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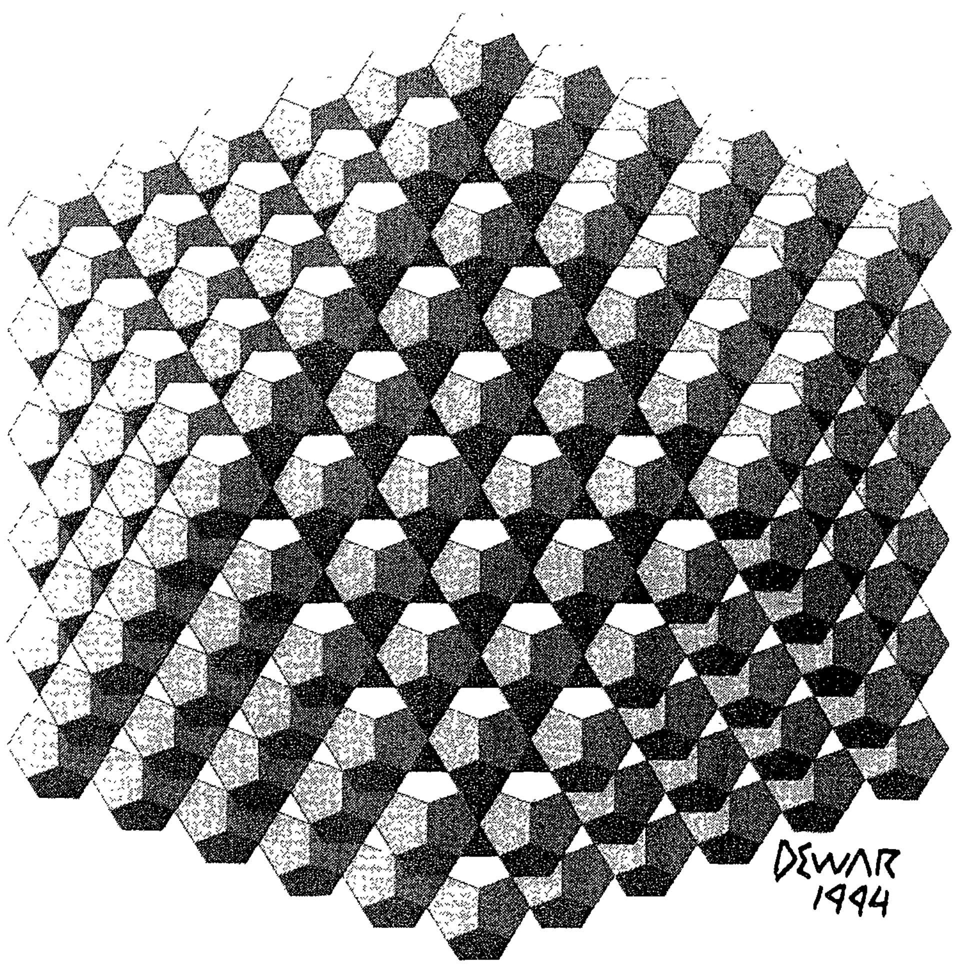

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Fractal Geometry in African Material Culture

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While my mathematical work describing the presence of fractal shapes in African architecture is quite orthodox, its social significance invites a variety of misinterpretations. On one hand, those of the 'zen physics' persuasion will see this as proof of an abstract, mystical intuition of the non-west -- thus supporting the essentialist stereotypes of orientalism. On the other hand, those who romanticize the 'eco-native' will see this as evidence for a concrete, unconscious expression of oneness with nature -- thus supporting the colonial construction of primitivism. My own approach is based on fractal geometry as a conscious, artificial construct in African societies, that is, as an indigenous knowledge system or technology. The methodology and theory behind such an approach has become well established in the recently developed discipline of ethnomathematics (Ascher 1991, D'Ambrosio 1985, Gerdes 1985, Crowe 1988, Zaslavesky 1973).

1) Fractals in African Material Culture

In north Africa fractals are associated with the feedback of the "arabesque" artistic form, particularly in the branches of branches forming city streets. In central Africa it can be seen in additive rectangular wall formations, and in west Africa we see circular swirls of circular houses and granaries. These structures are distinctly fractal in appearance and can be easily simulated by fractal geometric methods (e.g. figure 1, the settlement of Mokoulek in Cameroon). The fractal structure of African settlement patterns was confirmed by dimensional measure of digitized photos in Eglash and Broadwell (1989).

