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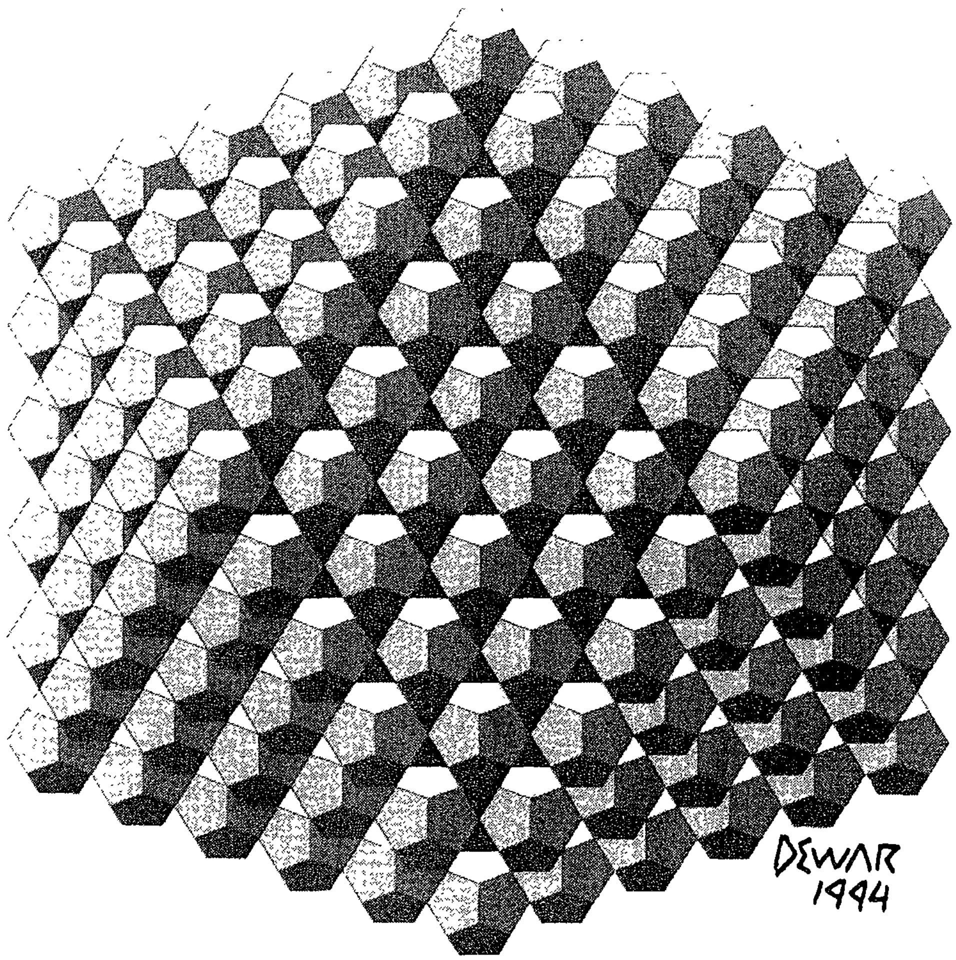

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Symmetry in the Biometric Description of Shape

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This presentation is not about symmetry of form but, instead, about symmetries of the tools for descriptions of form. This problem can be seen as arising out of the classic rules of perspective in art; early examples can be found in Albrecht Durer and in classic doctrines of caricature.

We connect this ancient concern to the themes of modern quantitative biology by exploiting the idea of landmark points as data. Landmarks are points that have names across a series of organisms: bridge of the nose, lip commissure, and tip of the chin, for instance, n a collection of human portraits in profile. It is often very useful to talk about biologically interesting phenomena (heartbeats, birth defects of the face, brain abnormalities in schizophrenia) in terms of how these patterns, causes or effects are manifest in changes of shape of a configuration of landmarks. For instance, a shape might change from circular to oblong, or one point might move on a background of all the others held fixed.

We would like our search through the space of descriptions like these to assess shape changes on a consistent scale of importance (magnitude) regardless of whether the change is, for instance, global or local. In ordinary geography, for example, if one is interested in talking about the length of a journey, the symmetry group one needs to know about is the rotations of the map plane, the passage from one definition of North to another. In perspective drawing, one needs to know how to preserve the symmetries of projective space, passing from one vanishing line to another.

Some new methods have just begun appearing in statistics that carry out programs in this same spirit for the more complicated question of relationships among many different points of many different figures (not just two points, as in journeys, and not just different views of the same object, as in perspective drawing).

My topic is the role of symmetry in the formulation and justification of these rules for landmark data. Some of the symmetries of the tools are obvious--for instance, every left-right descriptor has to be paired with an up-down descriptor. Some are a little less obvious: every description of how a square changes into a rectangle, for instance, is symmetric with a description of how it could change into a rhombus. Together these give us the plane of transforms that take squares into parallelograms. And others are quite a bit deeper yet: the full plane of those square-to-rhombus and square-to-rectangle descriptors, for example, is symmetric with a wholly analogous plane of how squares change into kite shapes, changes for which the diagonals of the square change their relative positions without extension or rotation.

In a variety of geometric diagrams and pictures of real organismal variation, I will try to lead the convention-goers to an intuitive appreciation of the way in which the notions of symmetry apply to these tools of shape description, including the description of symmetry itself. I will particularly emphasize the need for symmetry among analyses at all geometric scales, a symmetry that accounts for much of the intellectual power in taxonomy and evolutionary biology.