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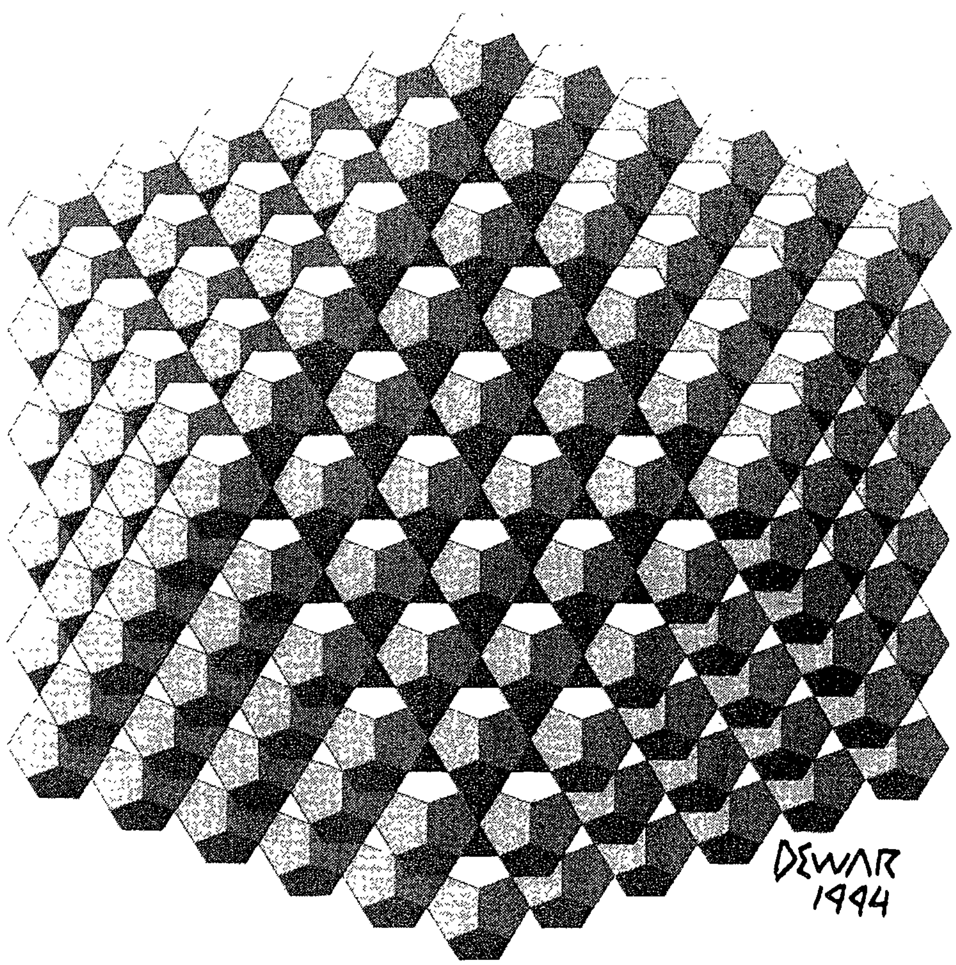

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RIDDLING: A PHENOMENON IN DYNAMICAL SYSTEMS

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Riddled and intermingled basins of attraction are a relatively recently noticed phenomenon in dynamical systems and differential equations. Although not necessary for its occurrence, riddling seems to occur most naturally in systems with some kind of (discrete) symmetry. A chaotic system exhibits sensitive dependence to initial conditions; however, usually the long-term average behavior is robust, except for initial conditions near basin boundaries. In a riddled system, the long-term average behavior is also infinitely sensitive to initial conditions. The name comes from the fact that the basin of an attractor is infinitely riddled with points which do not tend to the attractor. Riddled basins have no open sets–every point is a basin boundary. Intermingled basins are basins for different attractors which are dense in each other. Over the past several years, riddling has been observed in several scientific contexts: visually (originally), mathematically (rigorously), numerically (simulations), and experimentally (bench experiments). There are philosophical implications for replication of phenomena, which have been discussed in the popular scientific press under titles such as "Beyond Chaos." This talk is a survey of these developments.

A NEW EXAMPLE OF A FLEXIBLE POLYHEDRON

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1. Introduction. A surface in the 3-dimensional Euclidean space constituted by a finite set of polygons is said to be a *polyhedron*. The polygons are referred to as *faces* of the polyhedron and the sides of the polygons are referred to as its *edges* . We suppose that exactly two faces are adjacent to every edge.

