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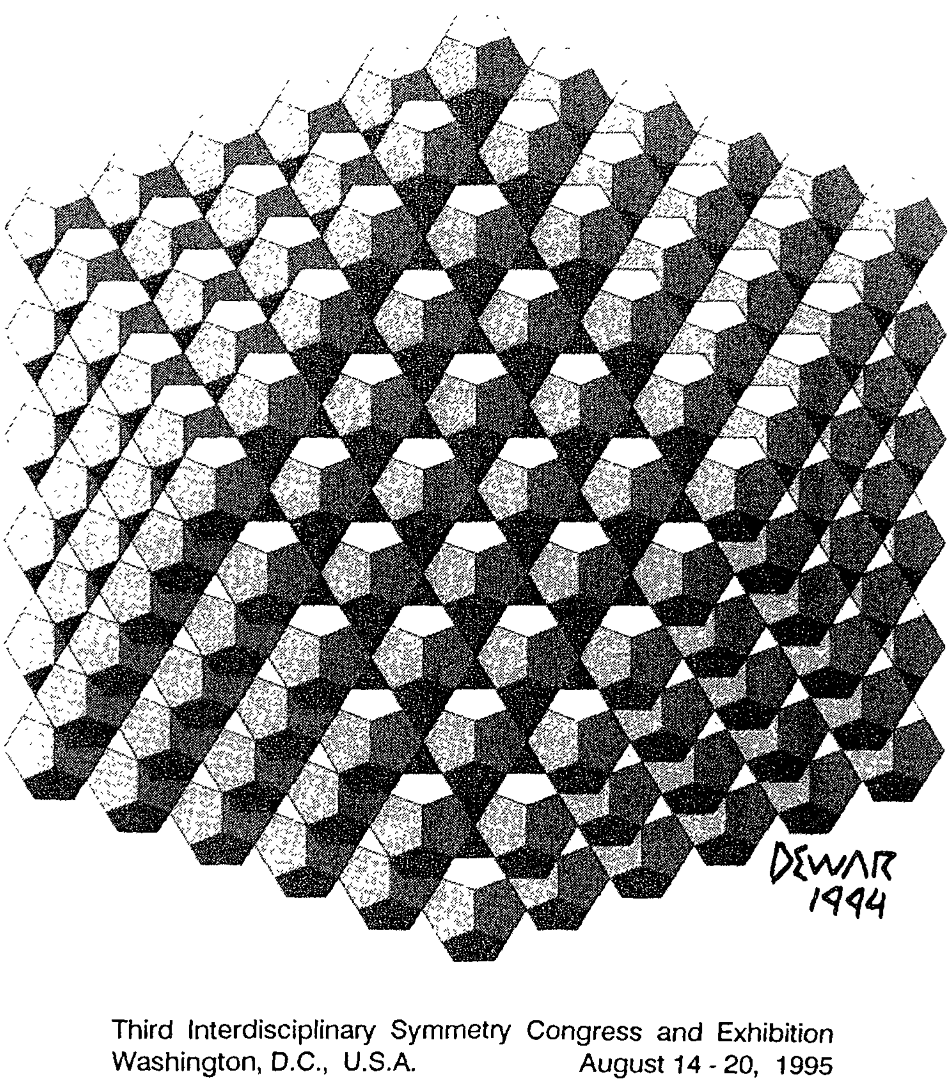

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WASAN (OLD JAPANESE MATHEMATICS): ART AND SCIENCE

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1 Briefly about *wasan*

Wasan (from *wa-* Japanese and *san* calculation, mathematics) was developed in isolation from the European mathematics (*yōsan*), which did not become dominant in Japan until the 1870s. Although *wasan* had Chinese influence in the beginnings, but the Japanese developed a special interest in some problems, especially in densest (close) packing of circles in larger figures, that are unparalleled in Chinese and other cultures in the same level. Another interesting feature of *wasan* geometry is the custom that many of the problems and their solutions were offered to gods. These problems were written on wooden tablets, called *sangaku* (from *san* mathematics and *gaku* framed picture), with beautifully colored figures. Among the circa 900 tablets that survived roughly twice as many appear in Shinto shrines than in Buddhist temples and these are distributed in both rural and urban areas (Fukagawa and Pedoe, 1989, p. xii). Considering these facts, *wasan* is obviously a relevant part of Japanese culture and may provide some interesting questions not only to mathematicians, but also those who are interested in religion or art. It is also an interesting topic for comparative studies: we may see various analogies between the Pythagoreans and some *wasan* schools, i.e., the unity of mathematical, aesthetical, and religious ideas, as well as about secrecy. In the Western world the Pythagoreans (6th-5th cc., B.C.) significantly contributed to the birth of modern mathematics: they provided the basic sources to various parts of Euclid's comprehensive work (3rd c., B.C.) that presents mathematics as a deductive system with rigorous proofs. It would be interesting to see the reasons of the missing 'Japanese Euclid' who could transfer the rich mathematical knowledge of *wasan* to major breakthroughs, instead of going to a deadlock manifested in the 1872 decree by the Ministry of Education that eliminated *wasan* from the curricula and required teaching exclusively of Western mathematics. Ravina (1993) gave some explanation of the failure of *wasan* focusing on the lack of its application to physical problems. Note, however, that the 'pure mathematics' of Euclid and his followers remained dominant through the ages without initiating any connection with physics or other practical problems from their side.

