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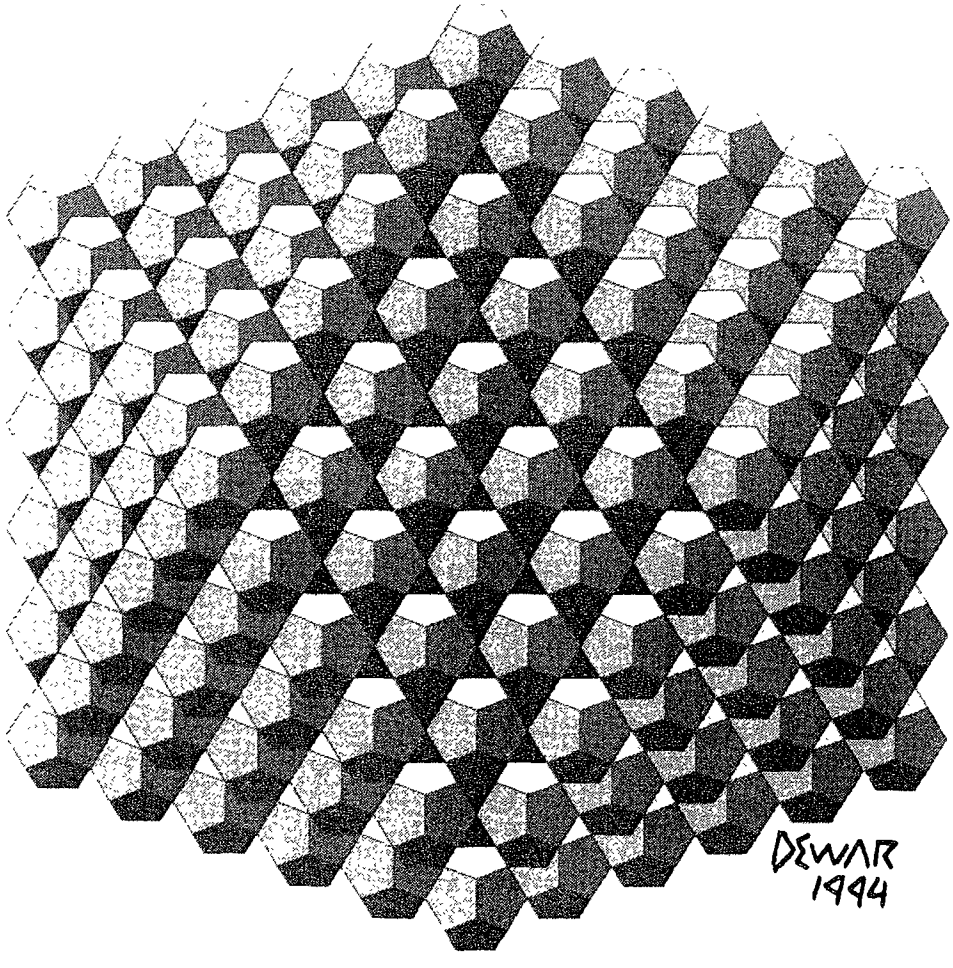

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MATHEMATICAL MODELS OF SYMMETRY IN MUSIC

POZZI ESCOT

Jacob Bronowski writes that discoveries of science and the works of art are both explorations and explosions of a hidden likeness, that we see only the surface beauty of nature and life. The Pythagoreans conclude that proportional number-patterns rule the essence of all things. These patterns are the basis of dynamic symmetry and the intuitive realization of them in the creative mind results in those explorations and explosions. Being responsive to the mathematics which connects and balances this symmetry means developing the critical mind and the aesthetic experience. Music has an internal logic or system of proportions governing its phenomena; music's events are of number and progressive divisibility affects the composition down to the smallest aspects and partitions of the overall structure. Again Bronowski writes that Leonardo Da Vinci was occupied with the logic of the processes he saw in people and machines; and he looked for the hidden structure because it expressed that logic and symmetry. This presentation discusses musics from Gregorian Chant to the modern Gyorgy Ligeti analizing the specific symmetrical formations of their respective designs. Slides are used showing original mathematical models determined by precise statistical information of the score.

