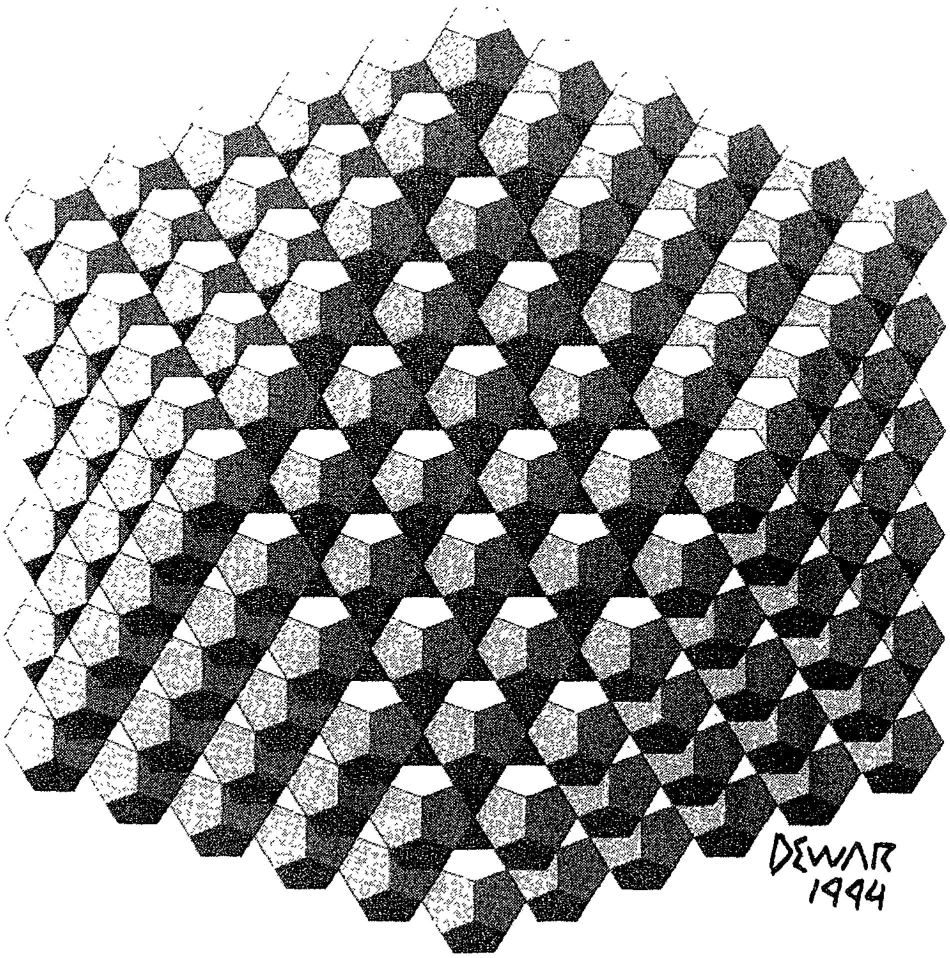
Symmetry: Culture and Science

Symmetry: Natural and Artificial, 3

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 6, Number 3, 1995



Third Interdisciplinary Symmetry Congress and Exhibition
Washington, D.C., U.S.A. August 14 - 20, 1995

Self-similarity, FASS Curves, and Algorithms for Musical Structures

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Almost all music analysis has proceeded from finished compositions backwards to underlying paradigmatic patterns: from individual symphonic movements to sonata "form," for example, or from individual fugues to "subjects," "counter-subjects," "stretti." Most of the patterns revealed through analysis of this kind are linked with particular historical or cultural traditions. Many of these patterns are also self-similar: micro- and macro-level "bar" forms in Wagner's music dramas discovered by Alfred Lorenz, recurrent melodic and harmonic "background" patterns in European art music of the eighteenth, nineteenth, and early twentieth centuries discovered by Heinrich Schenker, rhythmic patterns discovered by Leonard Meyer and his colleague Grosvener Cooper, and generative organizational patterns in tonal music discovered by Fred Lerdahl and Ray Jackendorff. With the exception of Lorenz, all of these individuals also believe that "anticipation/response" models of musical structure help us understand musical perception.

Only a very few analysts have suggested that other universal paradigmatic patterns may underlie the compositional styles of existing cultural traditions. Among such analysts are Kenneth and Andreas Hsu of the Technical Institute, Zürich, who have suggested that the key both to analysis and to composition lies in fractal geometry. On the one hand, some of the arguments presented by Hus and Hsu's arguments are not logically consistent, and their results to date can be used only to identify stylistic differences like those distinguishing the music of Bach and Mozart from that of Stockhausen. On the other hand, Hsu and Hsu have pointed out the significance of self-similarity in musical structures of several kinds. Moreover, computer scientist Przemyslaw Prusinkiewicz (together with several colleagues) has shown how self-similar curves, associated with fractal geometry, can be used to generate melodies and even complete compositions.

