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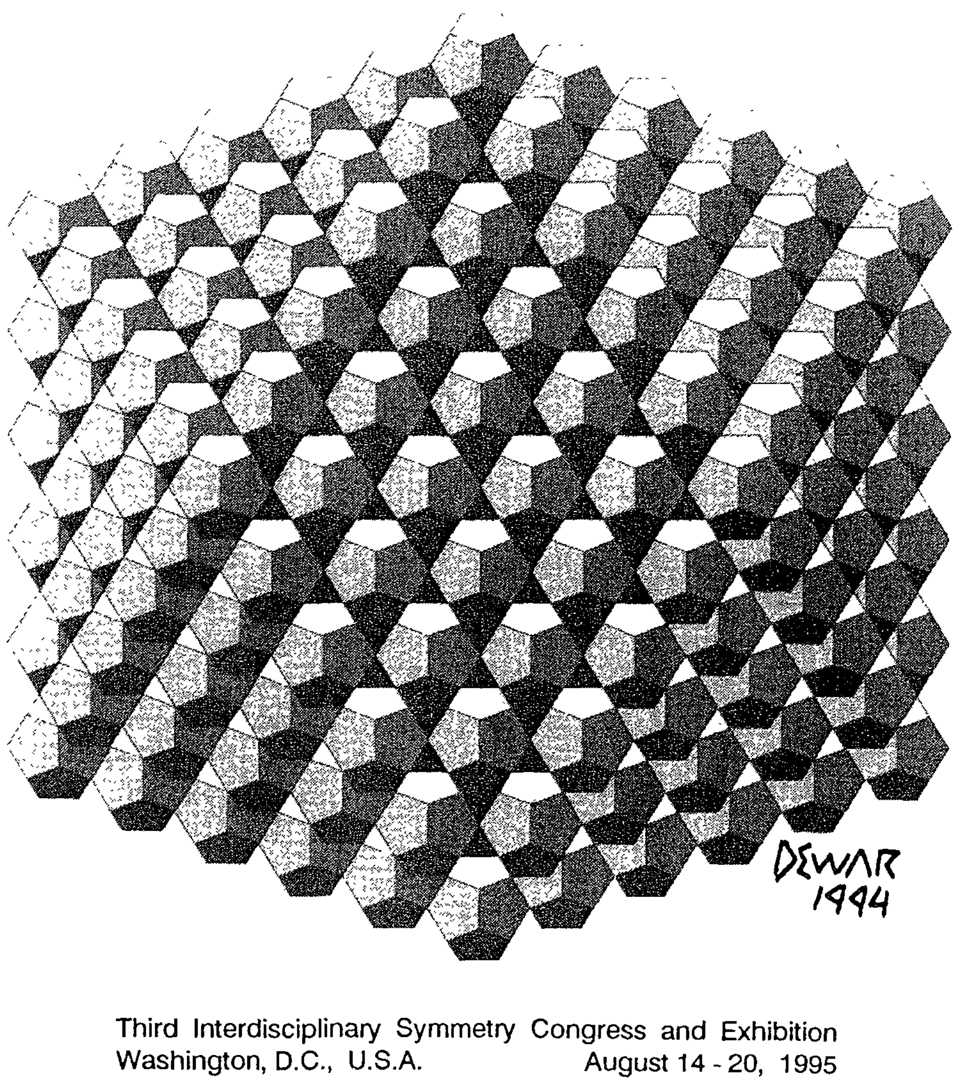

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SYMMETRIC AND NON-SYMMETRIC PROJECTIONS OF 4-SPACE OBJECTS

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Passport photographs and identification pictures usually are restricted to front views and profiles, but portrait artists more frequently depict less symmetrical but more appealing intermediate poses. Traditional front, top, and side views are still important for architects and machine designers, but artists generally prefer to render more interesting "general" views of buildings or other other objects. The same sorts of decisions are made by mathematicians and scientists using visualization techniques in their research and teaching.

For two-dimensional geometry, it is often possible to show an entire configuration at once, although the placement of the various elements of a diagram still challenges the artistic sense of the geometer. Rendering two-dimensional views of objects in the third dimension provides much greater challenges, and mathematicians have learned much from the experience of artists in the use of perspective, shadows, shading, and texture to make their pictures of abstract polyhedra and surfaces look like photographs of real objects. Frequently however designers have employed conventions that represent their objects in non-photographic terms, in stylistic projections that look like no photograph of an actual form. If one face of a cube looks perfectly square to us, we will not see any of the other faces! Yet most mathematics texts persist in drawing one face of a cube as a square and then translating that square in some oblique direction to form another square, with corresponding corners connected. Such a representation is useful for some mathematical purposes, to show the number of edges for example, but it presents a distorted picture of the cube and of anything inside the cube.

All this changes when geometry is approached through interactive computer graphics. If we give the instructions for drawing a cube, then the computer can show us what the cube would look like if we project it into the plane, either orthographically or in perspective. We can then build our other objects inside the picture of the cube, to get undistorted views of function graphs or more complicated three-dimensional mathematical objects.

Note that frequently the general views are less symmetric than various special views. The true appreciation of an object in three-space is probably best given by a "tour" that presents a series of views, including the particular projections that show the object at its most symmetrical. We get a much better concept of a saddle shape by looking at the general view rather than the projections into the three coordinate planes.

But what happens when we want to view objects in four-dimensional space? In order to gain a visual appreciation for phenomena that occur beyond our third dimension, we want to be able to relate the images we see to representations that we already understand. Unfortunately most artists are not directly helpful here, although by analogy we can imagine how they would handle some of the challenges of choosing the best views of basic four-dimensional objects. Several mathematicians have collaborated with artists to try to develop new ways of looking at phenomena in higher dimensions, extending traditional modes of design. David Brisson was one of the true visionaries in this effort, and there are several others who have made contributions.

It is with the advent of modern computer graphics that we can really begin to address the problem in totally new ways. Since the middle 1960's, researchers have been investigating techniques for rendering images of "benchmark" figures. In this presentation we will survey the progress made over the past twenty-five years, culminating in the current surge of activity in the Internet and the world wide web. How will this new technology change the way we view objects and the ways we communicate our insights?

What ways will the interaction between mathematicians and artists continue to enrich the experience of both groups?

The topics we choose to explore, in slides, videotapes, and interactive demonstrations, include some of the most symmetric objects in higher space: the four-dimensional hypercube, the flat torus in the three-sphere in four-space, the Veronese surface in a small four-sphere in six-space, the graphs in four-space of functions of a complex variable, and the Klein bottle in four-space.