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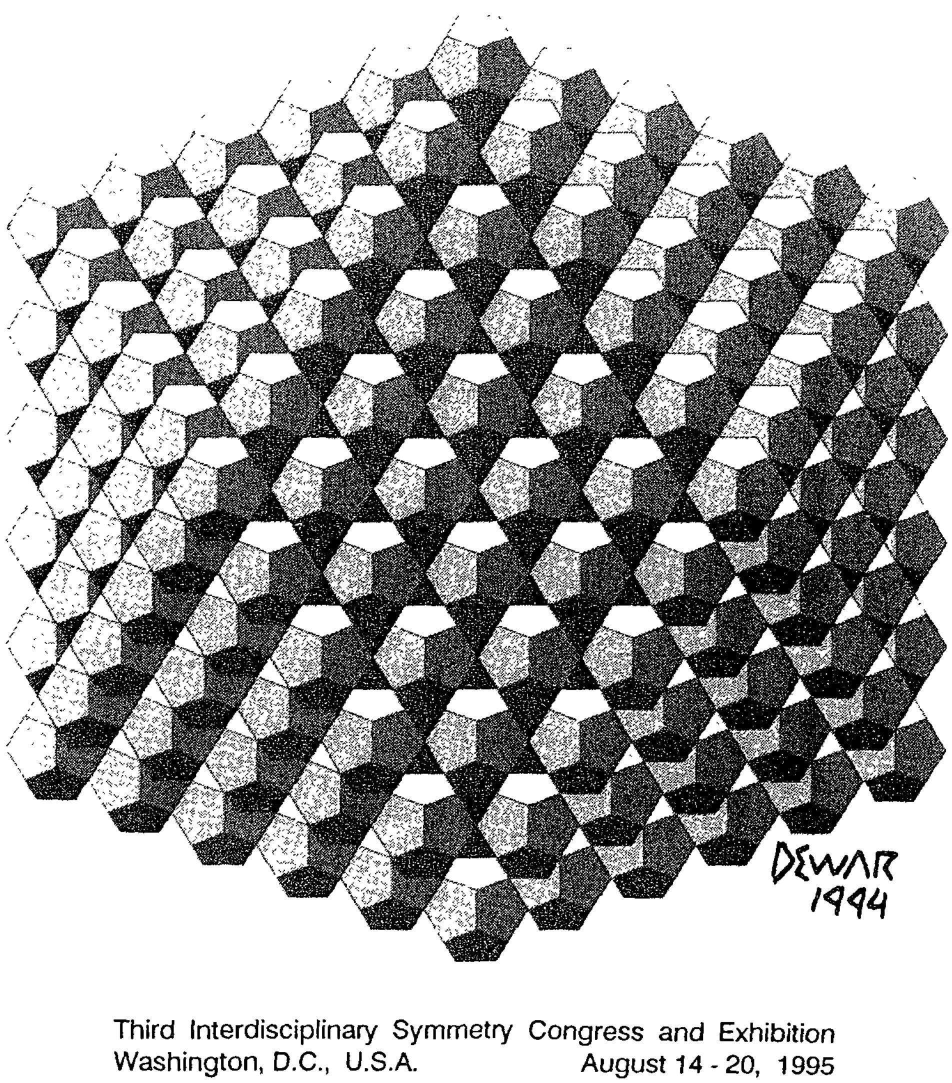

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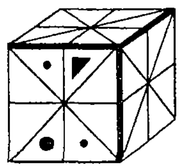
A SYMMETRY NOTATION THAT COACTIVATES THE COMMON GROUND BETWEEN SYMBOLIC LOGIC AND CRYSTALLOGRAPHY

Shea Zellweger
Mount Union College
Alliance, Ohio 44601 U.S.A.

The challenge before us is how best to share, and make use of, a strong commonality that feeds into an important interdisciplinary correspondence. It presents a classroom method that introduces the student to symbolic logic and crystallography in the same way, even if both introductory classes are not taken in the same semester. The surprise is that so much logic and crystallography fall out of each other, and so easily.