Despite this setback of *wasan*, it is still a significant source of the history of ideas and requires international attention. We have here a quite unique possibility to see another type of mathematics. *Wasan* may also have an importance in mathematics education. The ethnomathematical approach, i.e., using mathematics-related ideas of various cultures, instead of just referring to Western examples, has a success in education (cf., D'Ambrosio, Gerdes, Harris, Zaslavsky). Last, but not least, we also believe, that *wasan* - especially the geometry problems, practised widely in both urban and rural areas - had an indirect influence on Japanese culture after the Meiji restoration (1868): the skills gained in *wasan* could contribute, at least in some circles, to the quick adaptation and utilization of Western science and technology that requires some mathematical backgrounds. This traditional knowledge should not be forgotten in our age, but rather we ought to use it to widen our knowledge in the cultural history of mathematics and to enrich the often boring problem books used in mathematics education.

2 *Wasan* as science and art

Mikami (1913, pp. 166, 169, and 177) formulated an interesting thesis that *wasan* did not exist as science with rigorous proofs, but as art. However, very recently Horiuchi (1994, pp. 16-18) emphasized that the general characterization of *wasan* as art does not properly consider the developments in this field. The mathematical results by Takakazu Seki (1642?-1708) and Katahiro Takebe (1664-1739) are important tools to describe, for example, trajectories of astral bodies. The conclusion of the book emphasizes that the general techniques developed by Seki and Takebe in algebra and trigonometry gave a scientific status to *wasan*.

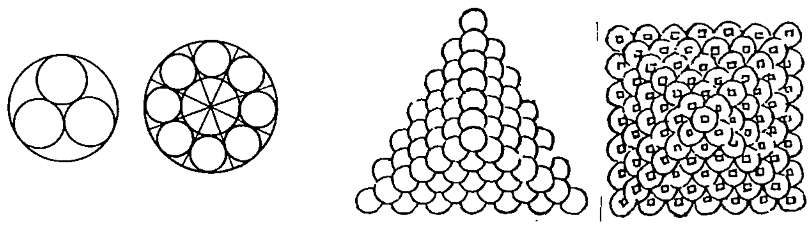

Figure 1: Densest packing of circles in larger circles (from *Sanpô futsudankai*, 1673) and densest packing of equal spheres (*Kokon sanpôki*, 1671). The systematic study of such questions in Western mathematics were started by Fejes Tóth in the late 1940s; also see the works by Coxeter, Rogers, and Delaunay [Delone]. An interesting result of these studies is the fact that density may induce order in a chaotic figure system. This topic also has a special importance in crystallography.

In this paper, making a compromise between the opposite views by Mikami and Horiuchi, we argue that *wasan* is both science and art. Horiuchi is right that Seki and his school lead *wasan* to such a level that it reached scientific status. Some of their results are beyond the level of the best results of contemporary Chinese and Western mathematics. However, we think that *wasan* was partly continued in an artistic way. We use this statement in a very positive sense: we believe that an artistic imagination is a very important aspect of mathematical creativity, especially in geometry-related fields. There are various confessions from leading mathematicians and physicists - from Norbert Wiener to Paul Dirac - that link their fields to art or to the search for beauty. Although these statements can be considered metaphoric, in the field of geometry it is obviously more meaningful. For example, G. D. Birkhoff dealt with aesthetic measure of shapes, George Pólya and Andreas Speiser initiated the group theoretic analysis of ornamental art in the 1920s. This was continued by the monographs written by Shubnikov, Fejes Tóth, and more recently by Crowe and Washburn. All of these books are illustrated by a large number of art works.

