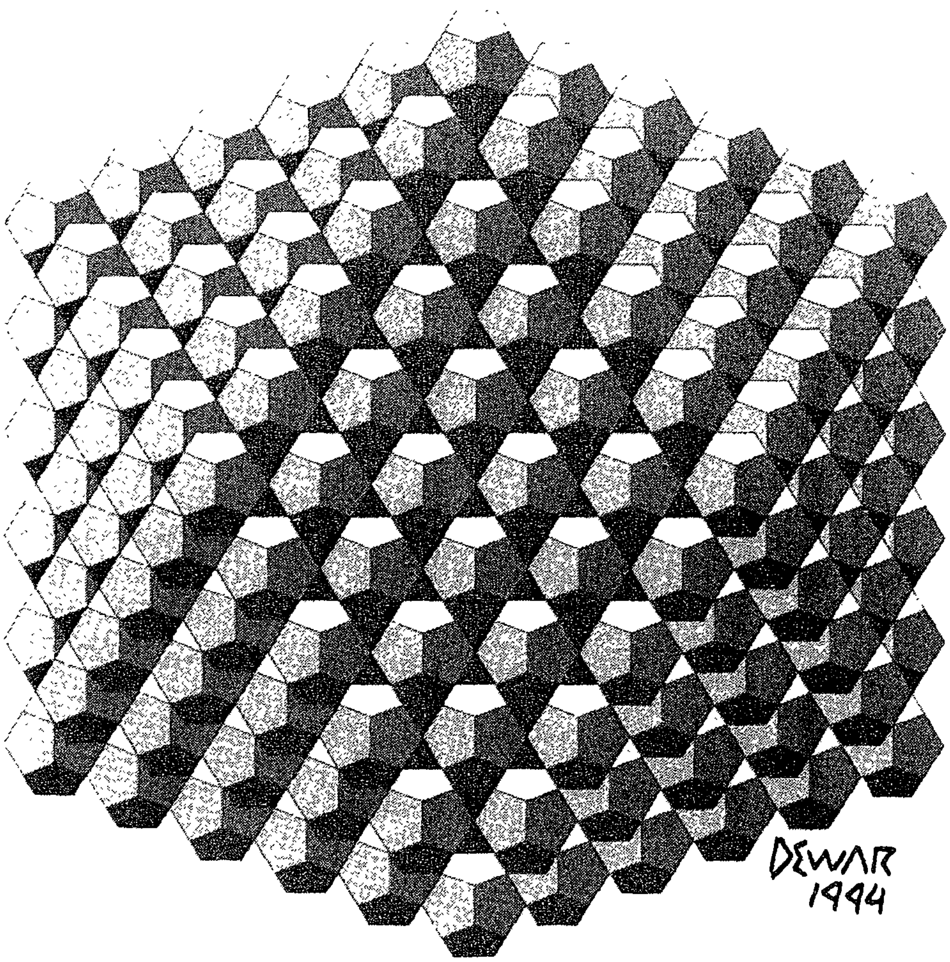
Symmetry: Culture and Science

Symmetry: Natural and Artificial, 3

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 6, Number 3, 1995



Third Interdisciplinary Symmetry Congress and Exhibition
Washington, D.C., U.S.A. August 14 - 20, 1995

DISCRETE PATTERNS ARISING FROM RESULTS IN JAPANESE GEOMETRY

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Japanese mathematics in the 17th-19th centuries is called Wasan, and left many results especially in geometry in the plane as well as in 3-dimensional space. These are concerning about elementary figures as triangles, quadrangles, circles, spheres, etc.. The results are stated as problems with their final solutions, where no explanation how to reach the final solutions is added in most cases.

Sometimes we can generalize such problems. Furthermore we can construct discrete patterns consisting of figures used in those problems. In this article we demonstrate such patterns in the plane arising from those results. In what follows we cite figures in Wasan problems and related patterns.

Figure 1 used in a problem in Fujita's book. Figure 2 is a pattern consisting of this figure (Okumura 1989).

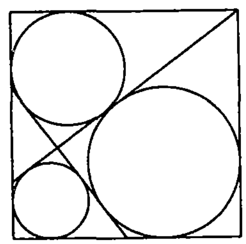

Figure 1.

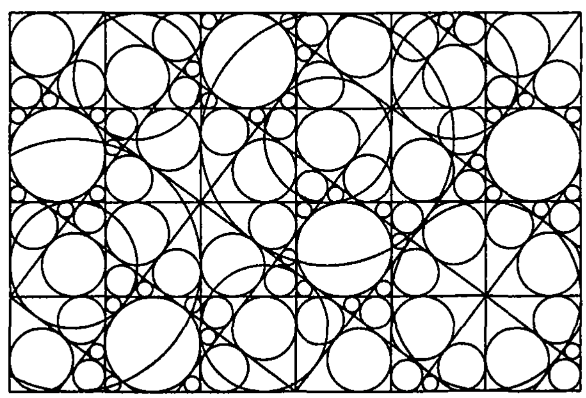

Figure 2.

