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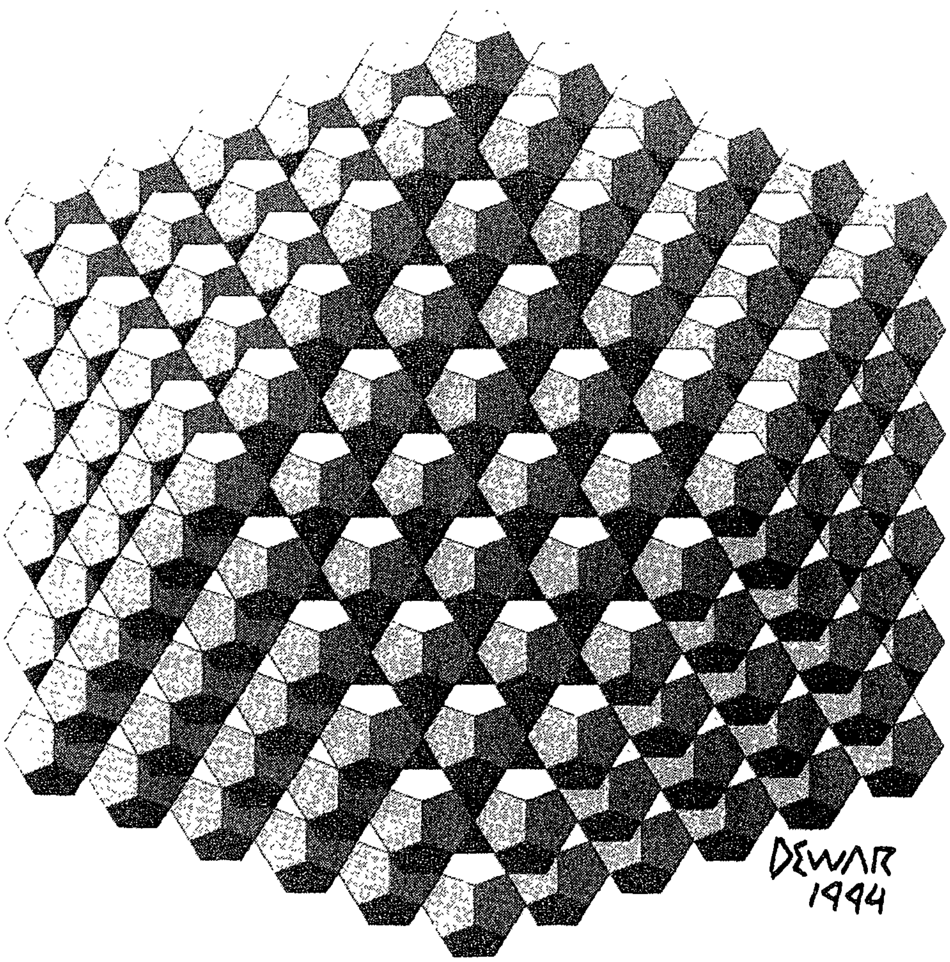

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ON THE SISTEM OF MUSICAL SOUNDS

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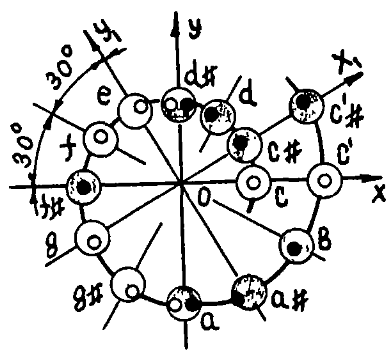
In nature there are no analogues for musikal sounds, according to Lord Rayleigh, a physicist, famous for his works not onli in the theori of the sound [1]. Having agreed with this premise, one has to suppose that the musical sound, just as the whole system of such sounds, is the creation of Man in line with his nature. It is through his perception that musical intervals are divided into two categories - consonant (consonanse - C) and dissonant (dissonance - D), supposing harmoni and chord in the first case, and opposite sensations in the second case. Not only professionals in music will always be therebi interested in the study of these fundamentals of music, adequate to human nature.

