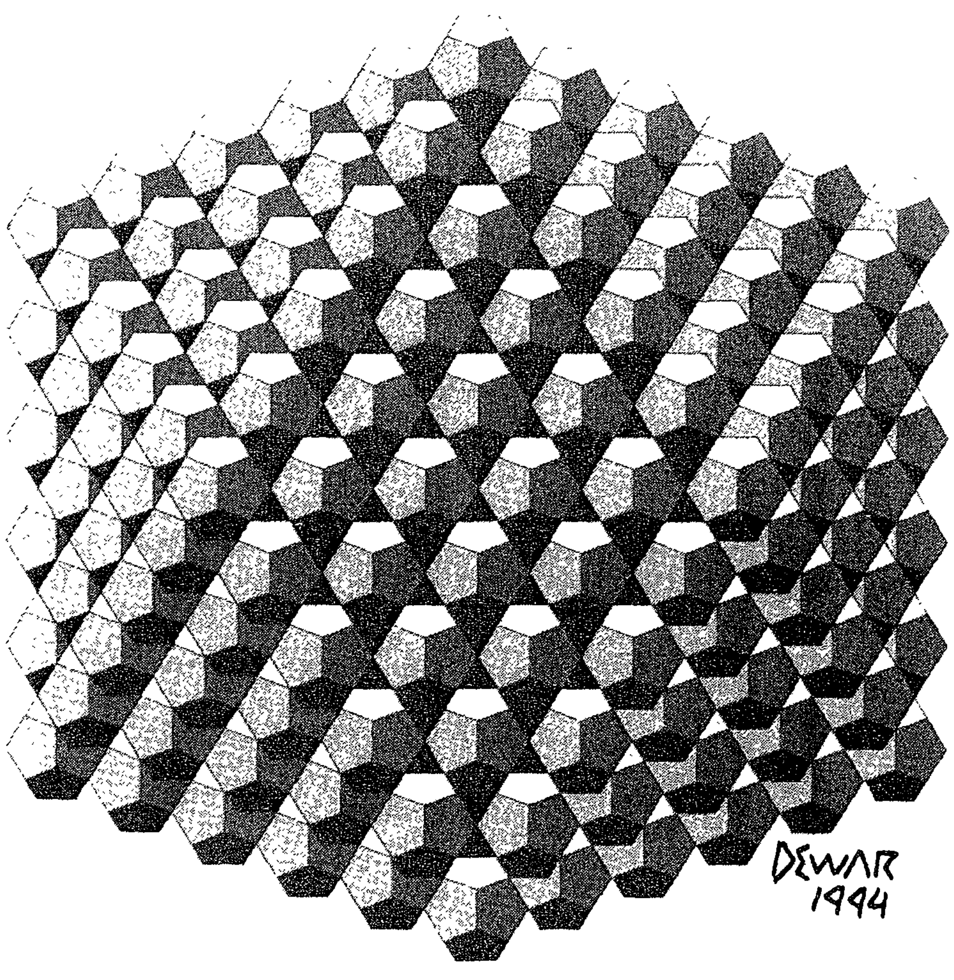
Symmetry: Culture and Science

Symmetry: Natural and Artificial, 3

The Quarterly of the International Society for the Interdisciplinary Study of Symmetry (ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 6, Number 3, 1995



Third Interdisciplinary Symmetry Congress and Exhibition
Washington, D.C., U.S.A. August 14 - 20, 1995

ORDER TO CHAOS TO ORDER
CHAOTIC BEHAVIOR IN FERROMAGNETIC RESONANCE

P. E. Wigen
Department of Physics
Ohio State Univ.
Columbus, OH 43210
E-mail wigen@mps.ohio-state.edu

The regular arc of a baseball or the periodic swing of a pendulum are examples in which nature determines a regular and predictable pattern of motion. Their position and velocity can be measured and predicted with a high degree of accuracy.

The rolling motion of the clouds in the sky or the turbulent gyrations of a drop of milk in a cup of hot coffee are examples of complex motion. But complex as the motion is, it too is determined by forces acting on the particles. While the behavior is complex to the eye and its future directions can be predicted with limited accuracy, the behavior is still deterministic.

In many complex systems, the deterministic behavior is observed to be chaotic. In order to see their beauty and symmetry, a new field of mathematical analysis has developed and with the aid of computers, characteristic parameters[1] have been developed that qualitatively categorize their behavior. Some of these measures include basins of attraction, Poincarè sections, fractal dimensions, Lyapunov exponents and Kolmogorov entropy.

Studies of chaotic systems tend to follow two approaches. In computer models the values of the variables in a set of coupled non-linear equations, such as the Lorentz equations, are evaluated as a function of the magnitudes of the various coupling coefficients in the equations. Using such analysis in a variety of systems, a search is made for the universality of the routes to chaos and the chaotic behavior.

