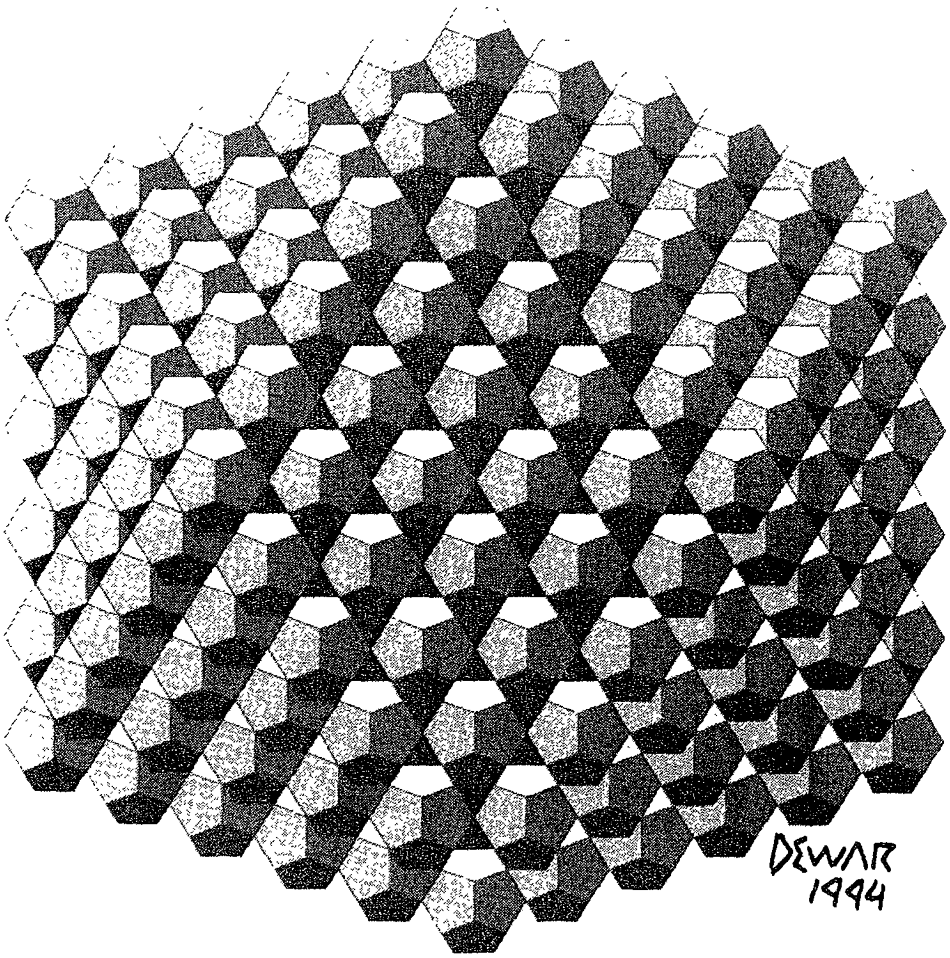
Symmetry: Culture and Science

Symmetry: Natural and Artificial, 3

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 6, Number 3, 1995



Third Interdisciplinary Symmetry Congress and Exhibition
Washington, D.C., U.S.A. August 14 - 20, 1995

SOME RESULTS IN THE BENDING'S THEORY OF POLYHEDRA

I.Kh. Sabitov
Faculty of Mechanics and Mathematics, Moscow State University, Moscow, 119899, Russia
E-mail: sab@sci.math.msu.su

1. It is known that: 1) almost all polyhedra are rigid and 2) there are flexibles ones. Thus having a given polyhedron we cannot say at once is it rigid or not and we need some criterion for a definite answer. For certain classes of polyhedra these criteria are known sufficient or necessary for the rigidity / nonrigidity (f.e. this is the case of convex polyhedra, suspensions etc.) but it is evident that we couldn't have a general and in the same time sufficiently effective criterion. In these conditions one can propose an algorithmical approach to the question: a polyhedron being given to indicate a finite algorithm for the verification of its bendability. We can give an algorithm of this kind for the polyhedra having only one linearly independent field of 1-st order infinitesimal bending (in its turn for the verification of this property it is sufficient to calculate the rank of a matrix defined by the coordinates of polyhedron's vertices), see (Sabitov,1994). The algorithm is realised for the computer calculation and is verified for certain known flexible / rigid polyhedra. For a polyhedron having two linearly independent infinitesimal bendings we (the author and O.Pavlova) can also propose an algorithm but this result yet no published.

An another approach is described in (Sabitov,1987) but the method is extremly depending on the polyhedron's combinatorial structure and is applicable only after a preliminary elaborate study of the polyhedron.

