flower was the sunflower, he liked to keep pine-cones on his table; both of these present exciting manifestations of Fibonacci numbers in nature. In addition to this, Lendvai used some impressive quotes from Bartók himself: "We are guided by nature in composition" , "folk-music is a phenomenon of nature [...] its formation has developed with the same organic freedom as other living organisms: the flowers, animals" . According to Lendvai's interpretation, Bartók's golden section system was originated in Eastern folk-music, while he used the harmonic system of Western music. These two components could provide a synthesis of East and West. Interestingly, a hand-written note where Bartók calculated some of the terms of a Fibonacci-type sequence (3, 4, 7, 11, 18) was discovered more recently in an East-West context. On this piece of paper Bartók analyzed the metric organization of a Turkish folk song, and this document was found by the Briton Roy Howat in the New York Bartók Archive (see the facsimile of this document in Howat's paper "Bartók, Lendvai and the principles of proportional analysis" , *Music Analysis*, 2 (1983), p. 86).

The fact that Bartók was not against making plans for the formal structures of his own composition is also supported by an interesting quote from him. Strangely enough, we found this source via some arguments against Lendvai's theory. The *Dictionary of Twentieth-Century Music* (edited by J. Vinton, London, 1974) includes A. Forte's article "Theory" . The discussion of Lendvai's work on the golden section and Fibonacci numbers in Bartók's music concludes that it is unlikely that any such construction is relevant either to the aesthetics or to the working methods of the composer. Forte gives a reference there to the paper by J. Vinton "Bartók on his own music" (*Journal of the American*

*Musicological Society*, 19 (1966), pp. 232-243). This article is an interesting collection of the very limited number of documents where Bartók spoke about his compositions. A quotation at the very beginning of this paper seemingly supports Forte's view (*The New Hungarian Art Music*, hand-written draft of a lecture, 1942-43, Bartók Archive, New York):

> "I never created new theories in advance; I hated such ideas. I had, of course, a very definite feeling [about] certain directions [I wanted] to take, but at the time of the work I did not care about what designations would apply to those directions, what sources they came from."

However, Bartók's next sentences give some support to Lendvai's approach:

> "This doesn't mean ... composing without ... plans and without sufficient control. The plans were concerned with the spirit of the new work and with technical problems--for instance, formal structure [as required] by the spirit of the work--all this more or less instinctively felt; but I was never concerned with general theories to be applied to the works I was going to write."

Indeed, we may say that the golden section in the organic world is not a "general theory" of musical composition, but a structural component that can be felt instinctively and applied in some plans consciously.

### 4.4 Howat's book on Debussy (1862-1918): from piano performance to musicology (1983)

Another milestone in the study of the musicological aspects of the golden section is Roy Howat's book *Debussy in Proportion: A Musical Analysis* (Cambridge, 1983, Paperback edition, 1986, xi + 239 pp.). Using a metaphor, the author represents a rare case of the 'golden section' between the performing artist and the musicologist. Perhaps the performing artist preferred intuitive music analysis, while the scholar also needed a theoretical basis, which was followed by further research in archives for evidence. Indeed, Debussy's own statements in letters and his revisions in the manuscripts of scores could provide much interesting information. The fact that an internationally known performer of Debussy's music went on to complete this wide-ranging study, an aspect which is not emphasized in the book, is a very exciting feature behind this monograph. His obvious motivation is a 'golden' answer to the question: Why should we study the golden section

in musicology? Howat presents partly his own theory on Debussy's methods and partly a rich set of documents on Debussy's thinking, which illustrates that the described theory could be very close to the composer's actual method. Both the theoretical studies by Howat and the documents related to Debussy could help a better understanding of his music, which is important for the performing artist and the devoted public alike. The opposite view that knowledge and science weaken the instinct led to some interesting debates in French artistic circles around Debussy, and many people would agree with Jules Laforgue's response: "In art there will always be, as there always was, instinct and reflection, inspirational or divining instinct and knowledge or science" (cf., the "Preface" of Howat's book).

Howat's goal, as he clearly declares it (p. 11), is not only to demonstrate the occurrence of the golden section and related proportions in music, but also to trace ways in which these are influential in the dramatic and expressive qualities of music. The author, after very careful investigations, decided to choose Debussy's *La Mer* (The Sea) as the central work for analysis, but he prepares the reader for this difficult task with the study of less complex works and later he also demonstrates some related structures in other compositions. These analyses in the book have many score examples, including some photos of the manuscripts located in various libraries and special collections, together with documentation of the sources. Howat's detailed proportional analysis very convincingly presents not only a few Fibonacci- or Fibonacci-type numbers, but a rich set of such systems.