However, *wasan* started this practice earlier. The fact that the results are often communicated in shrines and temples includes a special desire for beauty. We will refer here to some topics that are strongly connected with symmetry.

3 Some symmetry-related topics in *wasan*

3.1 A field of wasan *that is unparalleled in other cultures: densest packing of circles, spheres, and other round objects*

(See Figure 1.)

3.2 Theory of tilings

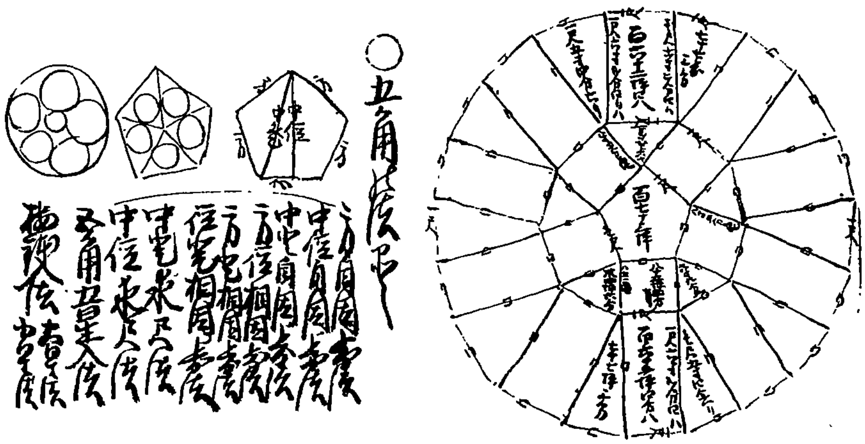

Figure 2: A tiling related to packing of circles (Dirichlet-Voronnoi cells) and an interesting pentagonal tiling (from a manuscript book of 1798, Symmetrion Library, Budapest). The topic of tiling (tessellation, mosaic, paving) has a great record in the history of culture, but its systematic study from a mathematical point of view was started in the West only in the 20th century (Heesch in the 1940s and more recently Grünbaum and Shephard).

3.3 Surfaces and polyhedra

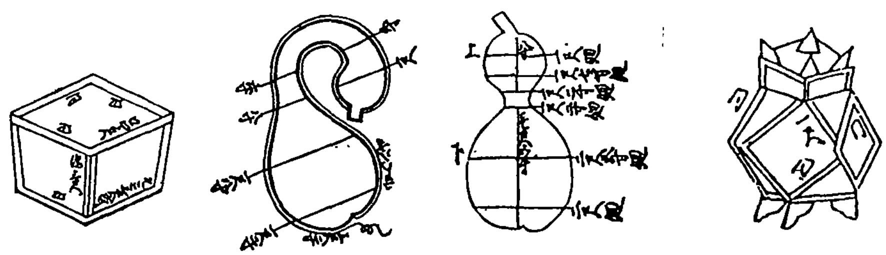

Figure 3: Surfaces and polyhedra, including a figure that resembles the Klein-bottle (from *Sangenki*, 1657). Although the roots of this topic go back to the Greek mathematics, but the systematic studies in the West were not started until the 19th century.

4 *Wasan* and Japanese culture

What were the sources or inspirations of this strong interest in packing problems and other symmetry-related topics? This question was raised by some distinguished historians of science, but they could not locate any relevant preliminaries in mathematics. Both Smith and Needham refer to Chinese connections; Needham adds, however, that he does not see the intermediate channel, and refers to some relation to the origin of calculus. Although a limited number of packing problems were discussed by Chinese mathematicians, but they never had such a strong interest in these questions as the Japanese. Here we suggest such aspects that are outside the mathematical activity in a narrow sense, but we may call it ethnomathematics:

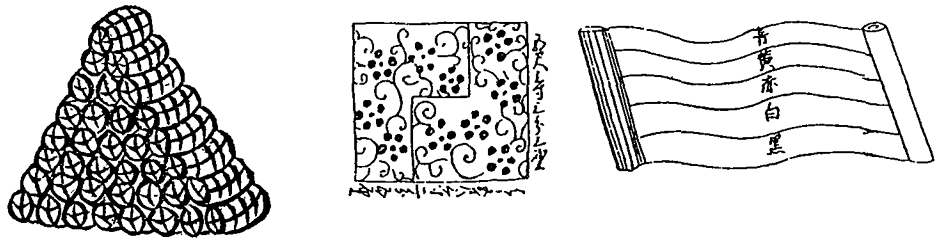

Figure 4: Practical problems in *wasan* books associated with densest packing of figures (see in many works) and patterns in art (from *Sangenki*, 1657).