L-systems (short for DOL, or "deterministic, context-free Lindenmayer systems") are used to generate FASS curves (short for "space-filling, self-avoiding, simple and self-similar" curves). For musicians, the shape of these curves is most important. "Reading" individual curves can generate new musical melodies. Certain FASS curves, including the quadratic Gosper curve, can even be used to generate existing melodies like those of Beethoven's sonatas, Verdi's "S'ancor si piange" from *Don Carlos*, the second theme from the fifth movement of Shostakovich's Op. 57 Quintet for piano and strings, and so on. Computer searches keyed to such curves could be used to study self-similarity and structure in many kinds of compositions: Bach inventions, twelve-tone works, collections of indiginous melodies and chants, and so on. More significant in the long run, however, may be the generation of new melodies and even entire musical scores: a method, to quote Prusinkiewicz, that yields complex results in spite of "the simplicity of the underlying production [method]" itself (Prusinkiewicz, "Score Generation with L-systems," p. 456).

Prusinkiewicz has described a method for creating music using L-system or FASS curves that involves reading the horizontal lines of curves as durations of notes and the vertical lines as pitch intervals between notes. Thus each horizontal "F" (or move forward horizontally) is read as a single unit of duration, e.g., a sixteenth note, eighth note, or any other duration desired. Each vertical "F" (or move up or down) is read as a single interval between notes, e.g., a half-step, whole-step, or any other interval desired within a given mode or scale. No distinction is made between left (π *direction)* and *right* (0.0 direction) in Prusinkiewicz's reading of curves, so the melodies generated always travel forward in time, even when the turtle is moving left. Each forward move up ($\pi/2$ direction) shifts the note being created up one interval from its predecessor. Similarly, each forward move down ($3\pi/2$ direction) shifts the note being created down one interval from its predecessor,

This method generates "new" musical melodies. Pre-existing melodies can also be modelled with an L-system, including the songs of aboriginal hunters, the plainchants of the Medieval Christian liturgy, the themes of Beethoven's symphonies, and popular song tunes. Counterpoint can be generated by reading two or more rotations of the same curve simultaneously to create canons, or by reading two or more different curves simultaneously to create multi-melodic structures. For example, if the second iteration of the quadratic Gosper curve is drawn with two different turtle headings—say, the 0.0 and $\pi/2$ directions—and the melodies generated from these iterations are played simultaneously, a "right-angle canon" is produced. Other modifications to L-system melodies are also possible. Individual curves, for instance, may be read in terms of unusual or "exotic" musical scales and modes, thereby giving the resulting melodies or contrapuntal passages various "historical" or "ethnic" flavors. Horizontal moves forward can also be assigned proportional durations of various kinds, producing some striking rhythmic patterns or even no "rhythm" at all if all horizontal moves are assigned the same duration. All of these generated tunes and contrapuntal structures are fractal productions.

Our initial work involved correlations between melodies generated from FASS curves and melodies associated with Western art and popular composers like Bach, Beethoven, Cole Porter, and so on. In order to listen to music produced by "natural" FASS curves, we wrote computer programs in which at least two curves or two turtle headings of the same curve were read simultaneously as music and recorded on magnetic tape, using Macintosh (C programming language) and MIDI software and technology. In most cases the curves were read in terms of a C-major scale and "long/short" rhythmic patterns in which one horizontal "F" was read as a *sixteenth note, two* "F"s as an eighth note, and so on. The results were quite effective musically, although not entirely "conventional."

In order to find out whether space-filling curves can be used to form larger musical structures, including short polyphonic pieces in, say, the style of Bach, we have also written computer programs that combine melodies produced from whole or fragmentary curves (most of them first iteration) according to some of the "basic melodic structures" discovered by theorists like Bence Szabolcsi, Bruno Nettl, and especially Eugene Narmour. We have employed models of melodic structure, like Narmour's "anticipation/response" models, in order to write computer programs that generate two- and three-voice compositions created entirely from FASS curves. Some of these compositions are generated in terms of the C-major scale; others are generated in terms of the C-minor scale, the Lydian and Mixolydian modes on C, and so on. A small number of rules involving maximum and minimum lengths of phrases, cadencing patterns, and repetitions of individual melodies or even entire polyphonic phrases allow our computer programs to generate hundreds of individual self-similar pieces, each 24-72 measures in length. Each group of pieces employs closely-related melodic materials; each piece within a group has a distinctive aesthetic or expressive flavor.

Details of the computer programs used to create such groups of pieces will be made available in Alexandria; we plan also to play at least three complete, short compositions on tape and distribute handouts that reproduce the same compositions in printed form.

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