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Japanese mathematics in the 17th-19th centuries is called Wasan, and left many results especially in geometry in the plane as well as in 3-dimensional space. These are concerning about elementary figures as triangles, quadrangles, circles, spheres, etc.. The results are stated as problems with their final solutions, where no explanation how to reach the final solutions is added in most cases.

Sometimes we can generalize such problems. Furthermore we can construct discrete patterns consisting of figures used in those problems. In this article we demonstrate such patterns in the plane arising from those results. In what follows we cite figures in Wasan problems and related patterns.

Figure 1 used in a problem in Fujita’s book. Figure 2 is a pattern consisting of this figure (Okumura 1989).
Figure 3 can be found in Ushijima's 1832 book, and from this we can construct Figure 4 (Okumura 1990).

Figure 3.

Our next pattern (see Figure 6) does not consist of figures which can be found in Wasan problems, but was found when the author was generalizing the following problem (Okumura 1994), which can be found in Yamamoto's 1841 book. It was also published in a recent publication by Fukagawa and Pedoe (see Figure 5).

Problem. In the circle $C$ of radius $r$ let $AB$ be a chord whose midpoint is $M$. The circle $C_0$ of radius $r_0$ ($r_0 < r/2$) touches $AB$ at $M$ and also touches $C$ internally. Let $P$ be any point on $AB$ distinct from $A$, $B$ and $M$; a circle $C_2$ of radius $r_0$ (equal to the radius of $C_0$) touches $AB$ at $P$ on the other side of $AB$. Distinct circles $C_1$ and $C_3$ of radii $r_1$ and $r_3$ touch $AB$, and each touches $C$ internally and $C_2$ externally. Show that $r = r_1 + r_2 + r_3$. 
Figure 7 can be found in a problem dated 1836 (Hirayama and Yamaki). The figure can be embedded in a pattern described in Figure 8.

References
Fujita (1781) *Seiyo Sampō*, Volume 3.
Ushijima (1832) *Zoku Sangaku Shōsen*.
Yamamoto (1841) *Sampō Jojutsu*. 