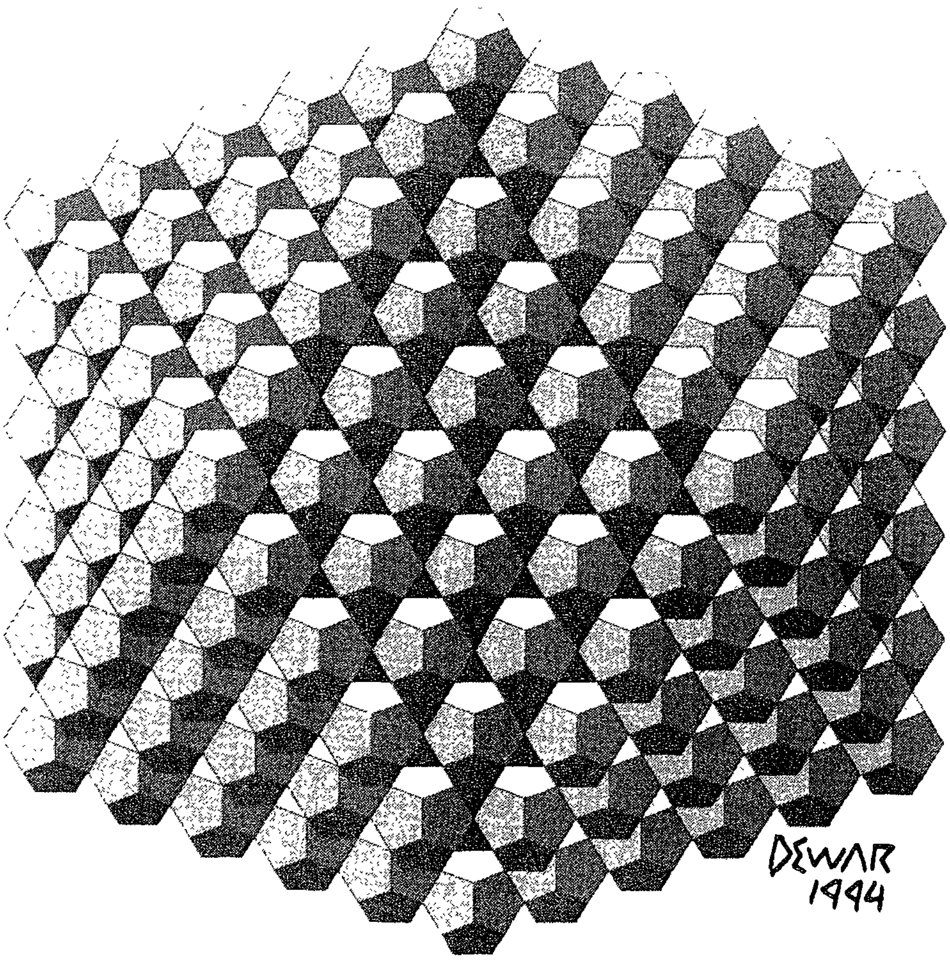
Symmetry: Culture and Science

Symmetry: Natural and Artificial, 2

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 6, Number 2, 1995



Third Interdisciplinary Symmetry Congress and Exhibition
Washington, D.C., U.S.A. August 14 - 20, 1995

HOMOLOGY: A KEY TO MORPHOGENESIS

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The paper will show that in many areas of nature, homologous patterns are formed from various substrata, organic or inorganic. This point to common principles of structuration in operation, to elementary rules of growth characterized by the phenomena of branching, gnomonic development, allometry, and spirality. In particular, the study of comparative morphology in its largest acception reveals homology of structure with phyllotactic patterns, which can through light in their origins. Given that this homology is found in areas where there are no genes, the origins of phyllotaxis must be sought beyond the botanical substratum. Genes only appear to bring tremendous speed in the formation of the patterns, thus their overwhelming presence in phyllotaxis, the study of the patterns made by similar parts in plants (e.g. organs, leaves, florets, scales).

GEOMETRY OF VENATION AND ORIGAMI MODEL OF LEAVES

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1. Introduction

Leaves of plants are most important organ with which we can learn to recognize those names. There are many different kinds of leaf types: entire type, digitate type and compound type. Standard computer graphics method that makes use of bicubic patches is suitable for modeling surfaces of simple leaves and not for digitate leaves.