Shape and size of the faces will be considered to be unchangeable, i. e. the faces will be considered as made from solid plates. On the contrary, suppose we can vary dihedral angles of our polyhedron. We'll call our polyhedron a *flexible* one, if it is possible to change dihedral angles continuously in such a way as to change the spatial shape of the polyhedron.

2. Survey of evolution of theory of flexible polyhedrons. One can formulate the Definition 10 from Book XI of Euclid's Elements [9] as follows: "Equal and similar solid figures are those contained by similar planes equal in multitude and magnitude". Some authors use this fact to prove that Euclid have passed through the notion of flexible polyhedron.

The first rigorous result on flexible polyhedrons was obtained by A. L. Cauchy in 1813. In particular he has proved that each convex polyhedron is not a flexible one [6]. Answering the question whether non convex polyhedron can be flexible, in 1897 R. Bricard have constructed examples of flexible octahedrons (i. e. polyhedrons with 6 vertexes, 12 edges and 8 faces) [5]. All of them have points of self-intersection. The problem on existence of a flexible polyhedron without self-intersection remains open for a long time even though it was interesting for such outstanding mathematicians as Henri Lebesgue [12] and

A. D. Aleksandrov [1]. Many mathematicians were sure that it has the negative answer. Nevertheless in 1976 R. Connelly [7], [8] have obtained the positive answer. Soon K. Steffen has simplified Connelly's example and constructed a flexible polyhedron without self-intersection with only 9 vertexes (only one more then the cube has) (see [3]). In 1994 I. Maksimov has announced that the number 9 can not be replaced by any smaller one [13].

3. Applications of flexible polyhedrons. In the paper [9] R. Connelly discusses applications of flexible polyhedrons to building mechanics. It is based on the intuitively clear reason that each construction made from prefabricated ferro-concrète items can be regarded as polyhedron with rigid items-faces and changeable dihedral angles at joints-edges.

In the articles [4] and [10] applications of flexible octahedrons to stereo chemistry are discussed. The idea is that the carbon skeleton of the cyclohexane molecule may be represented by a spatial hexagon with prescribed sides and angles. Replacing each pair of sides with common vertex by the rigid triangle with the same vertexes we obtain an octahedron. Thus the problem weather the spatial hexagon is rigid or flexible is equivalent to the problem weather the octahedron is flexible.

4. Open problems. By far the main goal is to obtain a criterion for flexibility of polyhedrons, i. e. to obtain a rule which will allows us to conclude after some finite set of operations involving finite number of sizes of our polyhedron whether it is flexible or not. As there are no direct approaches to this problem, we'll discuss the following partial problems:

- Does there exist flexible polyhedron in many dimensional space?
- Is the set of flexible polyhedrons semialgebraic one in the space of all polyhedrons of a prescribed combinatorial type, i. e. is it defined by a finite set of polynomial

equations and inequalities?

- Is it true that each flexible polyhedron preserves volume during the process of bending [9]? (The positive answer for a class of combinatorial one-parameter flexible polyhedrons was announced by I. Kh. Sabitov [14] in 1994. A flexible polyhedron is said to be combinatorial one-parameter if it fails to be flexible after we fix spatial distance between two its vertexes that were not joined by an edge.)
- Which functions except volume can be preserved by all flexible polyhedrons? Can the mean curvature play this role [2]?

Obviously, studying these problems it is useful to have examples of flexible polyhedrons. Connelly's and Steffen's polyhedrons are very elegant, but they are based on the Bricard's octahedrons. For better understanding of the problems it is desirable to have examples based on other ideas.

5. Formulation of the result. In the present report we'll explain a new example of a flexible polyhedron (with self-intersection), that is a piecewise linear realization (but not an immersion) of torus. The Bricard's octahedrons are not used in the construction. Flexibility of the polyhedron is deduced from the purely analytical reason — from the fact that every rational function can be expanded into a sum of proper fractions. We shall verify that, under some relations between parameters of the construction, the polyhedron is flexible. It turns out that precisely with these values of parameters our polyhedron preserves volume and mean curvature during a bending. For more details see [2].

References

[1]. A. D. Aleksandrov and S. M. Vladimirova. Bending of polyhedron with rigid faces (in Russian). Vestn. Leningr. Gos. Unive., Mat. Mekh. Astron., **3**, no. 13 (1962),

138–141.

[2]. V. A. Alexandrov. A new example of a flexible polyhedron. To appear in the Siberian Mathematical Journal in 1995.

[3]. M. Berger. Géométrie. Tome II. Paris, CEDEC/Nathan. 1978.