We should also praise Howat's delicate analysis of Debussy's possible interest in proportions: he frankly presents some negative facts, too. For example, he does not hesitate to make clear that none of Debussy's surviving manuscripts of scores contain any sign of numeric calculations concerning proportions. However, the lack of such documentation is not conclusive evidence against the possible use of proportions because Debussy was a secretive person and destroyed the large majority of his original notes (p. 6). Even in the case of a very positive piece of evidence, specifically the fact that Debussy used the expression *divin nombre* (divine number) in a 1903 letter, Howat avoids overstating it. Debussy, in this letter written in French (see it in English at Howat, pp. 6-7), made the following request to his publisher Jacques Durand:

> "You'll see, on page 8 of [the score of] "Jardins sous la pluie" , that there's a bar missing - my mistake, besides, as it's not in the manuscript. However, it's necessary, as regards number; the divine number, as Plato [...] would say [...]."

Howat, does not claim that the expression "divine number" is an alternative name for the golden number (cf., Pacioli's divine proportion and the legend that Plato had an interest in this topic). Instead, he explains that Debussy did not specify the meaning of this term, and he makes probable its interpretation as the 'golden number' in further studies of both the context of the quote and the cited score. Incidentally, our earlier note that the expression "golden section" was not a well-established term before the 19th century and its spread from German to other languages was relatively slow has an interesting consequence from the point of view of Debussy's terminology. The composer had obvious difficulties in finding an appropriate term for the golden section or the golden number, if he really had such an intention, since the French expressions *section d'or* and *nombre d'or* were not yet used in broad circles. Even without a definite solution of the puzzle of Debussy's cited term, we may state that the composer had a special interest in numerical considerations. In another case, Debussy, referring to his *Cello Sonata*, made the following remark: "I like the proportions and the nearly classical form, in the good sense of the word" (quoted in French, p. 9). Thus, Howat was right to investigate proportional relationships in Debussy's musical compositions.

We strongly recommend some chapters of this book even to those readers who are not interested in musicology. Beyond the aforementioned general survey of the golden section (Chap. 1), Howat provides a real *tour de force* by analyzing the broadest intellectual circles around Debussy, including the Symbolist movement, the artistic avant-garde, and some esoteric groups, from the point of view of proportional studies (Chap. 11). Similar to the preceding parts, Howat also uses many archive materials and lesser-known documents to illustrate that Debussy could have had access to ideas related to the discussed proportions, and could have used these in his compositions. These pieces of external evidence are reinforced by some quotes of the composer that show his affinity for mathematical ideas. He claimed: "Music is a mysterious mathematical process whose elements are a part of Infinity" (p. 171). Howat's rich set of data is amazing! Of course, when dealing with very many questions from the history of art to mathematics, it is almost impossible not to disagree with some points. We will discuss a set of problems, which are related to "external evidence" and have nothing to do with the main musicological theme of the book, in a following paper.

Locating documents that Debussy (1862-1918) dealt with the golden section consciously is even more difficult than in the case of Bartók (1881-1945). The fact that Debussy passed away many decades before Howat started his research made it much less probable that appropriate documents or witnesses could be found. Note that Howat was aware of

Lendvai's works on Bartók and refers to his first English monograph of 1971, but he also makes some critical remarks (p. 187). Unfortunately, Howat did not know about Sabaneyev's study of the golden section in his two-part paper of 1925 and 1927, which is almost totally unknown in Western musicological circles (even Sabaneyev's detailed bio-bibliography in the *New Grove Dictionary of Music and Musician*, Vol. 16, London, 1980, pp. 363-364, does not refer to this interest and the related work). In the first part of this paper, the Russian musicologist and composer considered 40 works by Debussy and he claimed that 34 of these, i.e., 85%, included the golden section (all together 67 times). This 85% is significantly higher than the average 75% that Sabaneyev obtained from a total of 42 composers. Although there are some methodological problems in claiming the presence of the golden section in one piece or another, we suggest paying attention to Sabaneyev's work. In the promised following paper, extending the scope of Howat's search for evidence on Debussy's possible interest in the golden section, we will return to Sabaneyev's approach and will discuss some lesser-known details about how artists and composers, including Debussy, could have learned about the golden section.

