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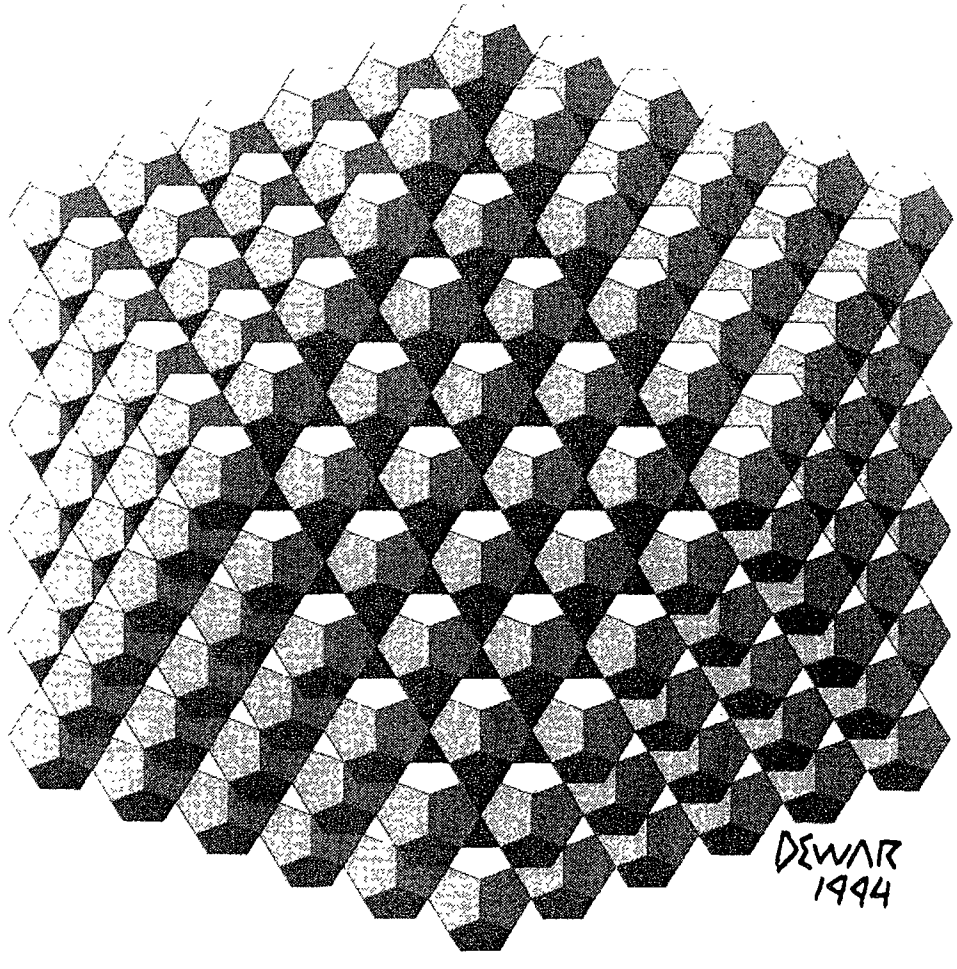

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SYMMETRY IN MATHEMATICAL THEORY OF CHESS

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Studying the geometry of chessboard, the criteria of valuation and movements of particular chessmans, combinatorial problems etc., the mathematical theory of chess strongly expresses the mathematical character of the game of chess. One of the especially important problems is the study of symmetry, even if it originates in the game, or if it appears in the chess problems or studies. The symmetry is unavoidable from the beginning position to the long pawn or other ending parts of the game, where the knowledge of the opposition, triangulation, the theory of correspondent squares, as the special forms of symmetry, it is necessary.

The most frequent forms of symmetry are the mirror and central symmetry. On the chessboard, the mirror axis usually coincides with the line dividing the chessboard into the left and right, or lower and upper halfboard. For the central symmetry, the center is the common vertex of the squares d4, d5, e4 and e5.

The first of all, the beginning position of the game is symmetrical. To this refers the question: what may happen, if the black strictly copies the moves of the white, wishing to preserve the symmetry of the chessboard. Interesting is the following example: 1.c4,c5 2.Qa4,Qa5 3.Qc6,Qc3 4.Q:c8 (checkmate). Because the black, playing in that way, can very quickly to loose a game, there is the chess-mathematical problem to construct such symmetrical games, in which the white checkmats the black in the minimal number of moves. Also, it is important by which chessman the checkmate is realized.

In the preceding example constructed by Samuel Loyd, the checkmate is realized by the queen in three moves In the symmetrical game, the checkmates by other chessmans in the minimal number of moves are realized by Russian chess-composers V.Pavlov and B.Sidorov:

1.Nf3,Nf6 2.Ng5,Ng4 3.N:h7,N:h2 4.Nf8,Nf1 5.Ng6,Ng3 6.R:h8
(checkmate by rook in 6 moves);

1.e4,e5 2.f4,f5 3.e:f,e:f 4.f6,f3 5.f:g,f:g 6.Be2,Be7 7.Bh5
(checkmate by bishop in 7 moves);

1.Nc3,Nc6 2.Ne4,Ne5 3.e3,e6 4.Ne2,Ne7 5.g3,g6 6.Nf6
(checkmate by knight in 6 moves;

1.g4,g5 2.h4,h5 3.Nf3,Nf6 4.Ne5,Ne4 5.h:g,h:g 6.g6,g3 7.g:f
(checkmate by pawn in 7 moves);

1.d3,d6 2.Kd2,Kd7 3.Kc3,Kc6 4.Kb3,Kb6 5.Ka3,Ka6 6.Be3,Be6 7.Bb6,Bb3 8.a:b,a:b 9.K:b4 (checkmate by king in 9 moves).