The cultural semantics of these architectural forms show that they are not merely a consequence of unconscious spatial optimization. Rather, concepts of scaling, recursion, and even infinity are symbolically linked to these material structures in close parallel to their usage in the western context of fractal geometry. This is by no means limited to architecture. Recursion, for example, can also be seen in many African art forms. Ethiopian crosses and ancient Egyptian cosmological icons use it to express religious ideas. In the Cameroonian bronze work we see it as a visualization of prestige (including a wonderful example of self-reference: a figure of the king smoking his royal pipe; the bowl of which is a figure of the king smoking his pipe, the bowl of which...). Fulani wedding blankets use scaling iterations to map an animist energy flow. Many other examples express fecundity, fertility, and unending health or well-being through the unending recursion of fractal forms.

Specific scaling techniques are particularly evident in Ghana, where the use of log spirals to represent self-organizing systems (biological morphogenesis and fluid turbulence) is common. Expansion/contraction of spatial form is also common in the textiles of this area, and again shows conscious use of this concept.

Symbolic mathematics in Africa also involves fractal concepts. Numeric scaling sequences, particularly doubling, can be seen in notational, gestural and linguistic counting systems. Doubling is used to model the forked patterns of lightning in the Shango religion, and can be seen in the mathematical records of ancient Egypt, where it was used for multiplication and division. Binary recursion is used in Bambara sand divination. The stylized snake symbol is used to depict feedback processes as the essence of morphogenesis in environmental, biological, and chemical systems, as well as in making the associations of recursion with infinity (e.g. the snake as symbol of eternal health or prosperity, as well as cosmological infinities). It may also have been the source for the infinity sign invented by cryptographer John Wallis in 1655; which brings us to the second area of this study.

2) Africa in the history of fractals

Recursive scaling in Egyptian temples can be viewed as a formalized version of the fractal architecture found elsewhere in Africa, and is most significant in its use of the Fibonacci sequence (Badawy 1965; see Petruso 1985 for additional Egyptian use of the sequence). The sequence is named for Leonardo Fibonacci (ca. 1175-1250), who is also associated with an unusual example of recursive architecture in Europe (Schroeder 1991, pg 85). Since Fibonacci was sent to North Africa as a boy, and devoted his years there to mathematics education, it is possible that this seminal example of recursive scaling is of African origin.

Benoit Mandelbrot, the "father of fractal geometry," reports that his invention is the result of combining the abstract mathematics of Georg Cantor with the empirical studies of H. E. Hurst. Cantor was a 19th century Rosicrucian mystic, who often combined his mathematics with his religious belief. His cousin Moritz Cantor was a famous scholar in the geometry of Egyptian art and architecture. Given these facts, and the similarity of this first European fractal to the Egyptian architectural structure symbolizing recursive autogenesis, an Egyptian origin is likely here as well.

H.E. Hurst was a British civil servant who lived in Cairo for 62 years. It was only because of the data set provided by the Cairo nilometer that Hurst was about to deduce the scaling law which Mandelbrot used to bring Cantor's abstract set theory into empirical practice; thus Hurst's work can be viewed as a direct outcome of this African tradition of searching for frequency domain patterns.

Does the characterization of euclidian versus fractal become yet another self/other dualism, or, as suggested by Gilroy (1993), can the properties of fractals themselves be used to displace such binaries? The boundary between self and other becomes quite different if we conceive of it as a fractal; and the diasporic history of African culture could be said to constitute such an example (c.f. Phillips 1990). Also unresolved are

Foucaultian structural questions. In colonial discourse, for example, the "chaotic" settlement patterns of Africans are clearly used as proof of inferiority. But colonialism also made use of the self-organizing aspects of African society, and valorized their "more natural" ways of living. Conversely, while there are many examples of how Africans have benefited from decentralized social processes, there are also instances where such self-organization becomes self-exploitation (c.f.. the voluntary caste system described in Stoller (1988)). How can we differentiate between these two effects, and what are the implications for the formulation of concepts like "autonomy?"

