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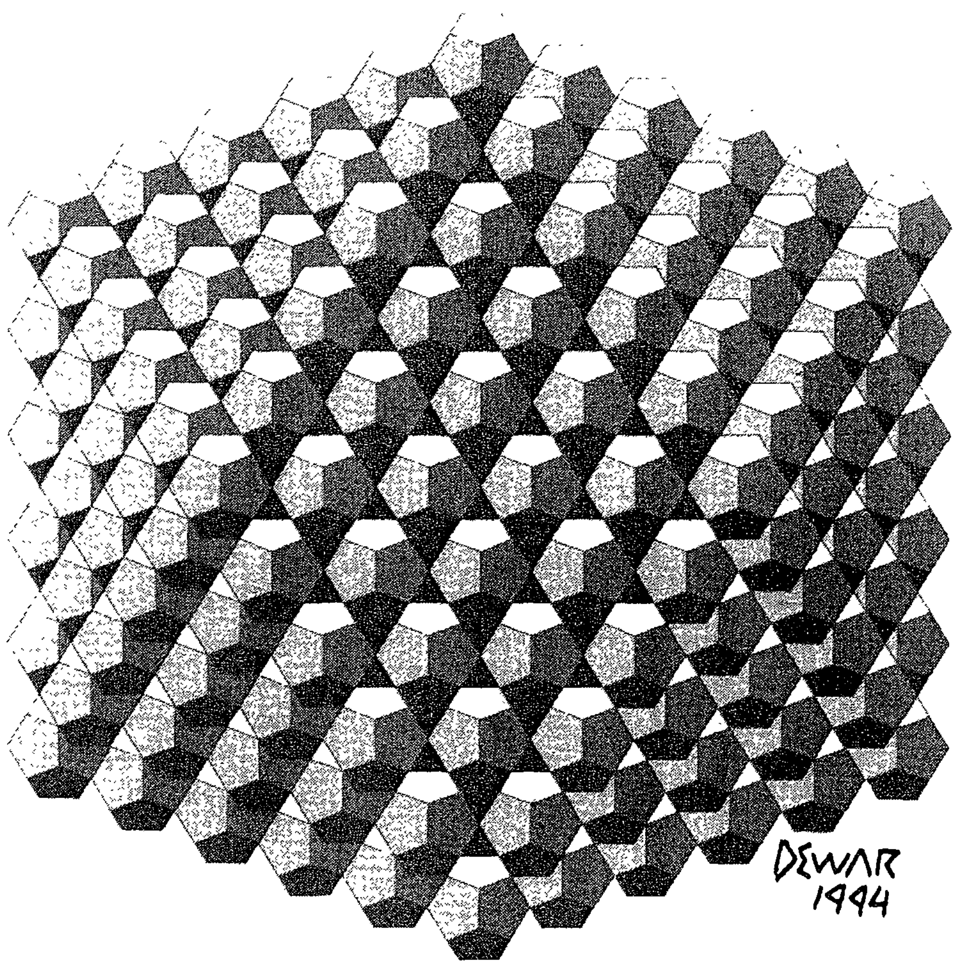

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The geometric stability of highly symmetric tensegrity structures

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A tensegrity structure is a collection of incompressible struts held rigidly in place by a collection of cables. Mathematically it can be modelled as a collection of points where certain pairs of the points are constrained not to get closer together (the struts) and another collection of pairs are constrained not to get further apart (the cables). A basic question is to tell if this structure is (globally) rigid (or stable) in the sense that the constraints determine the configuration up to congruence. One way to do this is to calculate a certain n-by-n symmetric matrix S, the stress matrix, where is the number of points is n. If S is positive semi-definite of maximal rank, then the tensegrity structure is stable. If further the tensegrity structure is so symmetric that the group of rigid congruences of the structure is transitive on the vertices, then S decomposes into much smaller symmetric matrices corresponding to the irreducible representations of the group of symmetries. In fact it is possible to catalog all the configurations of such highly symmetric tensegrities for all the point groups of three-space for all the combinatorial choices of connecting the cables. We will show some pictures of these stable tensegrities when the group is the group of rotations of the cube. They are quite pretty.

FRACTAL SYMMETRY IN NATURAL AND SIMULATED FRACTURES

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The recognition and measurement of fractal symmetry in natural rock fractures provides a method for analyzing the fracture patterns. Fractures are fast paths for fluid flow in the earth. This has importance for groundwater, which comes to the surface, through fractures, in springs, both hot and cold. Fractures allow the migration and accumulation of metals, forming economic mineral deposits. They also allow the migration of toxic materials from buried wastes and from leaking pipes and containers at the surface. When there is movement of the earth along fractures, (which we experience as earthquakes) these fractures are called faults. As an earth scientist, I have been given the opportunity to look inside the cracks of rocks, finding beautiful patterns that tell the history of the surface of the cracks. Fractal analysis of these patterns may be a means to translate these patterns into a history of sequential events.