What follows will focus on two things. The first is an (8 x 6) chart, one that is a special case of a "wreath product" (2^n x n!), when (n = 3), and one that describes the 48 symmetries of a 3-cube (Coxeter 1991: 31). The second is a special notation, one in which symmetry becomes a work slave and one that lets mirror reflection and the compounding of mirror reflections become the primary muscle (Zellweger 1982, 1996).

(1)  $\equiv (\dot{h}\ \dot{k}\ \dot{l}) \equiv (\dot{A} \mathbin{\dot{*}} \dot{B})$

What follows has three parts, each according to the three terms in (1). The first term connects the 48 symmetries of a 3-cube with both the (8 x 6) wreath product in Table I and the subgroup partitions in Table II. The second term connects the 48 symmetries acting on the Miller index (hkl) with the symmetry notation in (2). The third term connects the symmetry notation in (2) with the master equivalence in (5).

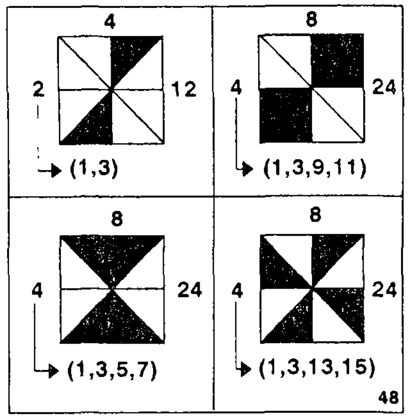
The first term in (1) introduces the group algebra for (abc), when the symmetries of a 3-cube are examined in terms of the (8 x 6) chart in Table I. Copies of this chart will serve as a standard form. The 48 symmetry elements are obtained from the Cartesian product of the *combinations* along the left and the *permutations* across the top. Selections of symmetry elements are then examined with respect to specified subsets. For example, and it will be used again, the cells in the first column (1-8) generate the (8 x 8) table for the 8-group variously called (C_2 x C_2 x C_2), (D_2 x C_2), and $(C_2)^3$.

The identity combination, 3 half-turns, 3 reflections, and central inversion (e, r,r,r, m,m,m, i) are acting on the identity permutation (abc) and three mirror planes (xyz).

Table I

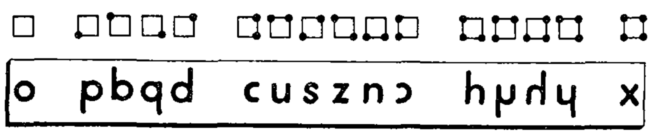
. . .	e abc	r bca	r cab	m cba	m bac	m acb
e OOO	1			9		
r ONN	2	I		10	III	
r NON	3			11		
r NNO	4			12		
m OON	5			13		
m ONO	6	II		14	IV	
m NOO	7			15		
i NNN	8			16		

Table II



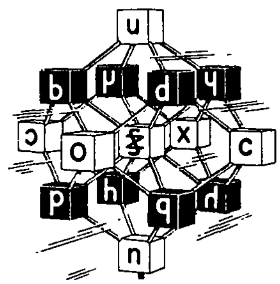
The first term in (1) also activates the subgroup partitions in Table II. The first 12-cell quadrant in Table I goes with the top-left panel in Table II. Then it combines with the other three quadrants (I-II, I-III, I-IV). Numbers to the left (2 4 4 4) are for subgroup patterns on the front face of the 3-cube; at the top (4 8 8 8), on the front and back faces; to the right (12 24 24 24), on all six faces. The 48 activates all of the cells in Table I. The 8 cells for $(C_2)^3$ are at the top-8 front-back in the lower left panel.

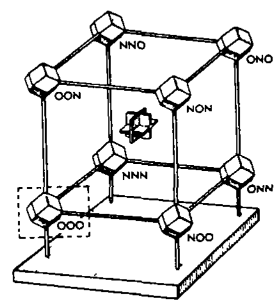
(2)



The second term in (1) introduces the crystallography of (hkl), when it is allowed to act on the symmetry notation in (2). The two rows depict 16-sets of dot-squares and letter-shapes. The letter-shapes have stems one for one where the dot-squares have enlarged corners. Instead of using letters of the alphabet to show single symmetry operations (mirror across a Y-letter, rotation of an S-letter, etc.), this notation constitutes a ***system of shorthand marks*** that is rich enough to isolate and examine the selected partitions in Table I and Table II. This classroom example on a grand scale covers 25 of the 32 symmetry point groups in 3-space (all but hexagonal).