The following extended abstract takes three examples of music. We begin with an Antiphon by 12th century composer Hildegard von Bingen. The medieval Gothic world had inherited the legacy of St. Augustine's thought. For him music and architecture were congruent forces since both were the result of a vision of number -- an attempt to repeat the harmonious consonances of an unattainable world. Geometry meant aesthetic perfection and was considered the link between God and earthly realm. It was Abbot Guy Charlieu (a pupil of the Abbot of Clairvaux, St. Bernard) who in his Regulae de Arte Musica writes that music must sound the great Gregorian virtues of symmetry and balance, namely the virtues of mathematics. The analysis of Antiphon "Sed diabolus" brings out its conmensuration.

There are clearly delineated six phrases divided by:

-onset/decay on the Final, priority pitch D, of the Dorian mode (phrases 1,2,5)
-onset/decay on the Final and Dominant pitch A (phrases 3,4)
-onset on the fifth D-A dyad (phrases 1,3)
-text semantics and syntax
"Sed diabolus
in invidia sua
istud irrisit
qua nullum opus dei
intactum
dimisit"

These six phrases of 9, 16, 9, 12, 6, and 16 attack-points each result on remarkable proportions:

-6,9,12 have arithmetical mean ([a+c]/2=b)
-6,9,16 have harmonic mean (2ac/[a+c]=b)
-the special Golden Mean, negative and positive, occur at salient points of the Antiphon's musical flow -- negative (.382x68 total number of attack-points), divides the Antiphon into two and four phrases; the positive (.618x68), at the second new B pitch in the fourth phrase after 2/3 of the phrase's 12 attack-points and just after the apex of the Antiphon has been sounded
-identical extended decay (E-D-C-D) for phrases 1,2,3, and 6

The fourth phrase, whose text mentions the work of God, presents a triangular graphic notation in its deployment literally suspended from the Final D^4 and does not end on the Final like all the other five phrases, but on the Dominant, repeating this A on the word God and sounding it on register four, five times. No other phrase sounds the B pitch and the A is only heard in phrases 2 and 3 in register three and the one A^4 is in the opening dyad of the Antiphon. Other significant symmetries are:

-the highest Final D5 is at exactly 26 equal attacks (negative Golden mean) distance from the very first lowest point and Dominant pitch A3
-the D-A dyad opening the first and fourth phrases divides the Antiphon into two exact halves of 34 attakcs each (9+16+9 and 12+6+16)
-the presence of the lowest pitch in the first half balances the presence of the highest pitch in the second half
-two phrases in each half are pentatonic (1,2,4,6) and phrases 3, 5 are each hextatonic and tetratonic
-the extended onset/decay of both halves are the same, only a slight register shift marks the second half extended onset (first half is D-A-D-C/E-D-C-D in register four; second half , the beginning D-A is followed by D-C shifted to register five, the decay is the same).

The first Prelude for piano by Frederick Chopin is divided into four phrases of 8 equal measures each plus 2 added to the last phrase to prolong the Prelude's final cadence.

Both the first and third phrases have cadential endings, the second however, runs into the third one to stress and enrich the Prelude's climax occuring here. The Prelude is a spiral of harmonious growth -- a gnomonic growth where an evident development remains homophetic; namely, a sonic design similar to itself. Through its 34 equal measures the Prelude presents an exquisite dynamic symmetry entirely dependent upon the Fibonaci additive series. Crucial musical events take place at measure 8 where the only lowest attack is sounded; at measure 13 where the first important and permanent change of the harmony is sounded and the precise moment where the chromatic ascent from register one begins; at measure 21 where the only highest and loudest climatic

event take place; at measure 34 concluding the Prelude. The Golden Mean (positive) occurs at the climatic point; the negative at the first change of harmony. Both lowest and highest poles are related spatially/temporally with a .618 ratio. It is at the particular moment where the ascension to the climax is finally given its thrust onward that the French pianist Alfred Cortot (who edited Chopin's Preludes) indicates a stretto (a sudden overlapping thematically). This coincides with precisely the division of two equal temporal parts of the Prelude (16+16) if the two extra measures of repeated arpeggiated Tonic (priority pitch) cadence at the end are not considered, since they are unnecessary to the Prelude in terms of language and gestural dimensions. However, these two measures are imperative to balance the temporal dynamic symmetry of additive Fibonaci numbers (8+13+21+34).

