Symmetry: Culture and Science

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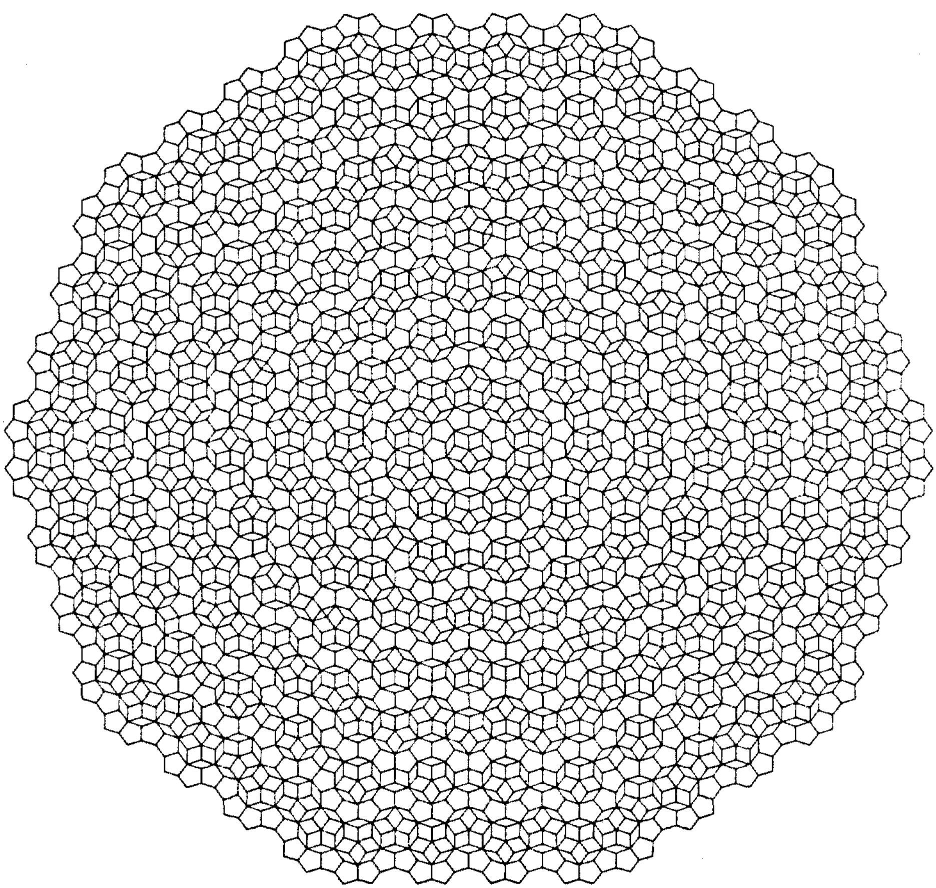

"SEEING THROUGH SYMMETRY" — A MULTIMEDIA COURSE*

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A recent NSF Instrumentation and Laboratory Improvement grant (No. DUE-9352670) enables us to provide a state-of-the-art interactive multimedia authoring platform (directed by Buckley, co-PI) for the enhancement of an interdisciplinary course, "Seeing Through Symmetry" (created by Gould, Project Director) that is for non-science majors and is focused around the concept of symmetry. That concept is used both as a link between subjects (such as physics, mathematics, music, and poetry) and as a method within a subject. Computer-based labs enable the student to further explore the concept by being gently led from the arts to mathematics and science. An explanation is given on how a variety of multimedia tools contribute to both the lecture and lab portion of the course. The aims of the course are as follows:

Symmetry is utilized as a basic concept through which many aspects of the scientific and artistic worlds can be better understood and appreciated. The course begins by illustrating pleasing patterns in the sciences and in the arts; this leads us to formulate a general definition of symmetry. We then characterize symmetry using elementary mathematical notions. Next we go on to see examples of symmetry in art, in poetry, and in music. Finally, we display the existence of symmetry in physics, chemistry, biology, and cosmology. A brief excursion into asymmetry ends the course.

