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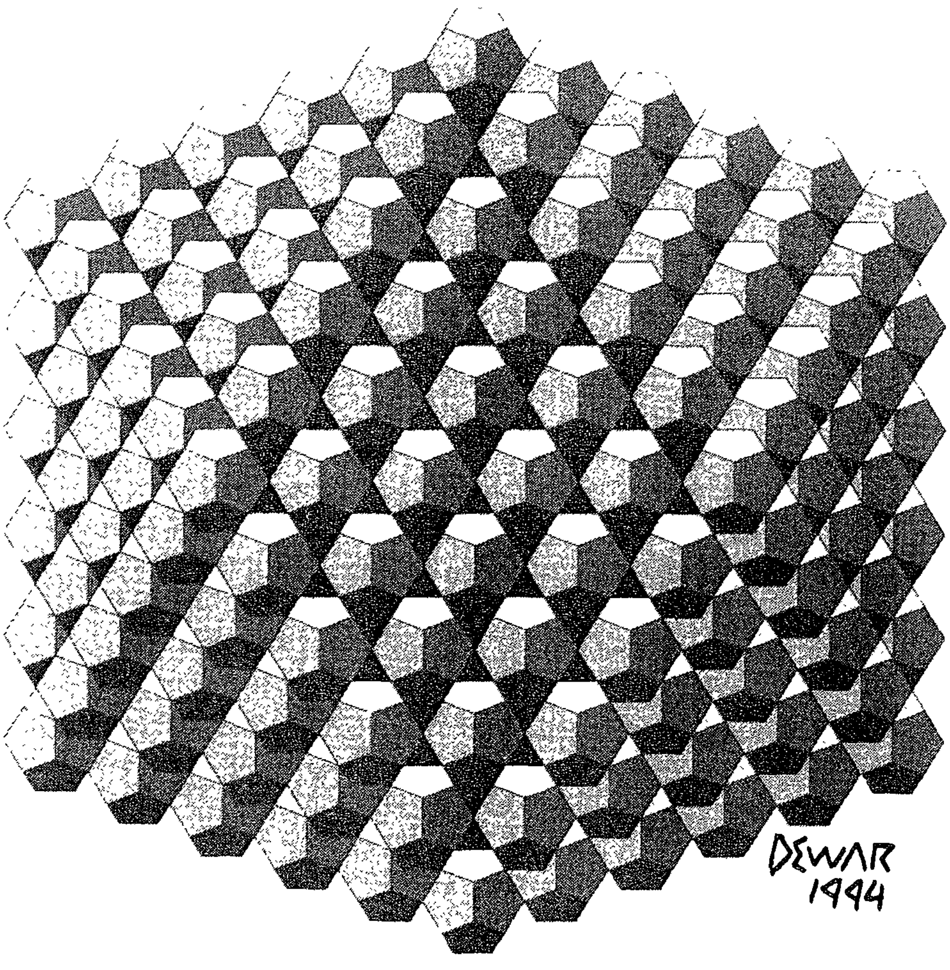

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SYMMETRIES AND ROBOTICS

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Human beings readily appreciate symmetries of objects. Would a robot be able to appreciate symmetries? And how relevant would this appreciation be in advancing modern technology? Our work [2, 4] shows that global and local symmetries play a crucial role in formalization and computation for one of the most fundamental problems in robotics, i.e. specifying the relative positions of solids that are in contact. If a robot could understand symmetries, instructing a robot would become much easier, especially for manipulating man-made objects.

The symmetry group of an object in N-dimensional space is an important descriptor of the *appearance* of the object. Moreover, symmetries serve as a pertinent descriptor of the *functions* of an object as well. In particular, when three dimensional solids in Euclidean space come into *contact* with each other, the relative positions or motions of the solids under such a contact can be expressed in terms of the symmetry groups of the contacting surfaces of the solids. For example, a cylinder can roll on a flat surface: the rolling motions are the product of the symmetries of the cylindrical surface and those of the plane; or a spherical joint where two spherical surfaces of two separate solids coincide and their relative motions are the symmetries of the sphere. The second example above also indicates that a *necessary condition* for two solids to have a surface contact (surface coincidence) is for the contacting surfaces to have the *same* symmetry group. This observation leads to a direct application of group theory in technical fields where the relative positions/motions of solids are of primary concern, such as robotics, graphics and mechanical design.

A detailed study shows that to compute the exact relative positions of solids in contact one needs to compute the *intersections* and the *products* of the symmetry groups of their contacting surfaces. Given the diversity of the symmetry groups of contacting surfaces (all of which are finite, infinite, discrete or continuous subgroups of the Euclidean group), the denotation and the computation of such groups on computers are non-trivial. A geometric approach using *characteristic invariants* to represent and intersect symmetry groups has been proven correct and efficient [3]. These results show that using group theory to formalize surface contacts is a general approach for specifying spatial relationships and forms a sound basis for the automation of robotic task planning. One advantage of this formulation is its ability to express continuous motions between two surface-contacting solids in a computational manner, and to avoid combinatorics arising from multiple contacts, especially from discrete symmetries in the assembly parts and their features.

Finally, we describe how these theoretical results are applied in an assembly planner called $\mathcal{KA}3$ [5, 6, 7, 8]. The input to $\mathcal{KA}3$ includes the Computer Aided Design (CAD) boundary information of a set of solids and a set of high-level instructions describing how the solids contact each other. The output of $\mathcal{KA}3$ is a precise set of assembly task specifications for mating surfaces and the order of assembly, that are especially useful for robot task-level planning. Besides group theory, $\mathcal{KA}3$ also combines geometric modelling and AI constraint satisfaction problem solving techniques into one coherent framework.

There are many other related fields where symmetry groups are important. A mechanical designer can use $\mathcal{KA}3$ to verify whether a product design can be assembled, if so then one can further find out how many different configurations it can be assembled into (desirable if the designed product is a puzzle). In model-based computer vision, recognizing a scene from a man-made environment can be viewed as an assembly task in the sense that a correct recognition is a successful "assembly" of a hypothesized model with the scene. Therefore symmetry group theory as used in solving assembly task problems can also be applied. CAD modelling of the childbirth process also bears parallels with the methods used in assembly/disassembly planning research [9].

Many open problems remain. The most immediate is symmetry detection, which is defined as: Given a set of 3D geometric entities S find all the rigid transformations G such that for each element g in G, $g(S) = S$. The inexactness existing in real data (e.g. the human body is symmetrical, but not exactly so) causes tremendous computational problems. Indeed, finding the nearest symmetry group of a set of points on a plane has been proven to be NP-hard [1]. On the other hand, taking advantages of global or local symmetries is an obvious choice to lessen the complexity of a complicated problem, and few researchers have systematically investigated algorithmic means for detecting inexact symmetries. These observations led to the theme of our current research: building computational tools for real-world symmetry analysis, which is potentially useful in fields such as mechanical design and manufacturing, robotics, graphics, computer vision and image understanding. One of our current research projects is to use image analysis techniques to recognize the seven Freize groups and the 17 wallpaper groups from scanned 2D patterns.

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