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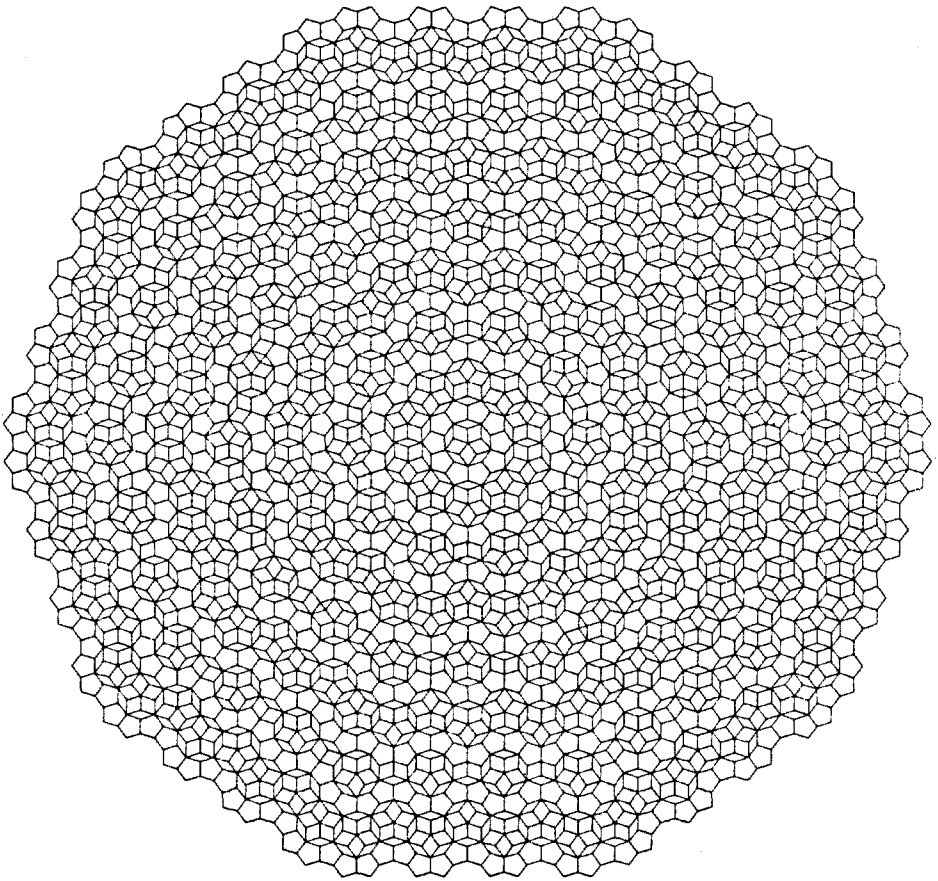

Information Originates in Symmetry Breaking

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Because symmetry interests us, and it is often an indicator of the deep structure of things, we are inclined to think it is informative, that it contains information. On the other hand, symmetries represent a sort of invariance under transformation, which implies that symmetrical things are redundant. This in turn implies a reduction in information content. We are faced with somewhat of a paradox: One the one hand, many symmetries that we find in the world are surprising, and surprise indicates informativeness. On the other hand, the surprise value of information arises because it presents us with the unexpected or improbable, but symmetries, far from creating the unexpected, ensure that the known can be extended through invariant transformations. How can this paradox be resolved?

In part, the resolution involves disentangling beliefs about symmetry from symmetry itself. Although the information content of a symmetry is relatively low, knowledge that some system is symmetrical reduces what we need to know about the system by eliminating possibilities that would be permitted if the system were not symmetrical. The reduction is greater the more pervasive the symmetry. Thus, the low information of symmetries is reflected in the high epistemic value of knowledge of symmetries. It would be hasty, however, to think that this completely resolves the paradox. Any new knowledge has much the same effect as knowledge of symmetry. Consider an experiment (a roll of a die, for example), with possible outcomes $O_1, \ldots O_n$, in which we discover O_i is the case. O_i reduces

the possibilities in much the same way that learning of a symmetry does. The account does not tell us in any way what is especially interesting about symmetries. Symmetries are interesting in themselves in a way that the outcomes of experiments generally are not. Another difference is that, while the outcome of an arbitrary experiment might be quite specific, and hence have a relatively high information content, symmetries are always of relatively low information content. We still need to explain the interest of symmetries, despite their relatively low information content. I will offer an answer that distinguishes between surface and deep symmetries, and argues that deep symmetries are interesting because of the way that information originates in symmetry breaking. I will first look at the mathematics of information and symmetry, and then at some specific ways in which information originates in thermodynamic, cosmological, biological and perceptual processes. I will conclude with some speculations on why we find symmetry interesting and attractive.

There are three (almost) equivalent approaches to mathematical information theory: combinatorial, probabilistic and computational (Kolmogorov, 1965). On all three approaches, redundancy is a reduction in information. A symmetry is fundamentally an invariance of a configuration of elements under a group of automorphic transformations (Weyl, 1952). This invariance implies that the configuration is redundant. The complementary mathematical character of the relation between information and symmetry suggests that information might originate in symmetry reduction, but it is possible that information might have other sources than symmetry breaking. I argue against this possibility on the grounds of a close tie between equilibrium and symmetry (Weyl, 1952: 25).