Figure 3 can be found in Ushijima's 1832 book, and from this we can construct Figure 4 (Okumura 1990).

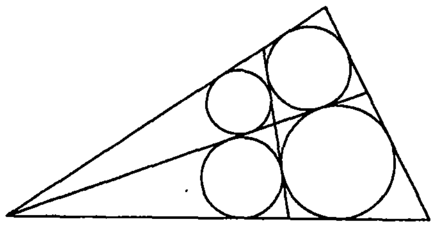

Figure 3.

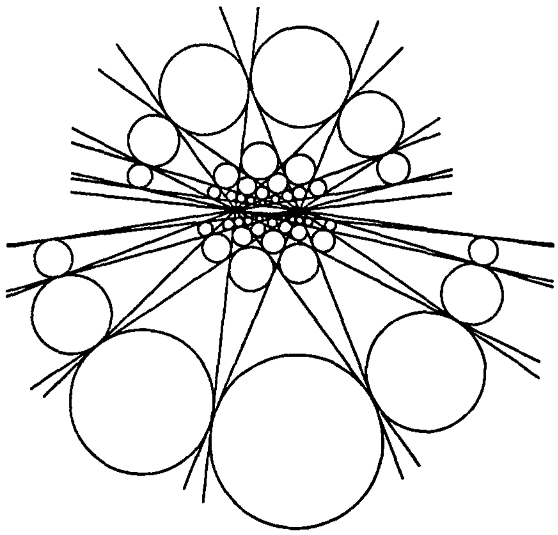

Figure 4.

Our next pattern (see Figure 6) does not consist of figures which can be found in Wasan problems, but was found when the author was generalizing the following problem (Okumura 1994), which can be found in Yamamoto's 1841 book. It was also published in a recent publication by Fukagawa and Pedoe (see Figure 5).

Problem. *In the circle C of radius r let AB be a chord whose midpoint is M. The circle C_0 of radius r_0 ($r_0 < r/2$) touches AB at M and also touches C internally. Let P be any point on AB distinct from A, B and M; a circle C_2 of radius r_0 (equal to the radius of C_0) touches AB at P on the other side of AB. Distinct circles C_1 and C_3 of radii r_1 and r_3 touch AB, and each touches C internally and C_2 externally. Show that $r = r_1 + r_2 + r_3$.*

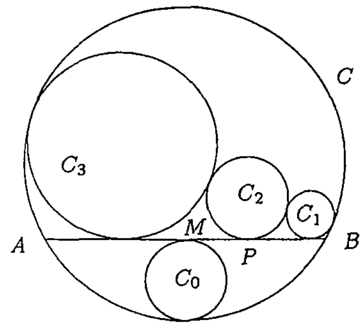

Figure 5.

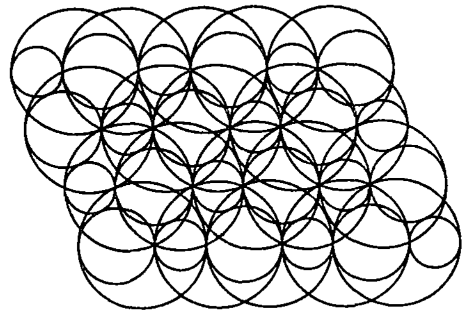

Figure 6.

Figure 7 can be found in a problem dated 1836 (Hirayama and Yamaki). The figure can be embedded in a pattern described in Figure 8.

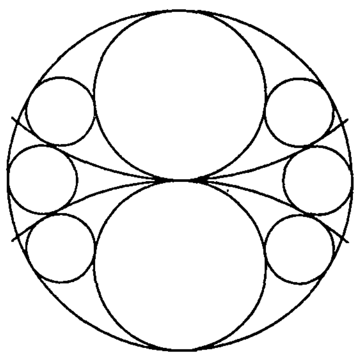

Figure 7.

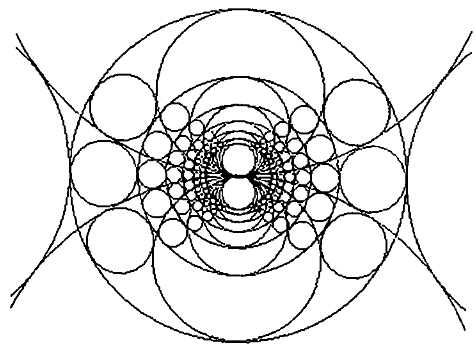

Figure 8.

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