What are some of the possible processes which might have created the patterns we see? We know that the rock has cracked, so there must have been an initial breakage which occurred because the rock was stressed beyond its strength. Pressure, temperature changes, chemical variations, and water are all very important factors determining the ability of the rock to withstand stresses. Next, there may have been many episodes of stress, from different directions, that may have caused movement along the fault, grinding and opening and closing the fracture. Then, fluids may have flowed through the fracture, depositing and dissolving minerals. The space created by all of these processes is the fracture aperture.

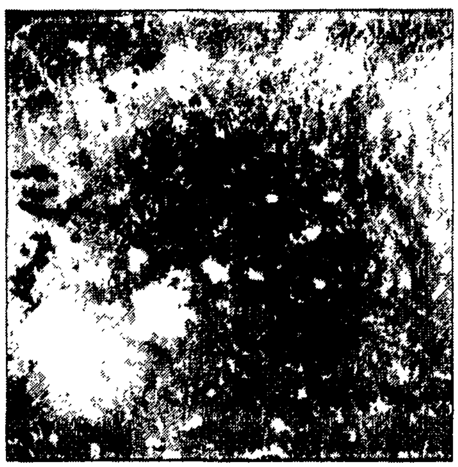

Figure 1. Stripa fracture.

Figure 2. Dixie Valley fracture.

Here are patterns obtained from fractures in different rocks (Figures 1,2,3, and 4). Each of these squares is approximately 7 cm by 7cm, and the average aperture is around 10 to 100 microns. Figure 1 is a granite from the Stripa site in Sweden, taken from a cylindrical rock core drilled perpendicular to the fracture. Figure 2 is an altered gabbro from Dixie Valley, Nevada, USA, taken from a faulted fracture outcrop near the surface.

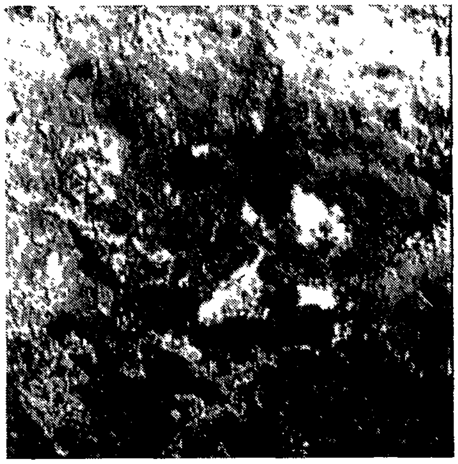

Figure 3. Yucca Mountain (YM3) fracture.

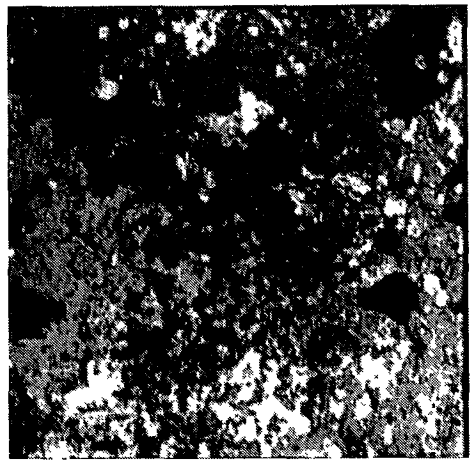

Figure 4. Yucca Mountain (YM4)fracture.

Figures 3 and 4 are volcanic tuffs from near Yucca Mountain, Nevada, USA, a potential site for a high level nuclear waste respository. These came from fractured rocks on the ground and are being used to help understand the fluid flow characteristics of the potential respository rock.

If we look at our natural fracture patterns, we can qualitatively see several distinct characteristics for these four fractures. These are not random patterns. A random pattern has a regularity to it that you don't see in these patterns. First, the Stripa fracture is relatively homogeneous, and has a semi-circular contact area (black) in its center. Second, the Dixie Valley Fracture, a fault, has elongated ridges and valleys along the direction of movement. This is anisotropic or directional symmetry. The ghost-like patterns have been translated at different locations and at different scales. Third, the Yucca Mountain fractures have a very patchwork-type pattern, again which has patterns translated at different scales and locations. YM4, however, has a diagonal trend in it, another type of anisotropy. In terms of symmetry, we can talk about mirror planes of symmetry on the two sides of a fracture, antimirror symmetry by the presentation of white as the most open (as in figures 1 through 4) versus black as the most open pattern, and translation and periodicity of the patterns(Hargittai and Hargittai, 1994).

These qualitative descriptions are useful, but we can also describe these fractures quantitatively, by measuring the fractal dimension. Fractal geometry is a relatively new approach to describing the geometry of irregular patterns (Mandelbrot, 1982). Fractals have dimensions which are fractional relative to the Euclidean dimensions of a line (1-dimensional), a planar surface (2-dimensional), and a volume (3-dimensional). The more irregular and rough the fracture, the greater the surface area, and the higher the fractal dimension.