C intervals, with the relative frequency characteristic of which ρ_1 being strictly determined, are the basis for the construction of the system of musical sounds, organized within the limits of the octave [1]. If perfect unison is set at $\rho_1 = 1$, than perfect octave will have $\rho_2 = 2$, perfect fifth - $\rho_3 = 3/2$, major sixth - $\rho_4 = 5/3$, minor sixth $\rho_5 = 8/5$, perfect fourth - $\rho_6 = 4/3$, minor third - $\rho_7 = 6/5$, major third - $\rho_8 = 5/4$. Acceptind as a tonic some musical sound, naming it do and designating with a beat the absolute frequency of its vibrations, we can then identifi the notions of the musical sound and the relevant interval. To construct the musical scale as a whole, within the bounds of one octave, it is sufficient to have the first five intervals, which, it should be emphasized, are equal to the first five approximations to the golden proportion by Fibonacci's numbers: $\rho_n = d_{1(n+1)} = 1 + 1/d_{1n}$; $d_{11} = 1$, $n = 1,..5$. The remaining C intervals follow from the inversion of perfect fifth, major and minor sixths: $\rho = d_{n+3} = d_{12}/d_{1n}$, $n = 3, 4, 5$. A further construction is linked with depicting C intervals as points on the logarithmic spiral; $\rho = \exp(\varphi \ln2/2\pi)$. One coil of the spiral corresponds to one octave, the length of the polar radius ρ_i - to the relatiye frequensy, and the polar angle φ_i - to the arrangement of the interval. φ_i (in the drawing - light small circles) are to be found by the known ρ_i. For a number of coils - octaves the sounds of the same name are placed on the beams coming through the constructed points.



Proceeding from the D notion, we shall determine C with points, diametrically opposite to those constructed (dark small circles). The frequency characteristics of C intervals are on the known values of the D angles. The constructed system of sounds quantitatively differs from the existing one (15 against 13 in the cromatic scale). In the chord, when the frequency of the sound ρ_c is equal to the product of the frequencies of all components, there arises a comma, i.e. the value ρ_c does not coincide with any of the points on the coils of the spiral, - a new sound, initially undetermined, is forming. The system is becoming wider, and cannot therefore be used in practice. The onli way of avoiding the comma is equal temperament [1], which reduces to adjusting the frequencies through regulating angular intervals. The system of sounds in this case is described by the same equation of the spiral with the constant spacing of angular intervals $\Delta\varphi = 30^0$. In the drawing, eqal temperament accords with big circles the arrangement of which with respect to the initial construction is in a way characterized by the terminogy accepted in music: fairly absolute C (zero deviation,- perfect unison and octave), absolute C (deviation by an order of 0,1% - perfect fourth and fifth), the non-absolute C notion calls for a separate discussion. It should be noted that D intervals have no such characteristics. The non-absolute C relates to major third and minor sixth, deviating from the Cinterval by 0,8%; minor third and major sixth, with two opposite intervals aligned as one. And if in the first case this definition looks justified, in the second case it seems to be more logical to give the name of "rather non-absolute C", or, equally,- "rather non-absolute D", since the geometrical mean of frequensy characteristics of the initially constructed C and D intervals exactly corresponds to minor third and major sixth. Apparently, this case measures up to the neutral (zero) qualitative characteristic of the intervals.

In the system initially constructed from C one witnesses diametrical asymmetry of C and D intervals; rigorous angular and qualitative symmetry of C and D with respect to the OX axis and the same asymmetry with respect to the OY axis. In the temperament the upper and the lower semi-planes symmetrically deformed with respect to the OX and OY axes, with the axial symmetry and asymmetry relationships being retained therefore,also retained is the diametrical asymmetry of the C and D intervals with due account for neutral exponents on the OY axis. If qualitative characteristics of C are extended to D according to the same principle, one comes to the definitions of twin, diametrically opposite C - D intervals the image of which on the drawing needs no explanation. Perfect unison - augmented fourth; fairly absolute C - D; minor second - perfect fifth: absolute D - C ; major second - minor sixth: non-absolute D - C; minor third - major sixth: is not characterized; major third - minor seventh: non-absolute C - D; perfect fourth - major seventh: absolute C - D; perfect octave - augmented fourth: fairly absolute C - D. This rounds off the octave-determining cycle.

The dominants, the second hold of the mode, is also, to a certain extent, linked with the notions of golden ratio, symmetry and asymmetry. Using another algorithm of approximation to the golden ratio: $d_{2(n+1)} = (1 + d_{2n})^{0,5}$, $d_{21} = 1$, one can be convinced that the axis of the OX_1 dominant divides in the third quarter the interval 2π with an error of 0,1% in the geometrical mean sense in the ratio equal to the third approximation d_{23}. Considering the zero characteristics of minor third and major syxth, which can be imparted any interpretation with the term "fairly non-absollute.." the axis of the OX_1 dominant for the tempered scale can be (conditionally) defined as the second axis of asymmetry, while OY_1, perpendicular to it, - as the second axis of symmetry for the subsequent octave interval, and this finalizes these notions in the sistem of musical sounds.

Reference

1. Стретт Дж. В. (Лорд Релей). Теория звука. - М.: Техтеориздат, 1955, т 1.