[4]. O. Bottema. Flexible hexagons. Proc. Kon. Ned. Ak. v. Wet., **70** (1967)=Ind. Math. **29** (1967), 151–155.

[5]. R. Bricard. Mémoire sur la théorie de l'octaèdre articulé. J. Math. Pures Appl. Serie 5, Tome III (1897), 113–148.

[6]. A. L. Cauchy. Mémoire sur les polygones et let polyèdres. J. Ecole Polytechnique, 16e Cah., Tome IX (1813), 87–98.

[7]. R. Connelly. An immersed polyhedral surface which flexes. Indiana University Math. J., **25** (1976), 965–972.

[8]. R. Connelly. A counterexample to the rigidity conjecture for polyhedra. Publ. Math. IHES, **47** (1978), 333–338.

[9]. R. Connelly. Conjectures and open questions in rigidity, preprint. Cornell University. 1979.

[10]. Euclid's elements. Books XI–XV (Russian translation from Greek). Moscow — Leningrad, GITTL. 1950.

[11]. H. A. Lauwerier. A note on flexible hexagones. Proc. Kon. Ned. Ak. v. Wet., **69** (1966)=Ind. Math. **28** (1966), 330–334.

[12]. H. Lebesgue. Octaèdres Articulés de Bricard. Enseign. Math. **13**, no. 2 (1967), 175–185.

[13]. I. G. Maksimov. Immersed flexible polyhedrons: minimal number of vertexes and volumes (in Russian). Abstracts of reports at the All-Russia colloquium on stochastic methods in geometry and analysis. Abrau-Durso, 25 September — 2 October 1994. Moscow, TVP. 1994.

[14]. I. Kh. Sabitov. Algebraic methods in the theory of immersions and flexible polyhedrons (in Russian). Ibid.

INFORMATION AND SYMMETRY IN THE CELLULAR SYSTEM

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1. INTRODUCTION

Very few approaches in theoretical biology have dealt with the cellular system in its entire functioning [1]. In this work we try to develop the concepts of Information and Symmetry within the framework of theoretical physics in order to be applied to the whole dynamics of the living cell. We make a closer examination of the selective action mechanisms of enzymes and enzyme networks ("the society of enzymes") and we study how the concept of symmetry for a stochastic dynamical system [2] can help us "to map" the information dynamics of the cell in relation to its environment. For this purpose, we work out a "symmetry transformation" and ponder the possibility of "conserved quantities" in the cellular system.

We also explore the interdisciplinary peculiarities this model-system shows within the framework of both Physics and Biology, and how the emergent "Information Science" can throw new light on the study of living beings (bioinformation).

2. THE CELL AS A "SOCIETY OF ENZYMES"

Certainly, the cellular system is one of the most complex entities --as we will discuss, it is a genuine "society". Not only because of the heterogeneity of its components, the complicated arrangements of interacting macromolecules in its subsystems, and the host of physico-chemical processes involved, but on other hand, because it is a system not quite amenable to experimental testing in its "in vivo" state [3]. Hence, simplified approaches seem necessary, in both methodological and formal aspects.

Biochemists and enzymologists have been taken as a matter of course that most metabolic activity of the cell results from the *superposition of action* of individual enzymes [4] dissolved in an aqueous phase, the dynamics being governed by simple mass-action laws and random thermal motion of metabolite molecules in weak-electrolyte solution. The cell (and each of the organelles therein) basically becomes a "bag" of enzymes and metabolites operating in homogeneous solution. The unitary space-time [4] events of the material transformations in such cell metabolism can be associated with localized enzyme-proteins.

In general, the action of enzymes entails two discrete stages: binding/recognition and chemical catalysis. The substrate molecules must diffuse to the enzyme "active center", where they bind selectively. The following step is the electronic transformation of the substrate, taking into account the impact of effectors (activators and inhibitors) at the binding sites and the influence of other thermodynamic variables (temperature, pH, etc.).

The above simplified approach allows us to see the cell as a "society of enzymes" [5] which exchange material and information among themselves and with the environment. It conduces us towards population-dynamics models not far away from the classical Lotka-Volterra approach. We can model the enzyme "population" as a stochastic dynamic system whose behaviour is described by trajectories within a state space.

