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1. INTRODUCTION

Here we shall examine the manifestations of symmetry in music, limiting ourselves for now to tonal music (which is based on predetermined schemes). The examination will extend to both the rules governing the raw material and the rules of the musical works, so as to shed light on the creative cognitive process in music and to introduce a new tool for musical analysis, taking the stylistic ideal into account (Cohen 1955). The type of symmetry influences the possibilities of musical organization in accordance with the ideal of the era, culture, or composer: simple or complex organization, short or long term on the immediate or overall level, and certainty or uncertainty regarding the continuation of the musical progression. Of course, maximum predictability causes boredom; and no predictability at all means no structure and no intelligibility. Music spans the entire area between these two poles.

Symmetry in music is manifested primarily in two concepts that are extremely fundamental to music and to cognitive activity in general: the scheme and the transformation, which are interrelated to some extent.

2. SYMMETRY IN SCHEMES

We divide the schemes into natural and learned schemes (which are not necessarily arbitrary) and into schemes of compositional rules and schemes of the raw material on a more preliminary level. Here we will just give a few examples of schemes.

2.1 Natural schemes of compositional rules: The convex curve and the division into 2^n

2.1.a. The convex curve

This concept is also known as the "arched form" (e.g., Green 1965). Here the term refers to the change along the time axis — a gradual increase to a single peak followed by a gradual decline. This curve can apply to any parameter that can be arranged on a scale, and it creates maximum predictability regarding the continuation of the progression.\(^1\)

A convex curve can occur on many levels. For example, for the parameter of pitch on the most immediate level, the large interval precedes the small while ascending and the small precedes the large while descending; for the parameter of duration, the rhythmic pattern starts and ends with a large value.

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1 In such a curve the rate of change is constant, unlike a zigzag, flat line (i.e., no change at all), or concave curve. Although the concave curve, too, is symmetrical, it is not bounded, and therefore we cannot predict where the musical progression begins and ends.
On the level of the musical phrase, there is only one peak, neither at the beginning nor at the end, thus avoiding both precise repetitions and numerous large changes. Interestingly, the compositional rules of Renaissance music, in which the ideal is tranquility and calm, require this convexity for the parameters of pitch and duration. On the other hand, some of the rules of Baroque music, in which the explicit ideal is excitement, in a sense deliberately break the rules of convexity (Buelow 1980): multiple repetitions of a small unit; substantial, frequent changes; etc. In the Romantic period, when the ideal of excitement was particularly prominent, convexity was violated even more noticeably. Convexity can also be manifested in the overall structures; the climax of this phenomenon is reached in Palestrina’s masses (fig. 1; this was only recently discovered by Guletsky).

Among the evidence for the fundamentality of the convex curve is its prevalence in folk music around the world (Nettl 1964) and its occurrence in the calm "syllables" of bird calls (Cohen 1983). In states of calm the parameters of pitch and intensity have concurrent convex curves, whereas in states of tension the curves are non-concurrent and deviate from convexity (fig. 2).

2.1.b. The 2nd scheme

This scheme — which can mean either a protracted division of units into two or a single, long repetition (1+1+2+4+8...), with varying degrees of precision, with or without transformation — is a natural process that, as with the convex curve, makes it possible to make predictions about the continuation of the progression. It comes as no surprise that this scheme appears in the classical period, the ideal of which calls for maximum clarity.²

² Most theoreticians of Western tonal music relate their analyses to this kind of symmetry (e.g., Koch 1782, 1983, p. 84; Lerdahl and Jackendoff 1983; Rosen 1972). This scheme was common in Far Eastern (especially...
2.2 Minimal Asymmetry in Learned Schemes in the Raw Material

Of these numerous schemes (e.g., scales, chords, and rhythmic patterns), we will only mention the scheme of the periodic scale system, from which the various scales are derived. In Western music, the diatonic system (only one) consists of two types of seconds, one of them double the other, or in units of semitones: $\frac{2}{2}, \frac{1}{2}$. Thus, the sole system in the West contains "minimal asymmetry," like the minimal asymmetry in biological systems (Arian 1981, 1987) and makes possible complex organization on various levels (in accordance with the Western stylistic ideal). Non-Western music, on the other hand, usually has multiple systems of scales that generally include more than two sizes of seconds; some of the systems have minimal asymmetry and others have no symmetry at all. Interestingly, minimum asymmetry also occurs with respect to duration, as in the skeleton of the cyclical metric pattern in African music (Arom 1988), which consists of a series of durations with the ratios $3, 2, 3, 3, 2, \ldots$. This skeleton makes extremely complex rhythmic polyphony possible. Minimal asymmetry is one of several aspects of the raw-material schemes through which the various schemes in the different styles can be examined and compared.