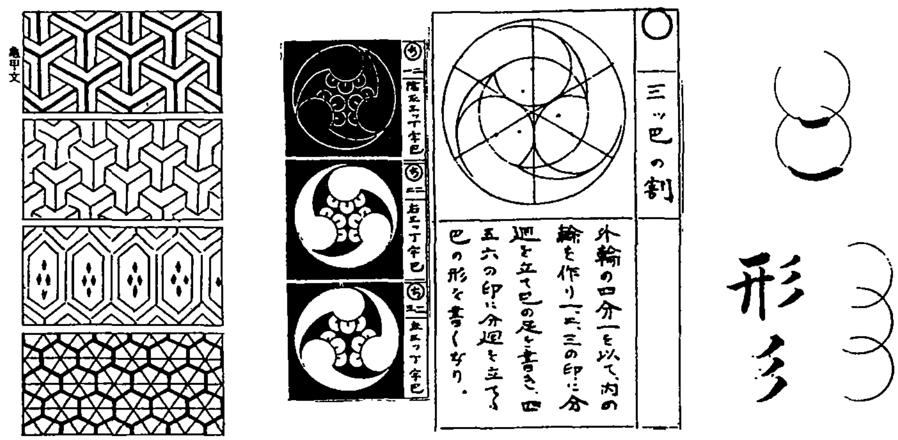

Figure 5: There are various fields in Japanese culture that could inspire related questions, including (1) geometric ornaments, (2) design of family crests, (3) the 'hidden dimension' of calligraphy (here the character *katachi*, i.e., form, shape, figure, is explained).

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Symmetry as a Fundamental Element of Physical Reality

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1. The Intelligibility of Physical Reality

Albert Einstein remarked "The ultimate incomprehensibility about the world is its very comprehensibility" [1]. The scientific method -- conceived in Greece, revived in the (thus appropriately named) Renaissance and having reached maturity with Newton -- has since successfully penetrated every domain of physical reality, including such as had earlier appeared (even in the eyes of many scientists) as beyond its reach, e.g. areas such as Life, Mind, (and very recently) Creation (this is one lesson of the new "Inflationary Cosmology"). There are two levels at which this 'intelligibility' of physical reality operates:

(1) first, the fact that *a mathematical description of the world fits observations and provides for predictability* (deterministic or at least probabilistic, whether statistical, chaotic or quantum);

(2) secondly, that even after we therefore conclude that *logic and mathematics do belong to the fabric of physical reality*, this description still appears to be more successful than would seem to be a priori justifiable. For instance, conditions in very dense states of matter represent extremely non-linear situations and are very hard even to characterize phenomenologically.

For lack of linearity, for example, one would not have been able to identify meaningful observables in the case of quarks. Characterization was only made possible through the existence of a different, "perturbative" (linear) regime, from which observables could then be extrapolated over the "non-perturbative" region.

This second success may be due to *evolution*: e.g., if there have been an infinite number of Big Bangs, as presently thought, it is very possible that other 'universes' are indeed *effectively unintelligible*. However, a universe in which the conditions evolve so as to have matter in separate bulks, i.e. in galaxies and stars (not to mention people), will automatically also lend itself to linearization and to the possibility of identifying and extracting useful variables from the phenomenology.

This explanation has been named [2] *"The Weak Anthropic Principle"*, a nomenclature linking it to the presence of *people*. Personally, I find this last point unjustified and dangerous, in that it tends to generate a -- Copernican notions -- making "people" important to the universe, when everything in science points in the opposite, Copernican, direction [3].

Returning to point (1), we remind the reader that this was what Sir James Jeans, well-known English theorist, summarized in his beautiful tract "The Mysterious Universe", around 1930, by the phrase "God is a Mathematician". The success of the scientific method implies that the mathematical description is more than just our own input, the way in which we can dissect the world. Had the world, for instance, been controled by the logic of dreams, the use of rational thinking would have failed. Logic and its extension, mathematics, must thus already be present in the fabric of reality itself. That logic has evolved within human thinking must indeed have been due to the fact that its presence in nature made it beneficial evolutionary-wise.