Students will be able to explicitly develop their quantitative abilities and analogical thinking by using concepts of symmetry both as a method within a discipline and as a bridge between disciplines. They develop these abilities, in part, through the medium of a highly graphical laboratory experience; this utilizes computers in order for them to explore the many scientific facets of symmetry (in such areas as geometry, the arts, music, and physics) and to generate their own patterns through the use of software packages and programming. In a maximum of 12 labs, each meeting once a week, discussion is encouraged between the students in order to promote a collaborative atmosphere for scientific investigation.

Written and oral communication are also emphasized. This is done through classroom and laboratory discussions as well as through a term project (approved by the instructor) which incorporates course concepts of science and math. For the project, every student must submit an end-of-term essay as well as demonstrate the project to the class ("show and tell"!). This serves students as a major way to display their creative and analytical abilities. For the project the student may choose a topic from one of the categories discussed in the course (e.g., an Art major might create a large colored poster employing concepts of mathematical design such as group theory applied to tessellations, while a Music major may write and perform a piece of his/her own composition that includes concepts from the physics of symmetry in sound such as the application of Fourier analysis to harmonic structure). In addition, students are encouraged to come up with their own research project, one not on a list they are given. An example of such a project was the report (based on a book for the intelligent layperson) by an Honors student on the application of mathematical groups to the study of "gauge theories" in physics.

The course is taught in an assembled lecture-discussion format meeting 3 times each week. During the lecture portion of the course discussion is strongly encouraged. In addition, every student is required to give a written report on their trip to a science museum (e.g., The Connecticut Science Museum or the one in Boston). Occasional guest lectures are given by faculty who can enthusiastically describe novel applications of symmetry to their discipline.

Seeing Through Symmetry, now going into its fifth year, is intended to give non-science students an understanding and appreciation of many important areas spawned by human creativity. In our society, where science is "at risk," the aim of this course is to have more people find pleasurable values in the areas of Science and Technology through interdisciplinary interactions.

*A brief description of this course has been published in the NLA [New Liberal Arts] News 7, 15 (May, 1991); Department of Technology and Society, State University of New York; Stony Brook, NY 11794-2250.

SPHERICAL MODELS OF POLYHEDRA IN WASAN

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§1. Introduction

It was in the beginning of seventeenth century when Wasan-ka began polyhedra. Their main study was to calculate the volume, but in the eighteenth century, they began to think about the problems--circles packed on the spherical surface--in which they used inscribed polyhedra on the sphere.

This thesis introduces studies on this field by a Wasan-ka, Shoko Kenmochi.

§2. Wasan-ka Shoko Kenmochi's Study

Wasan-ka Shoko Kenmochi 剣持章行(1790-1871) was an excellent and a discerning mathematician through the end of the Edo period into the beginning of the Meiji period (1868-1912). Through his life time, he left very fine mathematical works on the history of Wasan[1]. Now we will show one of them. In 1849 he published a mathematical text, "Sanpo Kaiun" 算法開蘊 Vol.1 - Vol.4 (Initiation of Principles of Mathematics). He set a problem in the addendum of Vol.4 with a figure as follows:[2]

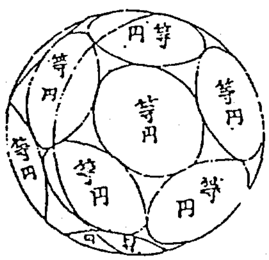


fig.1

Let us draw some circles, each having the same radius and circumscribing, on a spherical surface like figure 1. When the sum of area of regular spherical triangles which are constructed by three circumscribed circles are given, how many circles can we draw on a sphere ?

To this problem he wrote several solutions in " Kyumen Kaku Toen Jutsu Kai" 球面画等円術 (Solution for Finding the Numbers of Spherical Ball) and also in others, and gave them to his pupils[3]. The following figure 2 is a part of his solution. In fig.2 he said that n-circles surrounding a circle, each having the