In experimental systems the dynamic response of some variable is investigated as a certain drive parameter is varied. Again the routes to chaos and the chaotic behavior are determined and compared with any universal behavior patterns that have been developed from the models. However, it is often impossible to model these non-linear systems because they are so complex that the equations of motion cannot be derived or the coefficients of the various terms in the equations cannot be evaluated for comparison with the experiments. Rolling clouds and turbulent fluids are two examples that are much too complex for such detailed analysis.

A significant challenge in chaos research is the investigation of systems that have sufficient complexity that details of the route to chaos can be investigated but simple enough that a model can be developed and predictions of the behavior evaluated. In such systems it is possible to compare directly the model and the experiment, to evaluate the universality considerations and to develop methods that will allow the experimenter to influence the chaotic motion of the system in a controlled fashion[2].

Many "simple" systems have been developed in which the routes to chaos and chaotic behavior have been investigated in a controlled fashion. These include turbulence in thin fluid sheets[3], turbulence in rotating cylinders[4], vibrations of a magnetic ribbon[5], coupled electronic oscillators[6] and irregular oscillations in optical lasers[7].

Another "simple" system is that of magnetic resonance[8]. In ferromagnetic solids, the atomic magnetic moments are coupled via a coulombic "exchange interaction" and they align parallel to each other. In ferromagnetic resonance (FMR) the normal modes of the precession of the moments include small variations in the amplitude and/or the phase of the adjacent moments. Depending on the geometry of the sample, various "normal modes" of the magnetic moments can be excited. These modes would be analogous to a vibrating string on a violin or a guitar or to the more complicated patterns associated with the vibrations of the membrane of a drum head.

The FMR spectra in thin discs of yttrium iron garnet ($Y_3Fe_5O_{12}$) consists of normal modes that have an amplitude and phase of precession that have the same pattern as the amplitudes of the vibrations on a drum head. Since each of these modes have a different shape they will have different energies and therefore resonate at different frequencies. A typical spectrum is shown in Figure 1.

While each of these normal modes can be excited individually, there is a coupling between them giving rise to a non-linear interaction. As the perpendicular driving field is increased, any one of the modes is observed to progress from the steady state, precession at constant amplitude, to a state of auto-oscillation in which the precession angles gyrate in and out of their average values. As the driving field is further increased, the periodic oscillation is observed to follow a bifurcation route to chaos. In figure 1, the shaded region in magnetic field versus strength of the driving field are positions for which the angle of precession is non-constant or oscillating.

In magnetism it is possible to evaluate the energy terms (the spin Hamiltonian). From this expression it is possible to develop the equations of motion of the magnetic moments including the form and the magnitude of the non-linear terms. As a result it is possible to develop a model in which all of the parameters are fixed and a very careful analysis of the model with the experimental results can be made. The agreement between the experimental results and the predictions of the model are unusually close for such a complex system.

Having a chaotic system that is so well behaved is it possible to influence the chaotic response of the precessing spins by perturbing a system parameter? In this case the system parameter to vary is the applied magnetic field and three different techniques have been used to change the chaotic motion into a desired and predetermined behavior.

In the first experiment, a small sinusoidal time dependent variation is applied to the magnetic field[9]. As the frequency of the sinusoidal modulation is varied, there are certain frequencies at which the chaotic behavior is quenched and an auto-oscillation is observed that is at the frequency of the perturbing field or at multiples of it. Typical results of the experiment are shown in Figure 2.

In a second experiment, the goal is to prevent any oscillation in the precessing moments even when they are driven at amplitudes that are of sufficient magnitude that without the perturbation the system would behave chaotically[10]. Using a time delayed feedback technique, it is possible to control the chaotic motion by reducing it to a constant amplitude via a reverse bifurcation or de-bifurcation route. Typical results are shown in Figure 3.

In the third experiment, the goal is to have two samples both in the chaotic regime but to have them "phase-locked" or synchronized together. In the experiment the chaotic signal from the oscillator is stored in a computer and at a later time a feedback method is again used to control the behavior of the ongoing precession to follow the same behavior observed at the time when the signal was stored in the computer[11].

In each case, the model predicts the behavior out of chaos observed in the experiments with the same high degree of accuracy that was observed in the behavior of the system on the route into chaos.

Ferromagnetic resonance is a system in which the experiments are nicely controlled and details of the route from order to chaos can be investigated and compared to the model. Once in the chaotic state it is also possible to use perturbation techniques to influence its behavior returning from the state of chaos to one of regular order.