2. Symmetry and Group Theory

Group Theory, invented by a French highschool pupil, Evariste Galois, around 1830 (killed in a duel a year later), is the branch of mathematics which studies symmetry features. It is done,in particular, by "transformation groups". Example: clockwise rotations of 60°, in the plane, form a group. There are 6 such rotations, before we return to the original state (since we would have rotated by 360°. A group has to include the "identity transformation", i.e. leaving things as they are -- which is what we achieve by either 0 or 6 rotations of 60°. The group also has to include, for each transformation, an inverse, i.e., a transformation which cancels the original one. This is true here. For example, we can cancel the original 60° ("2-o'clock") rotation by adding one of $5\times60=300°$ (and thus returning to midday 0°). We have also thereby examplified another feature of a group -- the "group operation" (sometime called "multiplication" or "product"), namely a way of combining two elements and making a third. Here it consists in applying the two transformations consecutively, which is then equivalent to doing it by one single transformation, whose rotation angle is the sum of the previous two. Recapitulating -- elements, a group operation, the identity element and having an inverse to every element -- this is a group. A transformation group has in addition a "carrier-space" on which the transformations are performed -- here any planar "rotatable" object, such as a clock with no handles. With a clock, we would know when and by how much we have rotated it, because there are hour markings and it is customary to put the midday point up and in the centre, to start with. The markings break the symmetry and allow us to follow the sequence of transformations step by step and keep track. Should we however erase the markings, we would find it impossible to know if at all the clock has been rotated when we were not looking and by how much. We thus observe the relationship between symmetry and a transformation group: *symmetry is the state of affairs in which the action of the group leaves the carrier space invariant, i.e. when we cannot distinguish between the state of the carrier-space before and after an arbitrary application of the transformations*, (i.e., not just for some specific group elements).

3. Symmetries in Physics

We are used to observe and enjoy the spatial symmetry of crystalline minerals -- jewels have been applied to the decoration of the human body from the earliest phases of civilization; so have sea shells, i.e., samples of symmetry taken from organic matter. Modern watches use liquid crystals, but soap bubbles have already displayed the symmetries of liquids for centuries. As to gases, smokers enjoy producing elaborate spiral displays. Chemistry reflects very often the symmetries of a molecule -- Kekule's "dreamed up" benzene hexagon (the idea came to him in a dream..) being one such example.

In physics, the symmetries of crystals -- the space-groups -- are only one type of symmetry acting on condensed matter; more sophisticated symmetries have turned out to play key roles in phase transitions -- were the transition itself is related to a so-called *spontaneous breakdown of the symmetry*, as a result of a change in the value of an *order parameter*. One of the interesting features deriving from the recent advances in non-linear dynamics (*chaos* theory) is an understanding of symmetry patterns arising in dissipative processes, such as the Bénard Instability or the Zhabotinsky reaction, as described for instance in the book by Prigogine and Stengers [4]; remember -- all crystal symmetries are examples of equilibrium dynamics, whereas the 'chaotic' symmetries represent very-off-equilibrium dynamics. Another example of a symmetry displayed by a dissipative system is seen in the patterns produced by sea-waves breaking over a jetty. It has recently been claimed that the two very different systematics may sometime lead to similar symmetric outcomes.

4. Gauge Symmetries as Selected by Nature

A particular type of symmetry-carrying manifold is examplified by a coronet-like cylindrical ribbon and by a Möbius strip. Both are made by cutting out an elongated rectangular strip of paper and glueing together the two shorter sides of the rectangle. The two are similar, but in the Möbius strip, one of these sides is twisted and inverted before glueing it to the other.The twist cannot be gotten rid of, it is embedded in the geometry. In both cases we have basically a horizontal circle drawn on a plane, (e.g., that of the desk), along which a vertical line can travel and will then span the new manifold. In the case of the ribbon, the lign stays parallel to itself throughout, but in the Möbius strip, it gradually rotates, in a plane perpendicular to the circle. We can push away the strip locally, i.e. make a small region appear identical to the way it would be in the ribbon; but this cannot be done for a larger region or for the entire manifold, the twist will not go away.