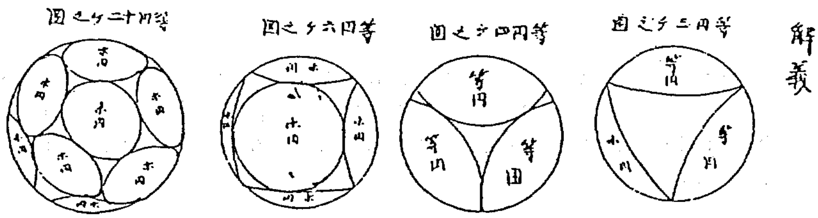


fig.2

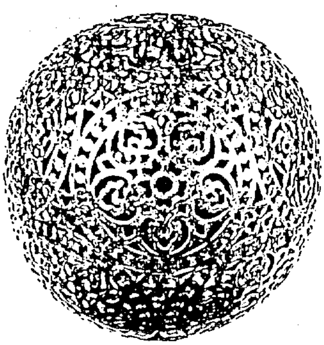
same radius and circumscribing, were defined by $2 \leqq n \leqq 5$,

$$(n=2 \rightarrow 3,\ n=3 \rightarrow 4,\ n=4 \rightarrow 6,\ n=5 \rightarrow 12)$$

because when n=1, we have no regular spherical triangle on a sphere. If n=6, the solid angle, which is made by jointing the pole of a circle and circumscribing 6 circles will be 360°.

Thus he demonstrated that the number of spherical balls drawn on a shpere would be 3,4,6 and 12. And he had realized that in the cases of 4, 6 and 12 if the poles of shperical balls are jointed respectivelly, polyhedron {3,3}, {3,4}, {4,3} and {3,5} were obtained inside a sphere[4], that is, if we joint each tangency of spherical balls, we have an icosidodecahedron [3, 5, 3, 5] inside a sphere.

§3. Conclusion



In Nara, Japanese capital during Nara period (710-784), Shosoin 正倉院, a very famous treasure-house, stands in Todaiji temple 東大寺. Shosoin was built to preserve treasuries of Emperor Shomu 聖武天皇(724-749). One of them is a spherical incense burner made by bronze 銅薫炉(diameter is 24.2cm). Shosoin record says that this incense burner was introduced from China. Otto Theodor Benfey said in his paper [5] that he discovered a figure, dodecahedral symmetry, on its spherical surface. Indeed it is packed by twelve circles as S.Kenmochi studied.

We are not sure whether or not S.Kenmochi and other Wasan-ka had known that Shosoin preserved the incense burner. But it is not quite imortant, because they could find out the polyhedoral, geometrical figures in the traditional arts such as bamboo works and handballs.

Before S.Kenmochi had an interest in such a problem, a similar problem, the arrangement on a spherical surface already appeared in the Wasan textbook as I stated in my paper[6]. However, no one studied as S.Kenmochi did, so he might have been the first mathematician in Japan who studied the packing problem of a spherical surface by any given circles .

Notes

1)See Tatsuhiko Kobayashi and others, Shoko Kenmochi; His life and Achievements, Edited byYoshimasa Michiwaki, Wasan-ka's Life and Their Works(Taga Publisher:

Tokyo, 1985), pp. 67-81, pp. 117-331.

2) ibid., p. 2.

3)These manuscripts have been preserved in Tohoku university's library, Japan Academy and others.

4)See Kaoru Tanaka and Tatsuhiko Kobayashi, On Shoko Kenmochi's "Kyumen Kaku Toen Jutsu kai" and regular polyhedron, Journal of History of Mathematics, Japan, No. 101, 1984, pp. 11-19. On this matter L. Fejes Toth also points out in his book that only four exist. See his book, Lagerungen in Ebene auf der Kugel und im Raum, Translated by Isao Higuchi and Masami Tanerura, Haichi no Mondai --Heimen·Kyumen·Kukan ni okeru--(Misuzu Shobo:Tokyo, 1983), p. 117.

5)Otto Theodor Benfey, Dodecahedral Geometry in a T'ang Era Incense Burner preserved in the Shosoin, Experiment of Science, Vol.25, No.13, 1974, pp.891-894. And also see Collected Edition of Japnese Art -- Shosoin and Ancient Painting -- (Kodansha : Tokyo, 1992), Vol.3, p.131.

6)See T. Kobayashi's paper, On the Study of Polyhedra in Wasan which is printed in this magazen. As another example see Sadasuke Fujita's "Seiyo Sanpo"精要算法(1781) and "Shinpeki Sanpo" 神壁算法(1789).