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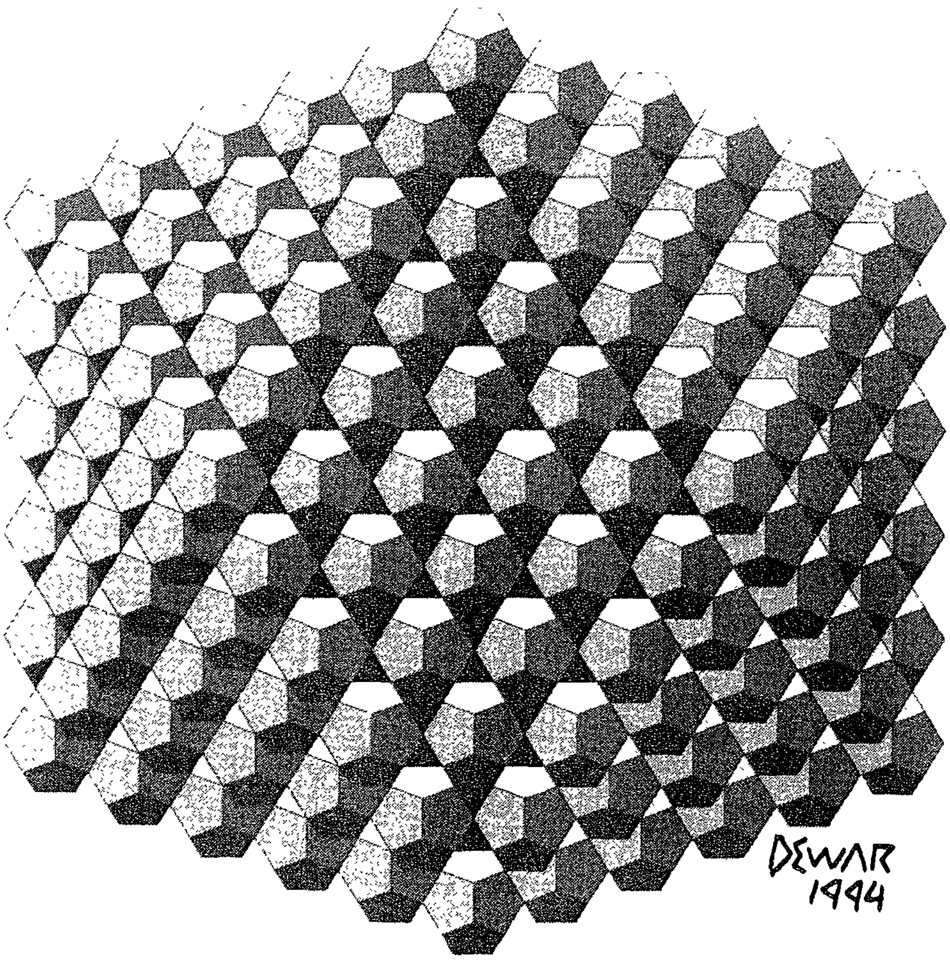

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ORIGAMI TESSELLATIONS

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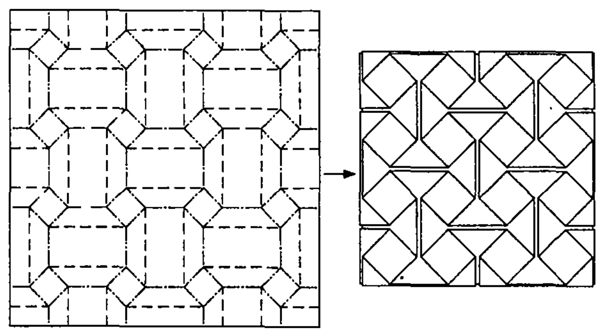
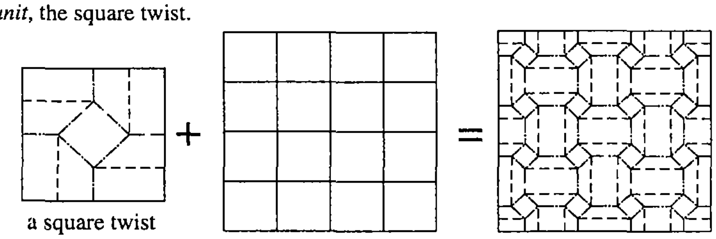
Origami is the Japanese name for the art of folding paper into intricate designs and objects. The structure of such paper folds can be studied by looking at the underlying *crease pattern.* That is, after folding a bird, for example, we can unfold it and examine the creases that produced the bird. These creases often exhibit a pleasing symmetry and various mathematical properties.