One simple method for measuring the fractal dimension of these fracture patterns is to measure all of the perimeters and areas of the shapes you see, by computer (Mandelbrot, Passoja, and Paullay, 1984; Cox and Wang, 1993a,b). This information is then plotted on a graph, on a log-log scale, and if it follows a straight line, then you can calculate the fractal dimension. Fractal symmetry means that there is a self-similarity to the patterns, that is, the characteristics shapes repeat at different scales. The fractal dimensions of the Stripa and Dixie Valley fractures, measured by slit-island method, are 2.3 and 2.4, respectively (Cox and Wang, 1993b). There is very little difference, yet, we see a distinct difference in the character of these two fractures. Obviously we are missing some information with our fractal dimension, but what is it telling us?

Here are two computer-generated fractal fracture patterns, using the random midpoint displacement method (Saupe, 1988). Figure 5 shows a pattern with a fractal dimension of 2.25, while Figure 6 has a fractal dimension of 2.50. Both of these patterns have equal amounts of white and black space, yet the spaces have a different distribution. As the fractal dimenson increases, the complexity of the pattern increases, and there is more perimeter relative to area. This simulation involved only 1 process, whereas the natural fractures were formed by many processes.

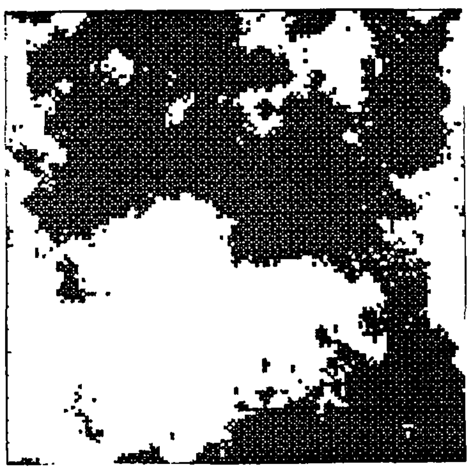

Figure 5. Simulated fracture , D=2.25.

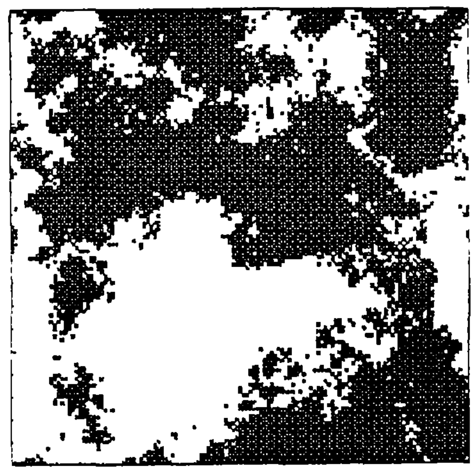

Figure 6. Simulated fracture, D=2.50.

Artists have, from the very beginning, received their inspiration from nature. Primitive cave paintings portray constellations, plants, animals and humans using scratching, burning, and painting on the natural rock surfaces of caves. Today we are able to observe nature with advanced technology which gives us the ability to see patterns at spatial scales from the subatomic to the astronomical. These patterns can inspire today's artists. If we take these fracture aperture patterns and make tiles from them by repeating the motif (Figures 7 and 8), which patterns will appeal more to artists, the artificial, or the natural? And why?

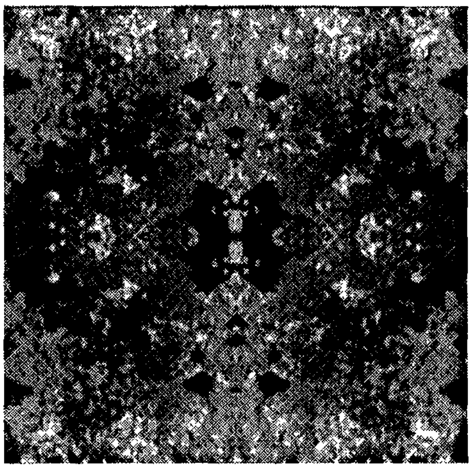

Figure 7. Tiling of Figure 4.

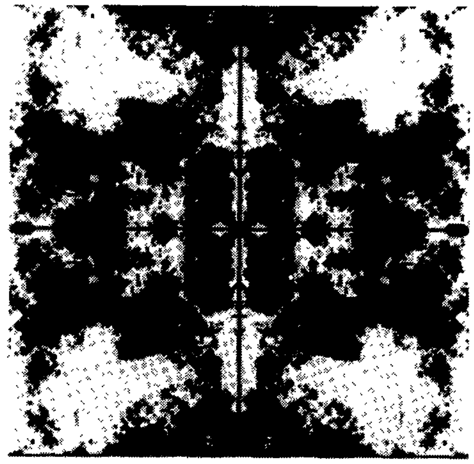

Figure 8 Tiling of Figure 6.

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