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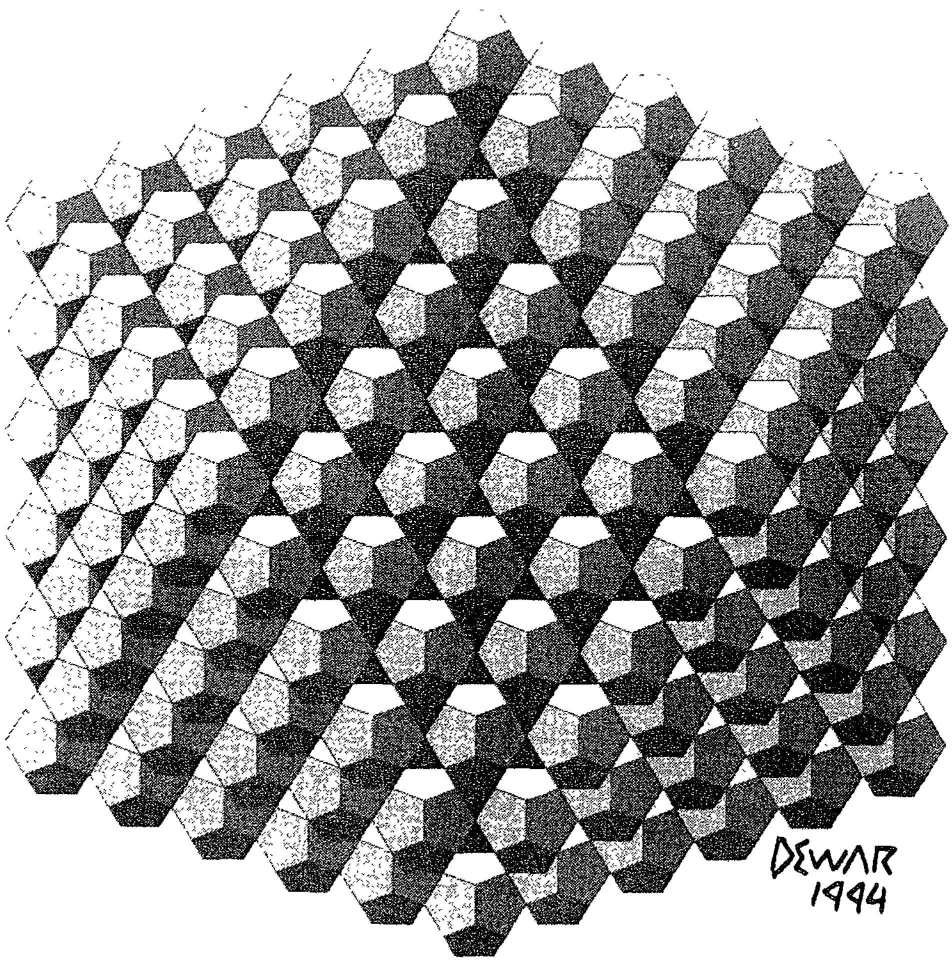

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QUANTITATIVE ESTIMATION OF SYMMETRY. POSSIBLE WAYS OF APPLICATION IN NATURAL SCIENCES

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The criterion for quantitative estimation of plane geometric figures with a convex contour "symmetry" is proposed. The characteristic of form range (a form factor) is used as such a criterion. Its use at the researching of some of the problems concerning the mechanics of a deformable rigid body proved such an approach to be promising for symmetry estimation in many continual problems of natural sciences related to the planar range. Researching the particular problems of natural sciences researchers often make attempts to classify symmetries in order to compare various geometric figures (ranges). For example, the number of symmetry axes (or planes) is determined by the order of symmetry (a square has the symmetry of the fourth order); spiral symmetry, which is manifested in stems texture of many plants, is characterized by a leaf cycle - the ratio of leaves number, lying on the generatrix and beginning from the first leaf, to the number of the spiral turns between two leaves on one generatrix (the leaf cycle of a fir-tree is $^5/_{13}$).

When comparing two geometric figures the researcher pays his attention, *in the first place, at the presence of symmetry* elements in them. If there is a great number of symmetry axes in one of them, it is considered to be more "regular". A circle is an absolutely perfect plane figure, the symmetry order of which is determined by infinity. If the compared figures are of one and the same symmetry order, then other geometric parameters, for instance - the proportions of rectangle sides, are considered additionally.