Abraham Moles, the French psychologist, refers to music perception as a musical message broadcast in a succession of packages of originality of varying size. With our last example, in this extended abstract, we touch the modern world. Opus 11, No. 1 of Anton von Webern for piano and cello is a chain of eight differentiated packages, all lasting barely one minute, held together by perfect ratios of dynamic symmetry and completely separated from each other by silence, thus obtaining a non-linear surface. There is an exact bi-partite division where every single detail of the sonic frame in the first half presents its mirror equivalence in the second although the music is different in each half:

-a total of 33 pitches each with
-27 equal durations of which 18.5 are sounded, 8.5 silent
-four chords
-four packages of originality of varying size (6+9+6+6, 3+6+6+12
-eight different pitches for the cello
-four packages having ranges adding up to 139 (100 cents from one adjacent frequency to the other or number of chromatic steps), 38+47+22+32, 35+29+47+28
-the same number of pitches for registers 2,5,6 while 3 and 4 balance each other
-onsets and decays on silence
-the same tritone-sounding interval for its four packages
-balancing at equal distances the lowest and highest pitches of cello and piano, F#2-Eb6 and Eb2-F#6

The varying size packages are in multiplesof three and the Fibonaci additive series from 1 to 55 guide the sonic motion through prominent musical events:

-at 1 of equal durational values there is silence
-at 2 there is again silence
-at 3 the lowest cello pitch is sounded
-at 5 there is again silence
-at 8 the highest pitch of the piano is sounded

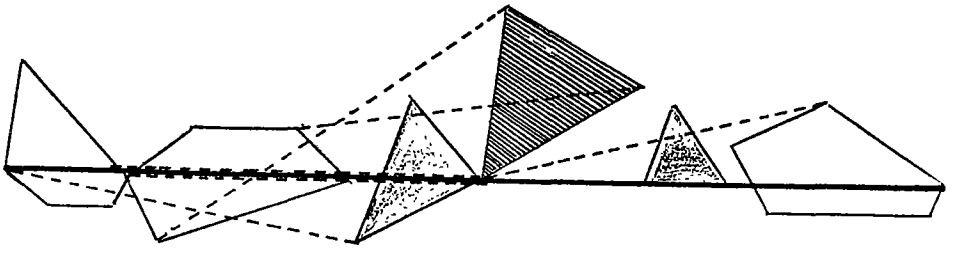
-at 13 the first slowing down of the musical flow appears
-at 21 the climax takes place
-at 34 the last slowing down and the sounding of the lowest piano pitch
-at 55 the work ends (although the entire Opus 11, No. 1 lasts for 54 equal durational values, there is however, 3 slowing down passages for 9 equal values and one speeding up for 6; the extra 3 slowing values adding one value to the actual 54)

Other symmetries are present in this extremely short work: the coefficient 8 summing up vital constructive features seven times:

-8 packages or phrases
-8 chords
-8 different total time durations
-8 velocity changes
-8 different dynamics
-8 different pitches for the cello in each of the two parts
-8 as the frequency interval separating the boundaries of the smallest package from the work's own boundaries!

The negative Golden mean happens at the most relevant event of the work, in package 4 where the climax is heard as a loud, noisy, highly draamatic accident that continues the speeding up of the previous package before abruptly changing to slowing down. The positive takes place in package 6 where the musical gestures are heard as if distributed in space and time; where the two most significant pitches of the work, the interchangable low/high Eb-F# are heard (the lowest Eb for both the piano and the work for the first time) in succession initiating an upward motion of almost two octaves within the limited time duration of the package itself.

Opus 11, No. 1, with less than a minute's span of time, creates a multiplicity of correspondences which outright gear the composition to perfect conmensuration, and thus a game of dynamic symmetry fascinating to hear.



complex plane of "Sed diabolus", abcissa = temporal flow of equal attack-points, coordinate = frequency
geometrical notation of the six phrases show vertices of onset/apex/nadir/decay
broken lines = Golden Mean equivalences among vertices (negative)
solid black line = the Final priority pitch D4