3. SYMMETRY IN TRANSFORMATIONS

Transformations constitute a large portion of the rules of musical composition, and awareness of them is even greater in post-tonal music. Note that studies on symmetry in music, a subject prominent in Lendvai's work [1977], do not always discuss transformations explicitly, and vice-versa. Here we would like to mention briefly the nature of transformations.

3.1 The variables that define the transformation are: (1) the operations; (2) the parameters upon which they operate; (3) the schemes and their realizations on the various levels on which the operations act; (4) the magnitude of the change produced by the transformations; (5) the direction of change, toward more or less stability, clarity, and directionality.

ancient Chinese) music, and currently appears mainly in gamelan ("orchestral") music in Indonesia, which is structured on layers governed by a cycle of $2^n$ beats (gongan) that recurs throughout the piece, such that the range of directionality is set mainly by the range of the cycle (which is generally 32 beats but can reach 256). The $2^n$ structure is rare in monophonic music (e.g., Arabic, Persian, and Indian), in which the ideal calls for focusing on the moment and avoiding complex, directional, overall organization; neither does it occur in African polyrhythmic music, which, on a certain level, can be viewed as consisting of $A, A, A, \ldots$.

3 In Arabic music, for example, the $\textit{ra`is}$ system has minimum asymmetry (here in units of quarter-tones): $\frac{4}{3}, \frac{3}{4}, \frac{1}{3}, \ldots$ and the $\textit{hejaz}$ system has no symmetry: $2, 6, 2, 4, 3, 4, \ldots$. These systems increase the possibility of complexity on the most immediate level and reduce the possibility of overall, complex, directional organization (in accordance with the stylistic ideal).

4 Researchers have looked at them from various perspectives: for tonal and atonal musical analysis (e.g., Reti 1951, Reale 1970, Cone 1987, Lewin 1993, Conner 1994), as a tool for musical composition, and even for illustrating symmetry (e.g., Solomon 1973 for atonal music, Wilson 1986 for tonal music).
(Berlyne 1974) and best known operation; (2) Shift operations that relate to cyclical patterns; (3) Expansion/contraction (or augmentation/diminution), which is accomplished either by inflating or contracting a complex event or scheme with respect to its various parameters or by making additions and deletions; (4) Unification/division (or fusion/segregation) into learned and natural units (inter alia, the 2ⁿ structure and the convex curve). This operation is essential for any perception and understanding (Bregman 1990); (5) Substitutability or equivalence, which is a prerequisite for every living language (Powers 1976).

4. AN EXAMPLE: Let us look at the operations that prevail in a simple Western folk song ("Lightly Row" [fig. 3]), the most obvious structure of which is A A' B A'.

(a) A B' A' B

(b) C

(c) A → b \(\text{(S_h)}\) \(\text{c} \rightarrow \text{d} \rightarrow \text{e} \rightarrow \text{f} \rightarrow \text{g}\)

(d) \(\text{Unification into harmonic scheme: I V I}

\begin{align*}
\text{measures 1,2 (A):} & \quad I V I I I I I \\
\text{measures 3,4 (A'):} & \quad I V I I I I I \\
\text{measures 5,6 (B):} & \quad V V I I I I \\
\text{measures 7,8 (A'):} & \quad I V I I I I \\
\end{align*}

\(\text{Unification into } 2^n: \quad 2 + 2 + 4 + 8 + 16\)

\(\text{(In units of number of beats)}\)

Figure 3: Various operations on units in a Western folk song: (a) The entire song (on the immediate level 0), with the large units denoted by capital letters (A, B), the smaller units by lower-case letters (a-g), and the harmonic degrees on the most immediate level; (b) The units on one level more abstract (level 1); (c) The operations, marked in parentheses above the operation arrow, in capital letters and with the number of the level (0 or 1): I = Inversion on level 1 (the category of contrast); Sh = Shift on the immediate level; A = Augmentation; D = Diminution; R = Retrograde (contrast); U = Unification of notes to form a chord scheme

REFERENCES: The list may be obtained from the author.

5 Some examples are "inversion," "retrograde," and many other transitions between members of pairs of simple and complementary contrasts, which are so numerous in music.

6 Some examples of shifts are "transposition" with respect to the system of 12; obtaining mode scales from the diatonic system; the "sequence" that relates to the scale degrees; and rhythmic patterns with respect to the metric cycle and even for the beat cycle.

7 For example, there may be equivalence between ways of accentuating a certain event in various parameters; between harmonic degrees; between "types" of tunes in a single mode; or between various realizations of a single scheme.