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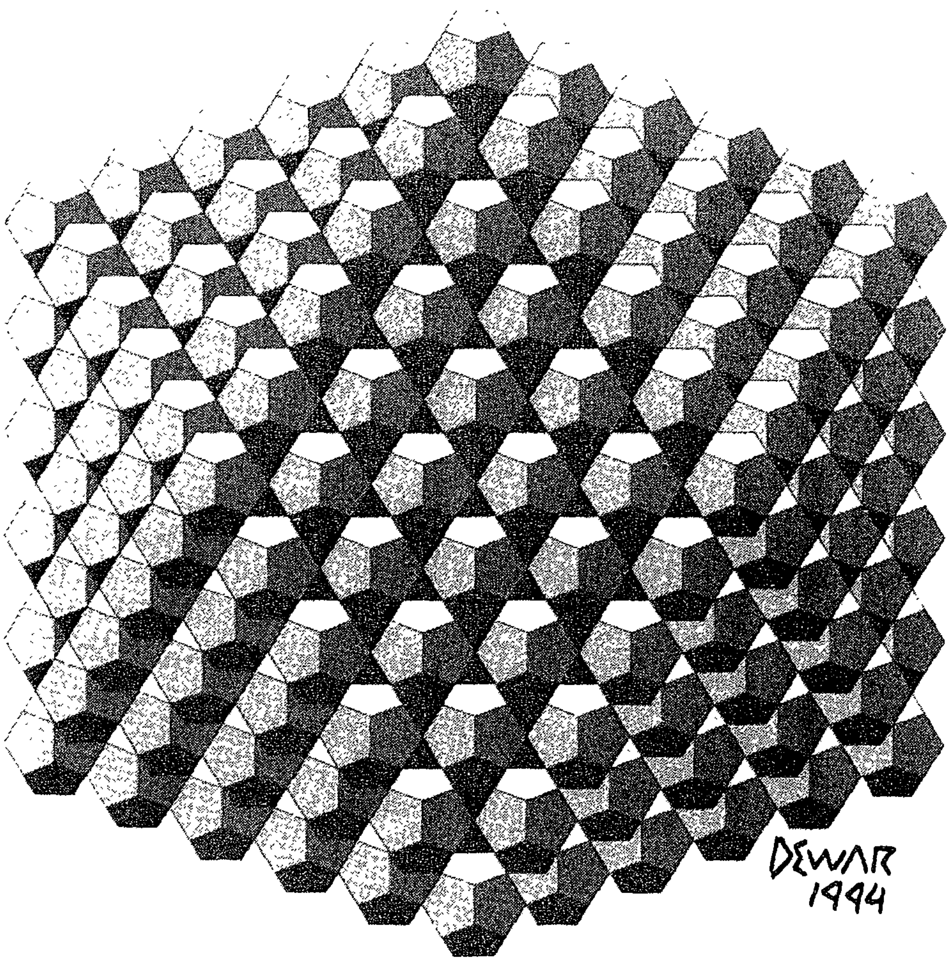

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BLOWOUT BIFURCATIONS

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Recently, it has been shown [Ott & Sommerer 1994] that two striking types of dynamical behavior, riddled basins of attraction [Alexander et al. 1992, Sommerer & Ott 1993] and on-off intermittency [Pikovsky 1984, Platt et al. 1993], are closely related phenomena, actually representing different aspects of a symmetry-breaking bifurcation. These phenomena arise in systems that possess chaotic dynamics *in a smooth invariant manifold of lower* dimension than the full phase space of the dynamical system. Both phenomena involve the sudden departure of a system trajectory from the vicinity of the smooth invariant manifold, so we refer to the bifurcation as a "blowout." Both of the phenomena have serious implications for application areas such as communications systems based on coupled oscillators, reaction diffusion systems that admit chaotic oscillations, and many other systems exhibiting this common form of symmetry.

In the case of riddled basins of attraction, the chaotic dynamics in the invariant manifold attracts a positive volume of points (the basin of attraction) in the full phase space. However, for *every* point in the basin, *any* ball centered on that point *always* has pieces of another attractor's basin in it, and those pieces have positive phase-space volume. Thus, the basin of attraction is "riddled" with pieces of another basin. This situation is illustrated with various sections through the phase space of a simple two-degree-of-freedom oscillator in Fig. 1.