Despite that such examples may look somewhat funny, certain parts and even the complete symmetrical games may occur in serious tournaments as well. Let us mention the game Rothlevy-Elyashov, from the beginning of our century: 1.e4,e5 2.Nf3,Nf6 3.Nc3,Nc6 4.Bb5,Bb4 5.0-0,0-0 6.d3,d6 7.B:c6,B:c3 8.B:b7,B:b2 9.B:a8,B:a1 10.Bg5,Bg4 11.Q:a1,Q:a8 12.B:f6,B:f3 13.B:g7,B:g2 14.B:f8,B:f1 15.Q:f1,Q:f8 16.Qg2,Qg7. In this position players agree to draw result.

From this we can conclude that the black by symmetrical doubing the white's movements, may hope in the best situation to draw result. But, the next game shows that it even may win: 1.e4,e5 2.Ke2,Ke7 3.Ke3,Ke6 4.Qf3,Qf6 5.Ne2,Ne7 6.b3,b6 7.Ba3,Ba6. In this symmetrical position the white plays 8.Nd4, and the only possible answer of the black is e:d.

In all this symmetrical games, after the final chackmate move, the position left to be symmetrical. But, in the case of stalemate, are possible the games with all the moves symmetrical, where both of the kings are stalemated at the end, and the final position is symmetrical as well: 1.Nf3 Nc6 2.Nc3,Nf6 3.Nb5,Ng4 4.h3,a6 5.Na7,Nh2 6.N:h2,N:a7 (the first and the last one exchange of chessmans) 7.g4,b5 8.Bg2,Bb7 9.e4,d5 10.Ke2,Kd7 11.Qg1,Qb8 (all the moves are centrally-symmetrical, and at this moment is established also the centrally-symmetrical position) 12.b4,g5 13.Bb2,Bg7 14.Rf1,Rc8 15.Bd4,Be5 16.f3,c6 17.Bf2,Bc7 18.Be1,Bd8 19.Kf2,Kc7 20.a4,h5 21.a5,h4 22.c4,f5 23.c5,f4 24.e5,d4 25.e6,d3. Now, the both kings are stalemated.

Symmetry motifs are always very inspirative for the composers of chess-problems and studies. There frequently happens that a slight deviation from symmetry shows the way to the solution. This illustrates the following study by T.Dowson. If we exclude the unnecessary vertical a-row, the position (Fig.1) is completely symmetrical, and the white wins by his next move. The solution in the main variant is the following: 1.Nd4,Ke3 N:f3,K:f3 3.Kf8,d2 4.e8Q,e1Q 5.Qh5, and the white wins.

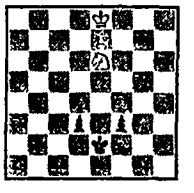


Figure 1

It is interesting that the symmetrical move 1.Nf4 gives the draw result, because of ... Ke3 2.N:d3,K:d3 3.Kd8,f2 4.e8Q,Kd2, and this is well known finish of the game: the queen against the f-pawn, giving the draw result.

Besides the symmetrical positions and symmetrical moves of the white and black, in the chess-compositions it is possible to find the symmetrical trajectories of certain chessmans. Wonderful example is the study of Paul Quindt, where the white plays for the draw result. The position in which the white is trying to get

saved is:

White: Ke4, Bd4, Bd5, Nd2, Ne2, Pb3, c5, f2, g3;
Black: Kh2, Bh1, Nd1, Na5, Nd8, Ne1, Ne8, Ng2, Nh5, Pc7, f7, g6.

The draw result can be obtained that way the white knights force the black king to the endless symmetrical circular movement: 1.Nf1,Kh3 2.Ng1,Kg4 3.Nh2,Kg5 4.Nh3,Kh6 5.Ng4,Kh7 6.Ng5,Kg8 7.Nh6,Kf8 8.Nh7,Ke7 9.Ng8,Kd7 10.Nf8,Kc8 11.Ne7,Kb8 12.Nd7,Ka7 13.Nc8,Ka6 14.Nb8,Kb5 15.Na7,Kb4 16.Na6,Ka3 17.Nb5,Ka2 18.Nb4,Kb1 19.Na3,Kc1 20.Na2,Kd2 21.Nb1,Ke2 22.Nc1,Kf1 23.Nd2,Kg1 24.Ne2,Kh2. By this, the magical circle is closed up, and the position is returned to the beginning. By two white knights is described the fantastic closed contour: h2-h3-g4-g5-h6-h7-g8-f8-e7-d7-c8-b8-a7-a6-b5-b4-a3-a2-b1-c1-d2-e2-f1-g1-h2, which is at the same time mirror- and centrally-symmetrical.