The point of each trajectory at time t is determined by the n-dimensional vector $x(q,t)$. We assume that the process $y_t=x(q,t)$ obeys a stochastic differential equation of Stratonovich type.

$$dy_t = \mathbf{B}(y_t,t)dt + \sum_{r=1}^{m} \mathbf{A}_r(y_t,t).dw_t^r \qquad (1)$$

Each component of this vector stands for the value of a population (occupation) number, which has a certain measurable property q_t that can change in time. The state of this population is therefore an element of a certain vector space. In the course of the dynamics of the system, the state vector should not leave this space.

$\mathbf{B}(y_t,t)$ and $\mathbf{A}_r(y_t,t)$ are non-linear functions of y_t; dw_t^r is an m-dimensional stochastic Wiener process. In (1) $\mathbf{B}(y_t,t)$ corresponds to the deterministic part of the process and $\mathbf{A}_r(y_t,t)$ to fluctuating forces which depend themselves of the function y_t; the latter can be interpreted as a "community matrix" too, its anti symmetric part corresponding to the web of effectors (activators and inhibitors) and substrate-product transformation (e.g. the transfer of free energy through the "society") while its symmetric part reflects a collective dissipation.

Within this framework, the whole cellular dynamics can be understood as changing the occupation number, e.g. entering (or acting) new points of q-space and leaving old ones by means of the action of specific enzymes and network of enzymes.

3. SYMMETRY, INFORMATION AND COMPLEXITY

Let us now attempt an interpretation of the q properties of this enzyme population. The exchange of information between the different enzymatic functions occurs through the sharing and networking of substrates, products, activators, and inhibitors. This complicated network of exchanges allows the emergence of an overall functionality for the whole population of enzymes which can be extremely rich. (Here, we will not enter in the complementary dynamics of protein synthesis, converter enzymes of the signaling system, and the protein degradation phenomenon.)

We can now formally discuss how every individual enzyme receives and sends [6] a message. We can clearly distinguish between the reception of information through the effectors and substrate, and the carrier of information, the product. This kind of "gradient field" inside the cell can be "measured" by the individuals of the enzyme population. Then, we take the q-space of the properties of enzymes, as the pattern recognition capacity of this population which has its dynamics described by the equation (1). Therefore, we can set the q's as a state vector,

$$q(t) = [q_1(t), q_2(t), \ldots, q_n(t)] \quad (2),$$

as for the stochastic differential equations, we can write them in the form

$$dy_t(\mathbf{q},t) = [\,\partial_t + Q_0\,]y_t(\mathbf{q},t) + \sum_{r=1}^{m} Q_r y_t(\mathbf{q},t).dw_t^r \quad (3),$$

where: $\partial_t = \dfrac{\partial}{\partial t}$, $Q_0 = \sum_{i=1}^{n} B^i \dfrac{\partial}{\partial t}$, $Q_r = \sum_{i=1}^{n} A_r^i \dfrac{\partial}{\partial q_i}$ (4)

are differential operators defined by (4).

We can see the equation (3) as a *symmetry transformation* for the process defined by the temporal change of a point in the q-space, consequently the dynamics of $q(t)$ must obey,

$$d\mathbf{q}(t) = \mathbf{B}(\mathbf{q}(t),t)dt + \sum_{r=1}^{m} \mathbf{A}_r(\mathbf{q}(t),t).dw_t^r \quad (5),$$

with; $q(t_0)=c(\alpha)$, $t\in[t_0]$, where α is a order parameter.

It can be demonstrated [2] that the condition for (3) being a symmetry transformation of (5) is that the function satisfies

$$\mathbf{B}(y_t(\mathbf{q},t),t) = [\,\partial_t + Q_0\,]y_t(\mathbf{q},t) \quad \text{and}$$
$$\mathbf{A}_r(y_t(\mathbf{q},t),t) = Q_r y_t(\mathbf{q},t) \qquad (6)$$

In this context we can consider the y_t as a "conserved quantities" for any system having its dynamics described by (5). In a similar fashion, this means (in our interpretation) that if the receiver properties of the enzyme can be described as a stochastic dynamical system (5) we can make a transformation to a population space the "form" of which (5) is invariant (under this transformation).

4. CONCLUDING REMARKS

The above mathematical description just maps an abstracted aspect of the "territory" of biomolecular information processing and control [7]. By following this approach, we can further formulate the notion of symmetry operator [2] which yields a more general conserved quantity for the system, which in its turn can help us to find other possible conserved quantities. The explicit form of the process $y_t=x(q,t)$, as well as its component functions $B(y_t,t)$ and $A_r(y_t,t)$, will be the subject of future enquires.

