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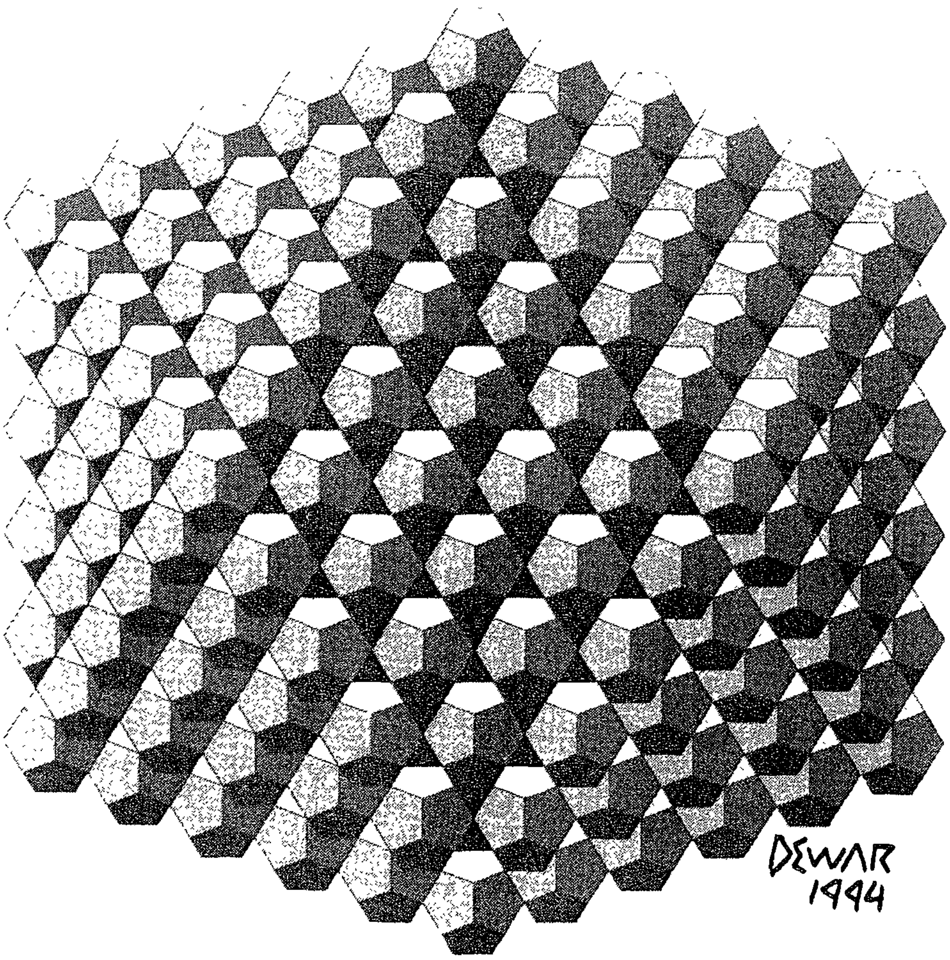

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THE USE AND LURE OF RULED SURFACES
- symmetric, more or less

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Developable surfaces are surfaces in which a family of straight lines (or segments) is wholly contained. A straight line (or segment), completely within the surface, passes through each point of the surface; such surfaces can be generated by an appropriate motion of a straight line (or segment) in space. They are said to be spanned by a one parameter family of straight lines. Ruled surfaces have 2 straight lines (or segments), wholly within the surface, through each point. They are said to be spanned by a straight spatial (non-planar!) grid on them.

Developable and ruled surfaces have played a considerable, at times surprising, role in art, nature and technology. They are generated by one or two families of straight lines, the simplest of geometric forms, yet include a rich variety of curves, surfaces and hypersurfaces. Sculptors (Pevsner and Gabo among others) have used them extensively in their art, architects and engineers have employed them in their designs of large structures; in nature they span membranes as minimal surfaces. In computer graphics and computer-aided-design (CAD) they serve as approximating patches (for which one has good error estimates, because of their simplicity). They are mathematically rich in discernible properties and come up in the various branches of geometry in 3 or more dimensions.

Developable surfaces can actually be constructed in space by connecting appropriate points, on 2 lines in space by straight segments. The mathematical construction follows (which will be explicitly shown) resulting in the appropriate formula for the desired surface, and the explicit possibility to display it on paper or screen in any desired way, to modify and manipulate it for any desired purpose.

Using symmetry arguments one may choose developable surfaces which are also ruled, i.e., which of the surfaces spanned by one family of straights admits another family of straight lines, passing through each point of the surface. One can than 'break' the symmetry of these symmetric ruled surfaces, in a way that preserves the linearity of the straight grid on them, thus enlarge the family of surfaces to include all ruled surfaces in three-space (the ordinary one, in which we have evolved). One can also deduce the analytic expressions for all such surface, and see that they must be quadratic in all the three Cartesian coordinates. Their can be described in canonical forms.

The much richer family of developable surface (non-ruled!), includes Moebius strips, and extension to other non-orientable surface, which will be demonstrated constructively and analytically, geometrically and algebraically. Symmetry considerations come up often, sometimes unexpectedly, with appropriate symmetry breaking yielding variations on the symmetric theme.

Ruled surfaces will be introduced via photographs and models. A simple mathematical analysis will be presented, which will result in a constructive formulation for the design of curves and surfaces. A few simplified proofs of theorems in plane and solid geometry will be given, with geometric extensions to higher dimensions, and ways to visualize them. Some of the latter is a new way to look at old stuff, some may be new.

Symmetry aspects in the mathematical theory and applications will be stressed. They are prominent in some of the art-for aesthetic reasons, in architecture and engineering uses for both beauty and functionality and in nature, expressing symmetry in the physical law governing the creation and stability of these ruled forms. In this context, like in many others, the occurrence of partial symmetry breaking results in a wealth of new forms, new aesthetics, new problems and new conjectures.