In some serious tournament games there are the smaller or bigger deviations from symmetry, but in the finish of the game the symmetry appears to be immensely important. The basis of the theory of all finishing parts are the pawn endings, where the main role play the notion of opposition, triangulation and the theory corresponding squares, which are the special forms of symmetry on the chessboard.

As the opposition, usually is considered the vertical, horizontal or diagonal facing of opposite kings, with one or more (obviously an odd number) empty squares between them. In this situation the player which moves first must leave the opposition, giving to its opponent the better possibilities for movements and for the occupation of important squares, which may decide the result of the game. In the next example:

White: Kc8, Pg2; Black: Ka7, Ph6;

the white wins using the opposition in the following way: 1.Kc8,Ka6 2.Kc6,Ka5 3.Kc5,Ka4 4.Kc4,Ka3 5.Kc3,Ka2 6.Kc2,Ka3. Till this moment, using the opposition, the white improves the position of its king with regard to the pawns, and now he is going in the decisive attack: 7.g3,Kb4 8.Kd3,Kc5 9.Ke4,Kd6 10.Kf5,Ke7 11.Kg6 and the white wins. From this example we can see that for the squares a7-a6-a5-a4-a3-a2 of the black king moves, the corresponding squares are c7-c6-c5-c4-c3-c2, respectively. Because that two series of squares are mirror-symmetrical with regard to vertical b-row, in the general case the opposition is the symmetry of two correspondent series of squares with regard to any vertical, horizontal or diagonal line.

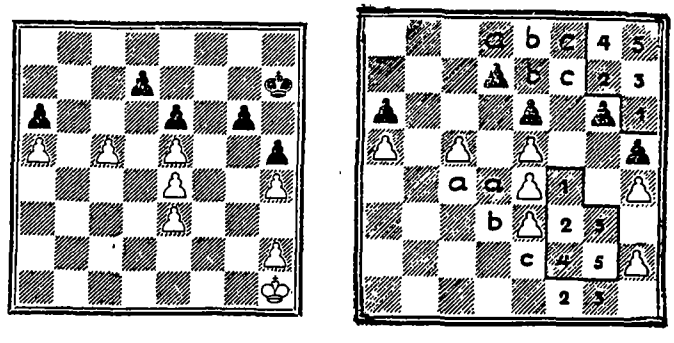
The notion of triangulation is closely connected with the opposition. The triangulation is a triangular maneuver by which the white, after three moves of the king, realizes the same position, but now is the black's order to to make a move. Therefore, the black is forced to drop the opposition and to leave the important squares for defence. Distinct from the opposition, which can be considered as the temporarily preservation of local symmetry, the triangulation is, in fact, their disturbance. In the position:

White: Ke2, Pb3, c4, f5, g3, g4; Black: Kd6, Pc5, e4, f6, g5, g7;

the move of the white king 1.Ke3 attacking the pawn e4 gives no result, because after 1....Ke5, the white must retreat back. Instead of the direct attack to the pawn, the white may before it make the triangular maneuver. It can use the triangle d2-e2-e3, and the black can use only the squares d6 and e5. Therefore, the white makes move 1.Kd2,Ke5 2.Ke3 and wins.

In the theory and practice of the pawn endings, the very important role plays the theory of correspondent squares, including as the particular cases the opposition and triangulation. There we must first to discern the key-square *K*, which is the center of the symmetry. Using it, the white king have to breach into the black's fortress. Also there is the set of the squares *B* for the white king, and the corresponding set *C* for the black king. As the weaker side, dependent from the moves of the white king, the black king always takes the appropriate, i.e. the correspondent square, hindering the breach of the white and mantaining some kind of opposition, which can be called the quasiopposition. From the other side, the determination of the correspondent squares of the both sides is a generalized symmetry which can be called quasisymmetry. The white may expect to win, by taking the quasiopposition. After that, moving on the corresponding squares in accordance with the moves of the black, he succeedes to conquest the key-square (the center of the quasisymmetry), from which can take the decisive breach.

In the position (Fig.2a) the white needs to find wining combination by the method of correspondent squares. First he must find the key-square g5 (the center of quasisymmetry), from which he can make the breach. After that, the sets of the corresponding squares *B* and *C* must be determined (Fig. 2b), by denoting the corresponding squares by 1,2,3,4,5 (or by a, b, c, if the breach is organized from the queen's side). Finaly, the white king uses the corresponding squares, going further untill he takes the key-square g5, when the victory is rendered. In the main variant, the way to the victory is the following: 1.Kg1,Kg7 2.Kf1,Kf7 3.Kg2!. Now the black cannot take the square h8, correspondent to the square h2, so he must play 3....Kg8 4.Kf2,Kf7 5.Kg3,Kg7 6.Kf3,Kf7 7.Kf4,Kg8 8.Kg5,Kg7 9.h3,Kf7 10.Kh6, and the white wins.



(a) (b)

Figure 2.