This property is one of the main characteristics of *gauge manifolds*, which play an important role in fundamental physics since *1975. They* display *global* properties (the twist, which exists *globally* even though it can be ironed away *locally* is such a global feature). Locally, gauge manifolds display *curvature*. Imagine a pipe, i.e., a circle rolling along a straight line. If we rotate the pipe globally, as if in one motion, it is as if nothing happened: it has the symmetry of the circle, like our clock in a previous example. But suppose we try to rotate the pipeby different amounts at different positions along its axis. We would be creating tremendous stresses in the material of the pipe, twists which have to be there in order to make such a *locally dependent* symmetry operation possible. Such a symmetry is called a *locally dependent gauge symmetry*. The stresses in the fabric of such spaces are known as *curvature* and *torsion*. The first such geometrical physical theory, with its peculiar locally dependent symmetries, was Albert Einstein's General Theory of Relativity in 1915. This theory, explained gravity in terms of the curvature of spacetime. The physical locally-dependent symmetry it propagates is that of rotations, accelerations and translations. It guarantees that while we might rotate this laboratory by some angle -- say 30 degrees clockwise horizontally, while at the same time rotating another lab, say in China, by 60 degrees clockwise in a vertical plane, the results of fundamental physical experiments should be the same. The laws of physics are invariant under such locally-dependent rotations, or also similar locally-dependent translations and accelerating boosts.

At the Budapest Conference I gave [5] a simplified non-technical explanation of *the relationship which exists in physics between symmetries and conservation laws*, a linkage first proven by Emmy Noether, a distinguished mathematician and theoretical physicist [6] who had to flee Nazi Germany in 1934 and died one year later in the USA, where she had been given a professorship at Bryn Mawr.

Noether's theorem ascertains that *to every continuous symmetry group, there has to correspond a conservation law for a related "charge"*. At this stage, the conservation law is *kinematical*, consisting in the existence of an observable whose total quantity is always preserved, as in the conservation of energy, momentum or angular momentum in classical mechanics. It is an "accountant's" conservation law.

I shall not repeat here the somewhat complicated argument I used in Budapest, to prove the theorem without appealing to a mathematical derivation. What is however relevant and important here is that *in locally-dependent gauge symmetries, the conserved quantity normally associated to that symmetry becomes "dynamical" rather than kinematical, i.e., like electric charge*, which is of course conserved kinematically too, i.e., in the accountant's meaning. The relevant charge thus also induces an interacting field around it, just as the electric charge generates an electromagnetic field. The *curvature* we mentioned as a tension in the fabric of space, resulting from the requirement of invariance under a locally-dependent gauge symmetry -- *this curvature is the field emanating from the symmetry's conserved charges*. In Einstein's theory of gravity, the conserved quantities are, e.g., energy and momentum, which were kinematically conserved in Newton's physics and in Special Relativity. In General Relativity, they become *dynamical* charges: the force of gravity itself is induced by their presence. *In the words of the theory, matter induces a curvature of the surrounding space.*

Einstein's Gravity is still a "classical" theory, in the sense that it is not a quantum theory and thus violates the Uncertainty Principle. After its publication, there was a rush of theorists who tried to "geometrize" the other known forces, mainly electromagnetism (the nuclear forces, first sensed in the discovery of Radioactivity at the end of the XIXth Century, were too new to be approached geometrically). Hermann Weyl, physicist and mathematician, discovered in 1919 the type of gauge symmetry which we introduced through the example of the Möbius strip. He tried to fit it to a description of electromagnetism and failed. Ten years later, after the discovery of quantum mechanics and after Fritz London had understood the mathematical structure of electrical charge in quantum mechanics, Weyl ammended his theory -- and electromagnetism was shown to correspond indeed to a locally-dependent gauge symmetry theory. The word *gauge* was used because the first version was a symmetry under changes of *scale*, i.e., *gauging* a size.

Between 1929 and 1958 there was no indication that the nuclear forces would follow the same pattern. The models introduced to describe them were of a very different nature, very non-symmetry-oriented and non-geometrical. And yet since 1958, experiments started to reveal new conserved charges, with newforces related to them, which ended up being at work "behind" the nuclear forces (behind both the so-called "weak" and "strong" interactions).

The picture kept changing between 1958 and 1974; meanwhile, already in 1953, C.N. Yang and R.L. Mills had developed the mathematical tools for an extension of the locally-dependent symmetries to "non-Abelian" groups, i.e., groups in which combining elements A and B is not the same as combining B and A. It turned out that this was precisely what was needed in order to understand what nature was trying to tell the physicists.

Since 1974/75, there is a grand-synthesis of the elements and forces we find in nature up to what can be observed or what has been explored to date. This so-called *Standard Model* is a quantum theory (except for gravity, which we do not yet know how to fit to the quantum world). Moreover *the entire Standard Model is built from locally-dependent gauge symmetry theories!*. Somehow, the message we got is that *symmetry makes the world go round*. It is thus deeply engrained in the very fabric of reality.In this sense, at least, *beauty is physical truth*. May be this is why we have acquired such strong aesthetic motivations through evolution.

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