1. Chaos and Nonlinear Dynamics", R. C. Hilborn, Oxford Univ. Press., Oxford, 1994.

2. E. Ott, C. Grebogi and J. A. Yorke, Phys. Rev. Letts. **64**,1996(1990).

3. A. Libchaber, C. Laroche and S. Fauve, J. Physique Lett. **43**, 211(1982).

4. A. Brandstater and H. L. Swinney, Phys. Rev. A **35**, 2207(1987).

5. W. L. Ditto, S. N. Rauseo and M. L. Spano, Phys. Rev. Letts. **65**, 3211(1990).

6. E. R. Hunt, Phys. Rev. Letts., **67**, 1953(1991).

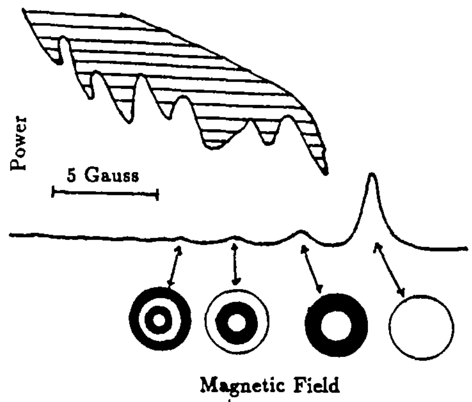
7. R. Roy, T. W. Murphy,T. D. Maier and Z. Gills, Phys. Rev. Letts.,**68**, 1259(1991).

8. "Nonlinear Phenomena and Chaos in Magnetic Materials", P. E. Wigen, Ed., World Scientific Press, Singapore, 1994.

9. R. D. McMichael and P. E. Wigen, Phys. Rev. Letts. **64**, 65(1990).

10. M. Ye, D. W. Peterman and P. E. Wigen, Phys. Letts.(to be published).

11. D. W. Peterman, M. Ye and P. E. Wigen, Phys. Rev. Letts. **74**, 1740(1995).



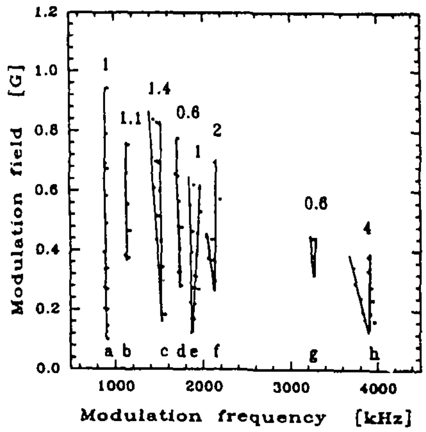
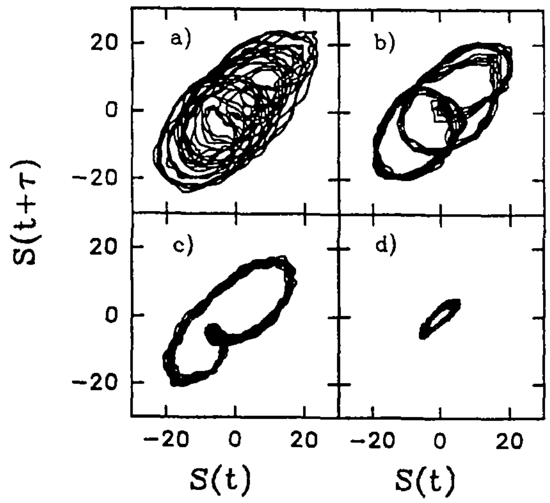


Figure 1 (left) The FMR spectra of a magnetic garnet film. The circular patterns indicate the variations of the phases of the magnetic moments in the various modes excited. The shaded region indicates the regions in power vs. magnetic field space where the moments have a time variation in the precession cone angle (periodic or chaotic).

Figure 2. (right) Regions of stabilization of chaotic signals to periodic orbits in the modulation field vs frequency plane. The numbers denote the frequency (in MHz) of the periodic oscillation after stabilization.



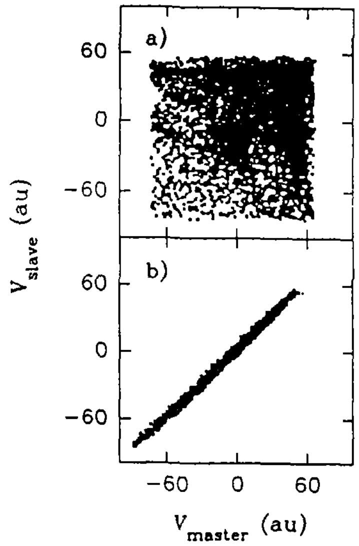


Figure 3 (left) a) Two dimensional phase diagram of a chaotic signal. With the application of a time delayed perturbation the signal is reduced to a period four b), period two c), period one d) and finally to the quiescent state (not shown).

Figure 4 (right) Signal amplitudes of the master (stored) signal vs. that of the slave (synchronized) signal, a) without the control perturbation and b) with the control