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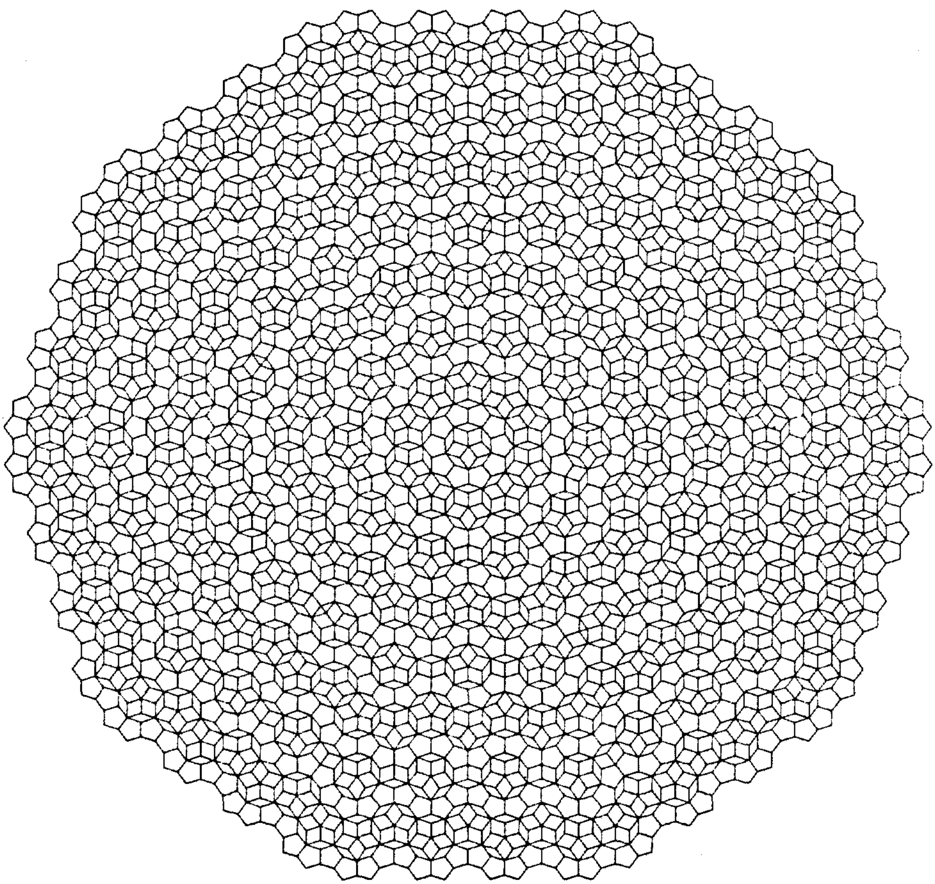

Vibrational Modes of a Spherical Drop and Minimum Dissipation Principle

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1. Introduction

Normal modes of small amplitude vibration of a spherical liquid drop is expressed by spherical harmonic functions (abbreviated as SHF), which was confirmed by an experiment (ARAI, ADACHI and TAKAKI, 1991). In the experiment, when the drop suspended in an immissible liquid was excited sinusoidally at one point on the surface, the instantaneous drop surface showed an axisymmetric shape of a single SHF (Fig.1) expressed as

$$R = R_0 + aP_n(\cos\theta)e^{i\omega_n t}, \tag{1}$$

where R_0 is the average radius and P_n is the Legendre function. The frequency ω_n is derived by one of authors (TAKAKI, 1991) as follows as an extention of the past theory (RAYLEIGH, 1879, 1902, LANDAU & LIFSHITZ, 1950).

$$\omega_n = \sqrt{\frac{\sigma}{\rho_1 R_0^3}}\sqrt{\frac{n(n-1)(n+2)(n+1)}{1+n(1+\rho_2/\rho_1)}}, \tag{2}$$

where ρ_1 and ρ_2 are the densities of the drop and the outer fluid, respectively, and σ is the interfacial tension coefficient. On the other hand, when the drop was excited at certain two points on the surface, the drop showed shapes similar to regular polyhedra (Fig.2). Note that the mode expressed by an adjoint Legendre function

$$R = R_0 + aP_n^m(\cos\theta)e^{im\phi}e^{i\omega_n t}, \tag{3}$$

or a superposition of SHFs with different directions of their axes has the same frequency as that in Eq.(1).

Then, there arises a problem what is the theoretical principle to select a particular mode for given frequency. One candidate of the principle is that a mode with the minimum viscous dissipation should appear. The minimum dissipation principle is valid for stationary states of linear systems, but it is not clear whether it can be applied to oscillating systems driven by external excitation.

2. Computation of Energy Dissipation

The flow field inside the drop is obtained from the Euler equation without viscosity. So long as the effect of viscosity Experimental apparatus is small, the viscous dissipation can be computed by substituting the inviscid velocity u_i into the formula for dissipation