An *origami tessellation* is an origami model whose crease pattern is a tiling, or tessellation of the plane. For simplicity, we only consider models which fold flat. An example is illustrated below. (The dashed lines indicate valley creases, while the dash-dot-dot-dash lines indicate mountain creases.)



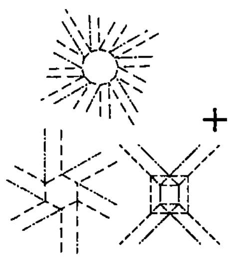
To understand the structure of such folds we classify them occurding to the *underlying* tiling on which the crease pattern is based, as opposed to the actual tiling formed by the creases. For instance, the above example can be thought of as the square grid tiling (4^4) where in each of the squares we've placed an *origami unit,* the square twist.



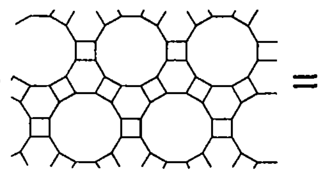
a square twist

(4^4)

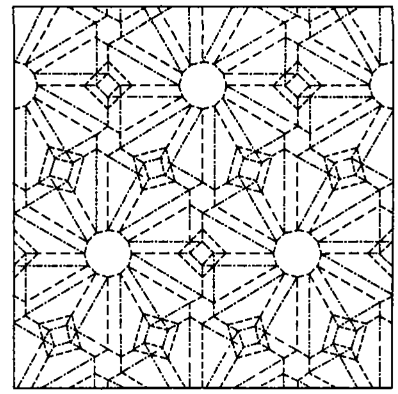
The "unit" which produces the above tiling is called a square twist because its creases actually twist and contract the paper. It can be tessellated in a square grid because (1) it has the same symmetry as the square and (2) placing two (mirror image) units together doesn't voilate any basic folding properties. This method of producing origami tessellations via twists can work for any of the 11 Archimedian tilings. A complex example of (4.6.12) is shown below.



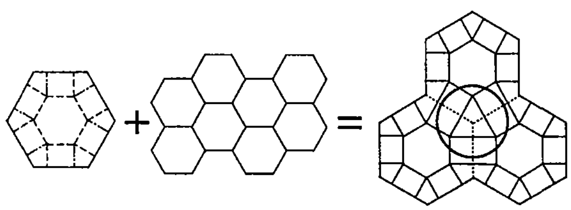
Dodecagon, hexagon, and partial square twists



(4.6.12)



From such examples it is possible to develop a more general theory of origami tessellations. That is, we define the boundaries (or sides) of two origami units to be **orimorphic** if when they are placed side-by-side we have that (1) their mountain and valeey creases line up and (2) any new vertices formed satisfy the 180° condition: the sum of every other angle about the vertex equals 180°. The idea is that origami units which fit into some tiling of the plane and have orimorphic boundaries will produce an origami tessellation. But this doesn't always work, as shown to the left. The hexagonal twist unit is completely orimorphic, yet when it is tiled we get the configuration of creases contained in the circle. It can be shown that this configuration is impossible to fold flat, which means this won't generate an origami tessellation. Nonetheless, this theory of orimorphic units has been used by the author to generate a wide variety of origami tessellations, including spiral and self-similar origamis.



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