The second term in (1) also activates the rhombic dodecahedron in (3). Here treated as an identity object, the 16 stem-shapes in (2) are placed at the vertices (two at the co-center). When the 48 symmetry operations act on this form as a total form, it generates a sunburst of 48 oriented dodecahedra. Nine mirrors would intersect at the center of (4). As one of the subgroups, the 3 mirrors and the 8 dodecahedra in (4) are for $(C_2)^3$.

(3) 

(4) 

The third term in (1) introduces the logic of (A * B), when the same shorthand in (2) is given another set of meanings. Consider the 2-valued propositional calculus. (A,B) is for any two primary propositions. The asterisk in (A * B) is for *any* of the 16 binary connectives (and, or, if, etc.). The 3 over-dots are for the 8 ways of negating (A * B) and the 3 under-arcs are for the 6 permutations of (A * B).

The logic code for (2) centers each dot-square on (x,y) axes, here called (A,B). The four corners (TT,TF,FF,FT) go clockwise from the upper right. Enlarged corners stand for (T)rue. The 4-dot square is for tautology (TTTT); likewise for all of the dot-squares. Each s(T)em also stands for (T)rue. The one-stemmed d-letter in (A d B) is for (TFFF), also called conjunction (A an-d B); likewise for all of the letter-shapes.

(5)
$$\left(\underbrace{\underset{R5\quad R6}{\overset{R1\quad R2\quad R3}{\dot{A}\,\dot{*}\,\dot{B}}}}_{R4}\right) \equiv \left(\underbrace{\underset{R5\quad R6}{\overset{R1\quad R2\quad R3}{\dot{A}\,\dot{*}\,\dot{B}}}}_{R4}\right)$$

The third term in (1) also activates the symmetry rules in (5). Rules (1,2,3) are for *combination mirrors* and Rules (4,5,6) are for *permutation mirrors*. Rule 4 changes the (A,B) order and Rules (5,6) change the position of the asterisk. Rules (1,2,3,4) belong to a game called "flip-mate-flip and flip" (f-m-f and f). R1 negates A. NA flips from left to right; (NA d B) becomes (A b B). R2 negates (mates) the letter-shape. N* reverses all possible stem positions; (A Nd B) becomes (A h B). R3 negates B. NB flips from top to bottom; (A d NB) becomes (A q B). R4 converts (A,B). This flip is along the dot-square diagonal that goes from upper-right to lower-left; (A d B) remains (B d A) because the dot in the d-letter dot-square stays in place.

Notation in logic (*F*)ixes the asterisk in three standard positions (*FFF*). The 48 subdivides into sets of 16-cells (D_4 x C_2) that cover pairs of columns in Table I (1-4, 2-5, 3-6) and pairs of faces on the 3-cube (front-back, right-left, top-bottom). The standard positions are ordinary in*F*ix (A * B), regular Polish pre*F*ix (* A B), and reverse Polish post*F*ix (A B *). This runs full fit into *the* perfect 3-coloring of a 3-cube.

Note especially that (5) is a master equivalence that is abstract enough, it has enough algebra in it, to unify three subsystems: propositional variables (A,B), connective variables (*,*), and operation variables (N,N,N,*F*,*F*,*F*). All rule-bound substitutions generate and become equal to exactly all well-formed atomic equivalences.

The challenge is to favor the student. The letter-shapes in (2) belong to a shape alphabet that introduces a symmetry notation that is robust enough to stand alike across the common ground between symbolic logic and crystallography. Unless there is one tucked away someplace in hiding, the claim is that no other notation is systems friendly enough to provide the analytic scope and unifying clarity revealed in (1).

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Zellweger, Shea (1982) Sign-creation and man-sign engineering, *Semiotica*, 38, 17-54. Displays (3) and (4) are repeats from this paper.

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