There are two things obtained by observation of leaves (Maekawa 1979). First, each leaf type is closely related to several main veins radially and straight running from base to edge. Second, young leaves that fold in leaf buds are shaped very similar to mature ones. In buds young leaves would grow under the condition of being folded and main veins would be weakly concerned with this folding. Thus the venation and the folding of leaves are closely related each other. After two blades of a young leaf are freed from the bud and opened up, they show the symmetrical V-shaped pattern of the leaf veins as seen in the Miura-ori (Kresling 1994). From this viewpoint an origami modeling method is proposed to generate digitate leaves (Kaino 1994). Using this method computer graphics image of plants with digitate leaves are generated successfully.

2. Venation and Folding of Digitate Type of Leaf

The way in which leaves fold in buds are a characteristic of plant species as well as leaf arrangements: opposite, alternate or whorled. A leaf folding of Acer is likely to fan because twofold arises about each main vein. Because leaves of Acer are placed opposite each other in pairs, valley folds are main situated toward the protective scales and mountain folds are on the plane that symmetrically bisect its bud. Figure 1 shows three examples for which digitate type of leaves have rather simpler form when they are folded up. The cutting folded leaves along a few simple curves is considered to determine complicated edges of digitate leaves. It is noticeable that main valley folds running radially from the base to edge divide leaf into several parts and deeply cut them.

Folding a sheet is determined by the Fusimi's inner point theorem. This theorem is generalized as follows. There are *m*'s folds in a sheet that run from a point O denoted by OA_1, OA_2, ..., OA_m. If this sheet is folded up into plane with those *m* folds, it is proved that *m* is even and

$$\begin{aligned}\pi &= \angle A_1OA_2 + \angle A_3OA_4 + \cdots + \angle A_{m-1}OA_m \\ &= \angle A_2OA_3 + \angle A_4OA_5 + \cdots + \angle A_mOA_1\end{aligned}$$

This theorem determines a local arrangement of folds that runs radially form and explains why for leaves folded into two with straight midribs their venation is almost symmetrical. Main leaf veins are naturally represented by the valley folds.

3. Origami Modeling Method of Digitate Leaf

The origami modeling method is to generate digitate leaf out of a sheet in the process of folding and cutting which correspond to *origami* and *kirigami* in Japanese. This method is suitable for the three-dimentional CG representation and the natural venation of digitate leaves, and generates a V-shaped surface of digitate leaf out of a sheet in the following way. (1) A sheet is divided by main mountain folds that run radially from a leaf base O. Let's denote two adjacent mountain folds by half-line OP and OQ. (2) A part of the sheet cut by OP and OQ is folded into two with a valley fold whick runs straight from O. This fold may branch according to the inner point theorem. (3) Folded sheet is cut along a simple curve.

For Acer palmatum main veins run straight from the base. For Acer tshonoskii a midlib divides into three and the other main veins usually divide into two. This leaf of palmate lobing and venation is easily generated by the origami modeling method. Figure 2 shows a developmental sequence generated from this model leaf. For Humulus lupulus (Hop) a digitate leaf is also divided into several parts by main mountain folds that run radially from base to edge. It is noted that each shape of two blades of this leaf is well represented by Archimedean spiral. This separability is likely to explain why H.lupulus leaves come in various shapes deeply cut around edges. Because H.lupulus is a quick growing vine, developmental sequences of leaves from bud to mature one are necessary to generate CG image of this plant.

Figure 3 shows such a CG image of a hop vine.

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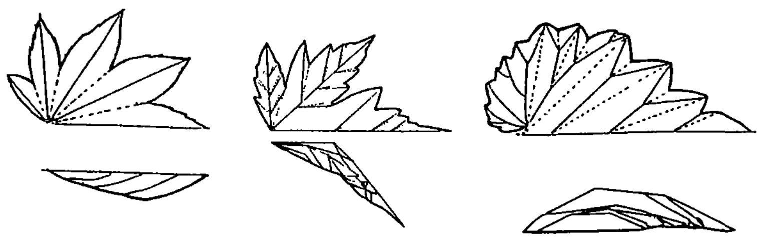

(a)Acer palmatum (b)Acer tschonoskii (c)Humulus lupulus

Fig.1 Digitate leaves and their folding. Solid lines denote valley folds, dashed lines mountain folds and bold cueves edges of leaf

Fig.2 Developmental sequence of a model leaf of A.tschonoskii

Fig.3 CG image of H.lupulus