$$E = \int \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 dV. \tag{4}$$

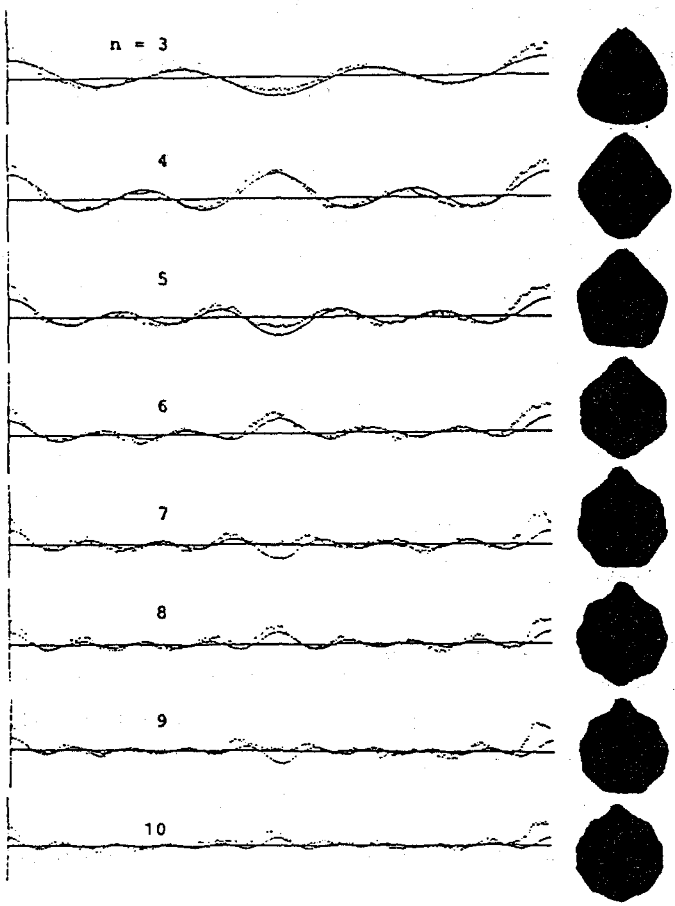

Fig.1 Instantaneous shapes of a drop with one-point excitaion at the top (side views). The left parts are comparisons of the measure contour shapes with the SHFs given by Eq.(1).

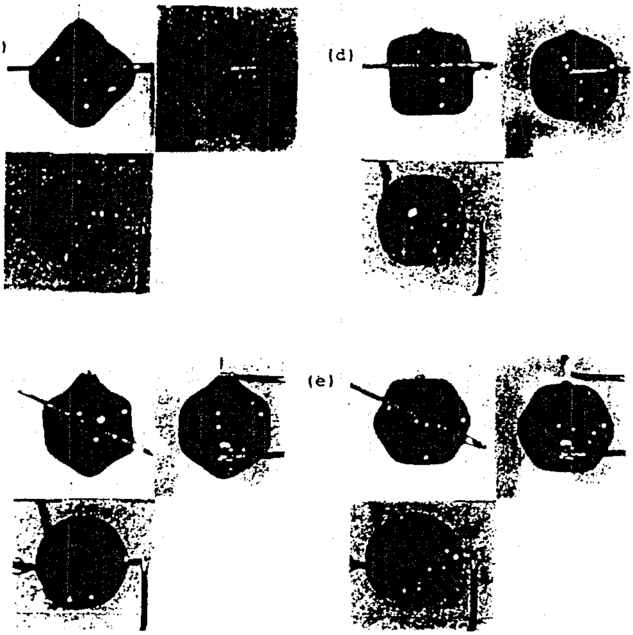

Fig.2 Instantaneous shapes of a drop excited at two surface points. They are similar to the cubic-octahedron and eicosahedron-dodecahedron modes. But, exact coincidences are suspicious.

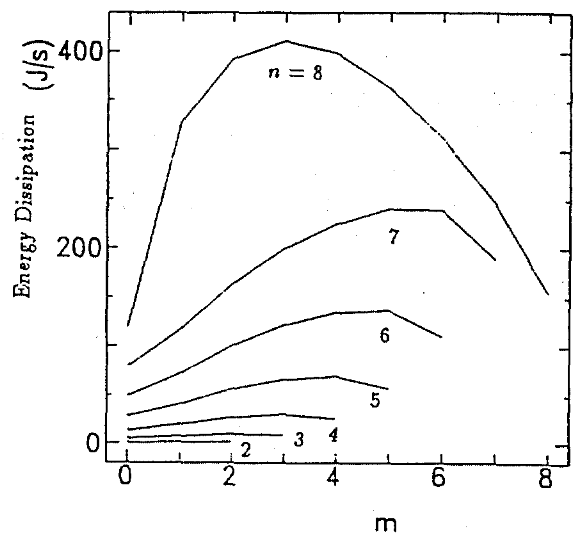

Fig.3 Computed energy dissipation for a mode $P_n^m(\cos\theta)e^{im\phi}$ for n=2-8.

For the case with one-point excitation the dissipations of modes given by Eqs.(1) and (3) are compared, where the maximum amplitudes on the surfaces (corresponding to the excitation amplitude) are adjusted to a common value. Figure 3 shows how E depends on the second parameter m, and that the mode with $m = 0$ (axisymmeric case) would be selected. This tendency is common for any value of n.

For the case with two-point excitation the dissipations of following modes are compared:

1) a superposition of two axisymmetric SHFs with axes directed to the points of excitations,
2) superpositions of other SHFs on the shape given by 1) to make higher degree of symmetry,
3) a single mode given by Eq.(3) which has the same amplitudes at the excitation points.

The results of computations for the cases $n = 4$ and 6 show that the case 1) leads to the minimum dissipation.

3. Discussion

From the above results one can conclude that
(*i*) the minimum dissipation is a useful principle for judging on mode selection,
(*ii*) the dynamical state of a vibrating drop does not necessarily prefer higher symmerty that is inherent in the exciting mechanism.

From the photographs taken by us we have been having a wrong impression that the drop prefers regular polyhedra (ARAI, ADACHI and TAKAKI, 1991). The reason of this misleading was that the actual drop shape was similar to regular polyhedra

We must be careful not to assume too high symmetries for real phenomena.

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