In summary, the new formulations of chaos and order in science are potent agents for cultural change, and it is at the intersections of the humanities and sciences where we are likely to find the most promising kinds of turbulence. Fractal geometry in African culture provides a particularly useful focus for such inquiry, while supporting questions of wider import in many areas.

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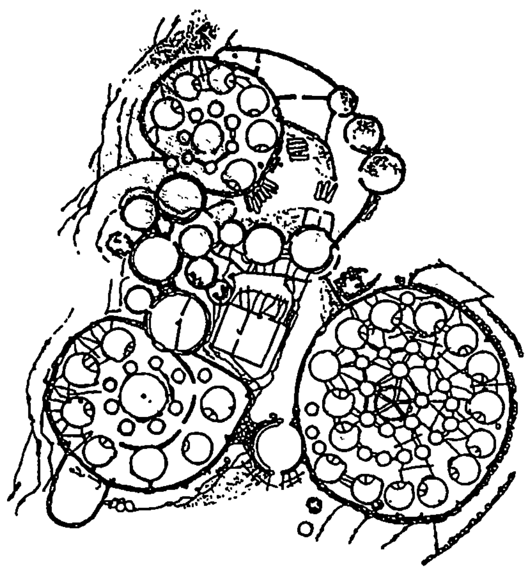
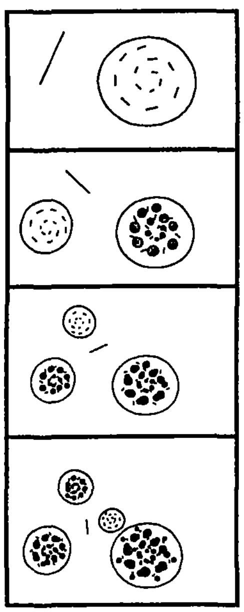
Figure 1: Mokoulek settlement diagram



architectural diagram for Mokoulek

Iteration



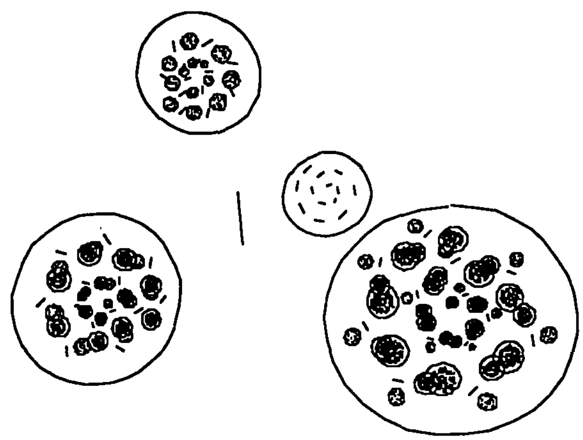
1st

2nd

3rd

4th

fractal generation



enlarged view of fourth iteration

BIOLOGICAL SYMMETRY AND INFORMATION-PROCESSING

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There seems to be a profound relation between the symmetry characterizing basic physical laws and that observed in biological structures. Two elementary symmetries of physical law are *i)* symmetry under translation, and *ii)* symmetry under rotation. Now the shape of many organisms seems to be related to *(i)* by the fact that all individuals of the same species often have the same shape, and to *(ii)* by the similarity of two or more sides of the organism's body. Is this affinity between the physical and the biological symmetries only coincidental or does it reflect a more profound relation?

An affirmative answer to the second option comes from the study of evolution as a computational process (Elitzur 1994, 1995). Central to this "proto-cognitive" model is the assertion that any population of self-replicating systems is capable, by the very dispersal of similar units in a certain environment, to extract, store and process information about that environment.

Within this framework, it is pertinent to examine the evolution of shape constancy and symmetry. As one follows the evolutionary tree onwards, from primitive unicellular organisms, through plants, to mobile animals, one encounters growing degrees of shape constancy and symmetry. We show that this evolutionary trend reflects growing cognizance of environmental invariants: The organism's shape should optimally adapt to those features of the environment that are not random, namely, to those environmental features that do not fluctuate from one site to another. Our hypothesis neatly reconciles two rival hypotheses concerning the biological role of symmetry: *i)* Møller (1992) argued that females' preference for symmetrical males, observed in swallows, reflects preference for healthier males, while *ii)* Enquist and Arak (1994) and Johnstone (1994) argue that this preference stems from the fact that symmetrical shapes are easier to recognize from various angles. In the proto-cognitive framework that we propose, both hypotheses are complementary in that they argue that evolution developes increasing abilities of recognizing invariant, as opposed to random, elements of the environment.