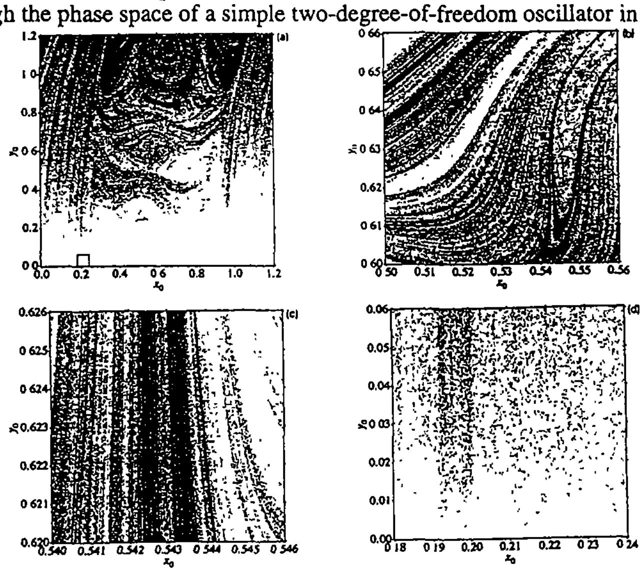

Fig. 1

The figure represents initial conditions going to an attractor off the system's invariant manifold as black dots; those initial conditions attracted to the chaotic set in the invariant manifold are represented as white. The basin riddling is manifested by there being black dots in any sufficiently magnified blowup around any white point. Basin riddling is a particularly pathological dynamical phenomenon. When encountered in a physical system, it means that experiments are essentially not reproducible. Unavoidable perturbations to serial repetitions of an experiment, no matter how small, can generically lead to qualitatively different outcomes. Riddled basins have been experimentally confirmed in coupled electronic oscillators [Heagy et al. 1994], so this symmetry-induced pathology cannot be dismissed as irrelevant to practical applications.

In the case of on-off intermittency, the chaotic set in the invariant manifold is not an attractor in the full phase space, but rather is marginally unstable. Near the bifurcation where the set first becomes unstable, however, nearby system trajectories spend large amounts of time extremely *near* the invariant manifold. It may then burst away from the vicinity of the unstable chaotic set. If the global dynamics far from the set can return the system trajectory to the vicinity of the invariant manifold, the scenario can repeat indefinitely, producing an extreme form of temporal intermittency where arbitrarily long periods of quiescence are intersperse with bursts, as in Fig. 2.

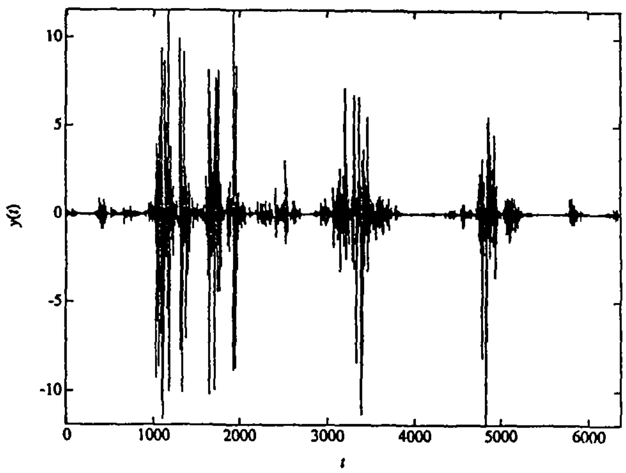

Fig. 2

The time series shown in Fig. 2 is taken from the same system as produced Fig. 1, but at slightly different parameter values, illustrating that the phenomena are closely related.

Although temporal intermittency of the type shown in Fig. 2 is not so deeply disturbing to physical intuition as riddled basins of attraction, on-off intermittency can have serious complications for application areas. For example, communications systems depend on synchronized oscillators. Covert communications systems using *chaotic oscillators have been* proposed. In this context, the state of synchronization between coupled oscillators implies an invariant manifold and the type of symmetry needed for blowout birfurcations. In fact it has been shown experimentally [Ashwin et al., 1994] that coupled chaotic oscillators can show intermittency of the type shown in Fig. 2 as a result of a blowout bifurcation. Finally, the importance of coupled oscillators in a biological context further supports the idea that this symmetry-breaking bifurcation has an important role in applications.

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