## 5 THE FUTURE OF THE GOLDEN SECTION

Lendvai and Howat gave new inspiration to musicological discussions about the golden section, while the Canadian mathematician Herz-Fischler revitalized the related studies in both the history of mathematics and the history of art. A relatively large number of new books demonstrate that there is a renewed interest in the golden section. This is also true in musicology which is illustrated by, among others, H. Reis's monograph *Der goldene Schnitt und seine Bedeutung für die Harmonik* (The Golden Section and Its Importance for Harmonics, in German, Bonn, 1990) and a trilingual collection of essays *Nombre d'or et musique - Goldener Schnitt und Musik - Golden Section and Music* that presents 16 papers, including nine reprints from six musicological, two mathematical, and an artistic journal (Frankfurt am Main, 1988, 186 pp. - cf., para. 2 in footnote 3 above). In parallel to these works, Russian and Ukrainian scientists and engineers extended the topic of the golden section with technical applications, from coding systems to robotics, that could give further impulse to artists and composers. On the other hand, the new publications did not correct some widespread misunderstandings. We have a very contradictory situation that is dominated by instances of

- claiming the presence of the golden section in such cases where there is an evidence that it was not used (see our analyses of the extant texts by Polykleitos, Vitruvius, and Dürer),

- ignoring some interesting documents on the golden section (see our discussions of Liszt's letter of 1859 which is not published in the original form and its available English translation misinterprets the golden section as "golden model" ).

Since some of the misunderstandings became 'well-known facts', their correction is a very difficult and painful task. Still, there is some hope of modifying this situation: the better means of distributing information and the growing interest in the topic may assist in it.

I think that proportional systems (strictly plural!) may have a new Renaissance in our age. There were periods when some artists needed the help of geometry, while others rejected it. In 1957, during a debate and then a vote at the Royal Institute of British Architects, the majority rejected the motion that "systems of proportion make good design easier and bad design more difficult" (*Journal of the Royal Institute of British Architects*, 64 (1957), 456-463). This result was commented on by the leading Italian journal of architecture, stating that "no one really believes any longer in the proportional system" (*L'Architettura* edited by Bruno Zevi; this statement is also cited by the art historian Rudolf Wittkower). Indeed, the new generation of artists, many of them with fresh degrees from universities, believed that the rigid proportional systems disturb their creativity. However, Computer Aided Design and Manufacture (CAD/CAM) provides new possibilities. In many cases, the computer needs some preliminary data to make a first sketch (rigidity), then this one can be replaced with hundreds of alternatives (variety). Earlier, let us say, an architect did not elaborate dozens of versions of his design for a client, but now it is possible to present such a set. In some sense, we may repeat Fechner's experimental-psychological test on the preference for the golden section and other proportions by using, instead of some rectangles, a large set of complete buildings in cyberspace. The client will be asked to choose from these and then the preferred one will have further modifications according to individual needs. In addition to this, there is a special case where we are almost forced to use design programs that are based on the golden section or other historic proportional systems. This is the restoration of historic buildings and other art works where we have clear evidence that the original plan was based on such systems. Even in those cases where a new building should be fitted to an existing urban block, the architect may have a desire to follow the proportional system of the latter.

Of course I am not attempting to re-introduce a belief in the universality of one or another proportional system as the old 'secret of beauty' and the only method for the 'great design', but I suggest accepting them as cornerstones whence we may start. Instead of looking for the proportions of the 'average man', we may restore the dialectics between

some objective principles and subjective aspects of beauty. This tradition goes back to, among others, the "eurhythmic" modification of a geometrical design (Socrates, Vitruvius, Hero) and the avoidance of the perfect symmetry (Plotinos). This idea can also be connected with the principle of dissymmetry formulated in the 19th century (Pasteur, P. Curie).

The knowledge of the golden section and other proportional systems, including their geometrical, psychological, and historical aspects, is important in our age. Furthermore, there are still many open questions in connection with the golden section (extreme and mean ratio, proportion having a mean and two extremes, divine proportion). The overstatements of the golden sectionism in the late 19th century and the first half of the 20th century led to anti-golden-sectionism. But the golden section is a sort of symmetry, is it not? We also need a synthesis between these two cases: more studies and survey works that are based on symmetry and balance, a 'divine' approach to a 'mean' between the 'two extremes'...

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The author is strongly indebted to John Hosack (Suva, Fiji) for his comments on the first version of the manuscript, to Souichi Mukaigawa (Sapporo, Japan) for many exciting discussions about Leonardo, and to Elisabeth Hoffmann (Bielefeld, Germany) and the Central Library of the University of Tsukuba for helping him to locate a large number of books and papers.

## INSTEAD OF REFERENCES

The most important references are included in the text, while some further ones are available in my paper in a musicological-pedagogical journal of the Mozarteum in Salzburg: "Dynamische Symmetrie (Goldener Schnitt) als Organisationsprinzip im Raum und Zeit" , [Dynamic symmetry (golden section) as an organizing principle in space and time, in German], *Polyaisthesis*, 4 (1989), pp. 88-102. I will continue the current discussion in another paper "Where could French artists and composers, including Debussy, have learned about the golden section in the late 19$^{th}$ century and early 20$^{th}$ century?" and I also plan an article on "Golden sectionism: A brief socio-cultural history".