But how to compare two figures having no symmetry axes (an unspecified triangle or quadrangle)? How to compare figures having the same number of symmetry axes (ellipse and rectangle)?

Researching the problems of mathematical physics the known mathematician G. Poyia proposed to make use of the integral characteristic of the form (which we later named a form factor K_f) as the geometric criterion characterizing the form of the range. This characteristic is determined as the contour integral minimum from the following ratio:

$$K_f = \min\oint(ds/h),$$

where ds - is the linear element of the geometric figure contour arc; h - is the height dropped from a random point "a" taken within the range onto the tangent line to a random point of the contour. For polygons this factor is to be determined by the following formula:

$$K_f = \min\sum^{n} l_i/h_i,$$

where n - is the number of the polygon sides; l_i - is the length of the i-th side; h_i - is the hight dropped from point "a" onto the i-th side of the poligon.

The more complete and detailed researches of this geometric characteristic are presented in the Author's monograph [1] and in the article of his disciples [2]. We present hereunder the main generality and properties of this characteristic.

1. K_f - gives the quantitative estimation to the form of geometric figures with a convex contour and may be the criterion for their symmetry estimation: the less is K_f, the more "symmetric" is the figure.

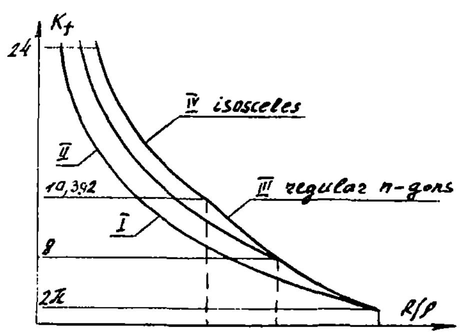
2. Of all the great number of convex figures a circle has the less value of $K_f = 2\pi$.

3. Of all the great number of n-gons a regular n-gon has the less value of K_f.

4. Of all the great number of polygons with the given direction of the sides the less value of K_f is noted for that one the sides of which are tangential to the inscribed circle.

5. Of two regular n-gons the less value of K_f is noted for that one which has the greater number of sides.

Analysing the above properties of the form factor, it is not difficult to take notice of the fact that all of them in a certain manner characterize the "regularity" of geometric figures and their "symmetry". For example, of two isosceles triangles (the both of them have one axis of symmetry each) more "symmetric" will be that one the K_f of which is less; it is more like an equilateral triangle. Let us present the form factor changing graphically depending on the R/ρ parameter (R - is the maximum radius of the circle inscribed into the figure, ρ - is the minimum radius of the circle circumscribed around it) (Fig. 1). It has benn proved [1] that the K_f values for all the great number of geometric figures (including the symmetric and asymmetric ones) are limited by curvex I, III, IV (Fig. 1).



Anlysing the plots shown in Fig. 1, we first of all note the remarkable fact that all the great number of the asymmetric figures are limited by the symmetric ones. Ellipses will be the most symmetric figures with respect to all the great number of convex figures (curve I). Their 2-nd-power symmetry in the limit turns for a circle into infinite power symmetry. Rectangles, the 2-nd-power symmetry of which in the limit for a square turns into the fourth-power symmetry, are the most symmetric with respect to the great number of triangular figures and tetragons (curve II).

By analogy with the above one can assume that any other great number of polygonal figures (for example the polygons having less than eight sides) will be limited from below by octagons having two axis of symmetry which will be turned in the limit into the 8-th power symmetry (for a regular octagon), etc.

Solving the problems of structural mechanics we managed to prove the functional connection of integral physico-mechanical and geometric characteristics of the objects under investigation with form factor [1, 2], i. e. the dependence on their "symmetry". In view of the fact that there is an analogy of mathematical description for a lot of nature's physical problems the used in these operations solution procedure may be carried over to the study of such problems, i. e. the proposed approach is of interdisciplinary character and offers ample scope for the scientists working in the most diverse trends of natural science.

References

1. Korobko V. I. Isoperimetric inequalities in structural mechanics of plates. - Moscow: "Stroyizdat" Publishing House, 1993. - 208 p.

2. Korobko V. I., Khoustochkin A. N. The use of the isoperimetric methods for the solution of plates stability problems. - Rostov-on-Don: The Publishing House of the North Caucasus Scientific Centre of the Higer School, 1994. - 146 p.