The above two symmetries appear also at another level, namely, the organism's sub-units. Interestingly, even non-uniform and a-symmetric organisms, such as plants, are comprised of highly uniform and symmetric units, such as leaves. Why do trees possess uniform and symmetric leaves? Branches grow more intensively towards the more illuminated side of the plant, thus being non-uniform and asymmetric. Why then not the leaf itself? True, symmetric and uniform leaves require smaller genetic programs for their production, but the fact that there are asymmetrical and non-uniform leaves shows that this explanation alone does not suffice. We therefore turn to information theory in search of a more appropriate answer.

Leaves are the main food resource of many insects and larvae. Luckily for the trees, many birds feed on insect larvae. The tree thus ought to inform the birds which of its leaves is harassed by the pests. Could leaves' uniformity and symmetry enable the tree to do so? Three observations (Heinrich 1979, 1992, Heinrich and Collins 1983) support this hypothesis: *i)* When the shapes of tree leaves were artificially damaged, the insect-eating birds frequented these trees or leaves more than undamaged leaves or trees. *ii)* Some caterpillars nibble the leaf margins in such a way that its shape is maintained, thus making it harder for the birds to locate the damaged leaves. *iii)* Many oak species, whose leaves are neither uniform nor symmetric, seem to be immune to leaf-eating pests. Notice that Møller's (1992) abovementioned hypothesis is complementary in this respect: Symmetry in birds is taken as an indication of well-being. Shmida (1992) suggested that flower symmetry is similarly correlated to high amounts of sugar.

Leaf uniformity and symmetry are thus efficient means of information transmission between the plant and other organisms. Could they also enhance information processing within the plant itself? We would like to suggest an affirmative answer, based upon the following observations. The leaf changes its angle throughout the day so as to get maximum sunlight. In order to best function as an "antenna," it must be symmetric in relation to the source of the radiation. It is the comparison between the light absorbed by the leaf's right and left halves that enables locating the source. Hence, identical halves would best perform this task. The explanation for the leaves' uniformity is similar. The formation of trunk and branches poses many demands to the tree: overcoming gravitational forces, optimal exploitation of light, avoidance of intersection between the brances, etc. Also, it consume much more resources than the formation of the small, short-lived leaves. Now leaves, besides fulfilling their known photosynthetic goal, are also capable of affecting the tree so as to grow towards those directions where light is abundant. Therefore it is vital for all leaves to be uniform and symmetric: Only then can the tree average all the environmental signals sensed by all the leaves, so as to grow the optimal shape. One way to test this hypothesis is to artificially affect leaves to have irregular forms and see if this affects the normal formation of the branch on which they grow in comparison to control branches.

There is an ethological analogue to this presumed mechanism: Increasing evidence suggests that flocking behavior among social animals enables the herd to reach optimal desicions by averaging all the individual movements (Deneunbourg *et al.* 1991; Kerlinger 1989).

The hypothesis that uniformity and symmetry serve information processing purposes rests on a principle well-known from information theory: Any medium or channel that stores, transmits or processes information must be in a low-probability state, i.e., highly ordered. Such, e.g., are the unexposed film or the clean writing-letter. The calibration of measuring instruments creates a low-entropy state in which the instrument is maximally unbiased and thus able to optimally receive incoming signals.

Uniformity and symmetry meet these very purposes. By growing a myriad of leaves that have the same shape and whose two sides are identical, the tree performs a very efficient form of measurements, comprised of numerous and repeating measurements that are later compared. Similarly, in order to notice local asymmetries in the environment, the leave itself must be symmetric in the direction measured. Leaf uniformity and symmetry are thus akin to the basic idea of replicability in scientific experiments: In order to rule out local, random artefacts, the replicated experiment should use identical, calibrated and unbiased measuring instruments.

Indeed, in is by now commonly accepted that many biological systems, besides carrying out their main tasks, also constitute information-processing systems. For example, the blood system in the brain serves not only to transport nutrients and oxygen to the brain, but also to inform the brain about the body's temperature, chemical balance, etc. The introduction of information-theory concepts to biology is likely to provide many further insights into the physical foundations of life.

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