2. Among the known embedded flexible polyhedra one of Klaus Stefen has only 9 vertices In (Maksimov,1995) it is shown that this number 9 is the least possible: if an

ımmersed polyhedron with 8 or less vertices is bendable then its generalised volume is 0 and it can't be embedded.

3. After the discovery of the existence of flexible polyhedra it was remarqued that for all known flexible polyhedra theirs volume is not changing in the process of flexion. Therefore R.Connelly in his report on the Helsinki International Mathematical Congress in 1978 has conjected quoting D.Sullivan that this property is general ("bellows conjecture"). The author has proposed in (Sabitov, 1992) and (Ivanova-Karatopraklieva and Sabitov,1995) an approach to this hypothesis from a standpoint more general: it is enough to show that the volume of any polyhedron is root of a polynomial with coefficients depending only on metric of the polyhedron. The proof of this extended Connelly-Sullivan hypothesis would give first a positive answer to the bellows conjecture second a possibility to calculate the possible volume's values of a polyhedron by its intrinsic metric and combinatorial structure without even its realisation in three-space. This approach is realised in (Pavlova, 1995) for suspensions. In a more general case this is made in the author's work (Sabitov,1995).

A sketch of proof. We say that a polyhedron with triangle faces possess the property B if its volume is root of a polynomial coefficients of which depend only on the edge's lengths. Let a polyhedron P of any topological genus with n vertices have a vertex of degree 4. We eliminate the open star of this vertex and close the obtained 4-gonal hole H by two triangles: in the first time we consider as given a diagonal of H (and by this manner we ob tain a polyhedron P1 with n-1 vertices) and in the second time we ta ke as known the other diagonal of H and obtain an another polyhedron P2. A lemma says that if polyhedra P1 and P2 possess the property B then P is so too. Thus if P is situated on the base of a graph-tree which has final elements (may be of different level) possessing the property B then the extended hypothesis for P is true. For example, it is the case for all polyhedra of genus 0 with $n < 12$ vertices. It is true also for so-called combinatorially one-parametric polyhedra (a polyhedron P is said combinatorially one-parametric if its verti ces may be found successivly by fixation of the length of a diagonal of P; f.e. all suspensions are combinatorially one-parametric).

A polynomial Q for the volume of a polyhedron P being obtained a question arises about the minimality of degree of Q. In the work (Astrelin and Sabitov, 1995) this question is solved for the octaedra: the minimal degree of the volume polynomial for an octaedron is 8 and more this number is the best possible.

4. It is known that in the classic definition of the higher order infinitesimal bendings there is some logic defect(Sabitov, 1992a); also the definition of the n-order infinitesimal rigidity requiers some refiniment (Connelly and Servatius,1992). We can propose for the discussion a definition of n-order rigidity (Sabitov, 1992b) naturally related with the definition of the high order infinitesimal bendings from (Sabitov, 1992a).

The work is supported by RFFI, projet No 93-01-00154.

Astrelin, A.V. and Sabitov, I.Kh. (1995) Minimal degree polynomial for the volume calculation of an octaedron by its metric, *Uspekhi matematicheskikh nauk,* Vol.50, No 3 (to appear).

Connelly, R. and Servatius, H. (1992) Higher Order Rigidity - What is the Proper Definition? *Preprint, Cornell University, Itaca,* 10 p.

Ivanova-Karatopraklieva, I. and Sabitov, I.Kh. (1995) Bending Bending of surfaces, Part II, *Journal of Mathematical Sciences,* Vol.74, No 3 (to appear).

Maksimov, I.G. (1995) Bendable polyhedra and Riemannian surfaces, *Uspekhi matematicheskikh nauk,* Vol.50, No 2 (to appear).

Pavlova, O.V. (1995) Suspension's volume as a function of length of its edges, *Uspekhi matematicheskikh nauk,* Vol.50, No 2 (to appear).

Sabitov, I.Kh. (1987) An algorithmical verification of the bendability of suspensions, *Ukrainski Geometricheskii Sbornik,* No 30, 109-112.

Sabitov, I.Kh. (1992a) Local theory of bendings of surfaces, in: *Encyclopaedia of Mathem. Sc.,* Vol.48, Geometry III, Springer-Verlag, 179-250.

Sabitov, I. Kh. (1992b) On the relations between the infinitesimal bendings of different order, *Ukrainskii Geometricheskii Sbornik,* No 35, 118-124.

Sabitov, I.Kh. (1994) On an algorithm for the bendability verification of polyhedra, *Vestnik Moskovskogo Universiteta. Ser.1. Mathematics and Mechanics,* No 2, 56-61.

Sabitov, I.Kh. (1995) On volume invariance hypothesis for bendable polyhedra. *Uspekhi matematicheskikh nauk,* Vol.50, No 2 (to appear).