Another approach is to examine where information originates in the universe. If all nature were entirely lawful, then nature would be fully symmetrical, reflecting the symmetry of the laws (Weyl, 1952). In order to account for non-symmetries, we need contingency, which arises either from initial conditions, or else from symmetry breaking processes. On the "no boundary conditions" cosmology that I favour, there is no information in the initial conditions (they are entirely symmetrical) (Layzer, 1990), so all information must arise through symmetry breaking. This typically occurs in processes in which microscopic fluctuations are promoted to macroscopically detectable structure. The classic example is Bénard cell convection in a fluid gently heated from below. Similar processes in cosmology during the expansion of the early universe led to the separation of matter and radiation as the universe went out of equilibrium. Speciation can occur via similar processes, as can tissue differentiation during biological development. It has been proposed that at least some perceptual systems (smell and hearing) involve chaos in the transducer and associated neurons (a condition of symmetry) that is driven by a sensory signal into a particular harmonic orbit of the many quasi-harmonic orbits that compose the chaotic condition. If all perception works similarly, then perception is a form of symmetry breaking that produces perceptual information.

Given that information production is symmetry reduction, the problem of the interest of symmetries becomes acute. Not all symmetries are equally interesting. Many symmetries are purely superficial, being either accidental or traceable to equilibrium conditions, which wipe out information. Deeper symmetries, however, are left after symmetry breaking process produce information at higher levels. These symmetries are much more interesting. Their interest, I propose, is in what they reveal about the common source of different informative

structures that arise from them. The deep symmetries highlight the individuality of derivative structures, and also make their common nature more clear. Their recognition allows us (and forces us) to see the sorts of symmetry breaking – information – that make up the different structures emanating from a common source. The symmetries behind natural laws don't merely provide a convenient shorthand to compress our empirical data; they also help to highlight both the particular and the general in that data in an especially insightful way.

The aesthetic value of symmetries (aside from their artistic use to point to other things) is a reflection of the basis of their scientific interest (a symmetry underlying symmetry!) Shallow symmetries are boring and repetitive; too much symmetry is bland. When surface characteristics are underlain by a recognisable deep symmetry, however, the symmetry ties together the individual parts without denying them their individual interest or identity. At the same time as it gives an overall harmony and unity, deep symmetry highlights the individuality and unique interest of the parts of a composition or natural phenomenon. Appreciation of deep symmetries allows a much richer experience than would be possible otherwise. Although evolutionary biologists have tried to explain our taste for symmetry in terms of the health of potential mates, I suspect that its basis is much more deeply embedded in our nature, and in our relation to the world at large.

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IMPERFECTLY COLORED SYMMETRICAL PATTERNS

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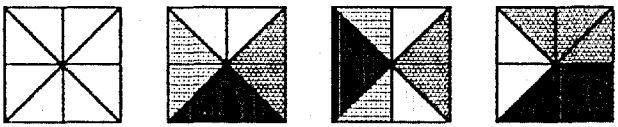
In most studies of colorings of symmetrical patterns, the colorings considered are what are termed perfect colorings. These are colorings in which each symmetry of the associated uncolored pattern effects a permutation of the colors of the pattern. For example, of the colored patterns below obtained from the pattern in Fig. 1-a, the pattern in Fig. 1-b is perfectly colored



(a) (b) (c) (d)

Figure 1.

while the other two are not. The uncolored pattern has eight symmetries, four of which are reflections in the indicated lines through the center of the square and the other four are counterclockwise rotations about the center of the square at angles measuring 90°, 180°, 270°, and 360°. It may be checked that each of these symmetries results in a permutation of the colors in Fig. 1-b. For instance, under a reflection in the horizontal line through the center of the square, colors "black" and "white" are interchanged while the other two colors are fixed. The colored patterns in Fig. 1-c and Fig. 1-d are are not perfectly colored because under a 90° - counterclockwise rotation about the center of the square, color "black" goes to two colors and not just one color.

In this paper, we present a framework for the analysis and creation of colored symmetrical patterns, whether perfectly colored or not. We show the role played by three groups- the symmetry group G of the underlying uncolored pattern, the symmetry group K of the colored pattern (which is the subgroup of elements of G which fix all colors) and the group H of color transformations

which consists of the symmetries in G which permute the colors of the pattern. We treat a coloring as a partition of G or a decomposition of G into the union of mutually disjoint non-empty subsets wherein a unique color is assigned to each set in the partition with different sets getting different colors. It is assumed that the uncolored pattern is obtainable as the set of images under symmetries in G of a motif in a fundamental domain for G.

Perfect colorings are colorings where the group of color transformations H is equal to the symmetry group G of the corresponding uncolored pattern. It is well known that a perfect coloring corresponds to a partition of G into left cosets of a subgroup of G. In the general case, where H may be different from G, we show that a coloring corresponds to the partition of G into sets which are unions of right cosets of K, where K is the symmetry group of the associated colored pattern. Using the theory of permutation groups and group actions, we devise a method for determining all colorings of a symmetrical pattern where the subgroups H and K of the group G are specified. For example, if it is specified that the only symmetries of the pattern in Fig. 1-a that will permute the colors are the reflections in the diagonal of the square and the 180° and 360° -rotations about the square's center and with only the 360° -rotation and the reflection in the diagonal sloping upward to the right fixing the colors, we show how to arrive at all the non-equivalent colored patterns satisfying this condition. These are shown in Fig. 2.

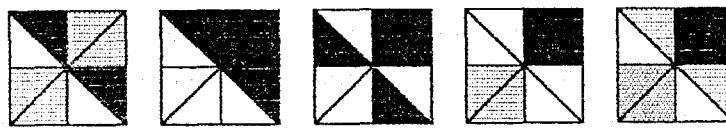


Figure 2.

The method described is applicable to symmetrical patterns in general. In particular, we exhibit illustrations for wallpaper patterns or two-dimensional crystallographic patterns.

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