The use of theories based on the intrinsic *symmetries* in the space-time structure of the natural world has been a fruitful strategy of physics in its attempt to unify its various levels of complexity. Among the symmetry principles of physics, the most abstract one is the set of "gauge symmetries". In this respect we can speculate how "form" (structure) and function are invariants under a gauge transformation (e.g., specific recognition).

It is a reasonable approximation to consider the populational species as being in quasi-stable equilibrium of growth with respect to its environment. The "chemical fields" (effector and substrate web), which are invariant to the gauge transformation, compensate for any local change of any "enzymatic field" within the populational number, and preserve (save) the global invariance. A local input in this system is a factor for a change in "local condensation" - nucleation. Through this nucleation process, the symmetry of the system suddenly and spontaneously lowers, and it does so in a non-predicable, random event. The symmetry of the system is "broken". In this context the enzyme populational action emerges as a "thermodynamic coordinate"--because of a fundamental symmetry principle grounded in the concept of broken symmetry.

5. REFERENCES

[1] ROSEN, R. (1991) *Life itself,* Columbia University Press, N.Y.

[2] MISAWA, T. (1994). *New conserved quantities derived from symmetry for sthochastic dynamical system.* J. Phys. A, **27**, L777-L782.

[3] LUZZI, R. AND VASCONCELLOS A. R. (1991). *Statistical mechanics of dissipation and order: an overview.*Ciencia e Cultura, nov./dec., **43** (6), 423-431.

[4] WELCH, G. R. (1992). An analogical "field" construct in cellular biophysics: History and present status. *Prog. Biophys. molec. Bio.,* **57**, 71-128.

[5] MARIJUÁN, P. C. (1994). *Enzymes, automata and artificial cells.* In: Computing with Biological Metaphors. R. C. Paton (ed.). Chapman & Hall. London.

[6] HAKEN, H. (1988). *Information and self-organization,* Springer-Verlag. Berlin

[7] CONRAD, M. (1992). *The seed germination model of enzyme catalysis,* BioSystems, **27**, 223-233.

FUNCTIONAL BRAIN ASYMMETRY IN PAINTING CREATIVITY AND RHYTHMS IN PAINTING EVOLUTION

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In modern psychology of creation is conceived a new scientific trend - a study of creative processes and their results in the light of the functional assymetry of human cerebrum hemispheres. A detailed study of functional pecularities of each hemisphere gives evidence that a prevalence in psychological activity of a left hemisphere is characterized by the features of analytics, rationality, verbalization, whereas at prevalence of right-hemispherical processes - a creative activity is characterized by intuitiveness, figurativeness, synthetical character [4]. Thus the motives are provided for separation of the two polar types of creative process, which one can conditionally call the left-hemispheric (L-type) and the right-hemispheric (R-type). Certainly, the relative prevalence of one of these characteristics is apparent, but not the absolute domination of one process by another.

A predominance phenomenon of one from two (L- or R-) types was examined upon material of creation in such different spheres as music [21], architecture [4], and others [5]. In recent investigations [1], characteristics have been found in artists' work that can be interpreted as indicators of the domination (in the creativity of these artists) of one of two components: drawing style or colour. From the neurophisiological point of view, these two styles can be attributed, respectively, to L-type (analytic) or to R-type (synthetic) domination of brain activity.

Ten parameters of painting were chosen (throgh several methods, including factor analysis) to describe the domination of the processes discussed above:

DRAWING STYLE PARAMETERS	COLOR STYLE PARAMETERS
1.Inclination toward normativity	Inclination toward originality
2.Rationality	Intuition
3.Strict form	Free form
4.Limited express means	Diversity of express means
5.Dominance of grafic features	Pictureusqueness-dominance of coloristic features
6.Steadiness, static features	Expressive, dynamic features
7.Discrete elements	Continuious transitions between elements
8.Inclination toward cool part of spectrum	Inclination toward warm part of spectrum
9.No gradation within each color element	Much color gradation
10. Smooth painting	Texture painting

Domination of color style under drawing style means that the number of features referring to drawing style (n_l) more then the number of parameters, referring to color style (n_r).

In our investigations with Vladimir Petrov, 240 artists (both Western European and Russian) from the fifteenth to the twentieth centuries were studied. Each artist may be characterized by "index of asymmetry":

$$K=\frac{n_r-n_l}{n_r+n_l}$$

This index changes from +1 ("pure" drawing style) to -1 ("pure" color style).

Twenty-two experts defined the stylistic features of the artists thrugh the use of these 10 paremeter.

These date were used in a large-scale investigation concerning evolution of Western-European and Russian scools of painting during 1430 - 1930 years from the point of view of two types of mentality. In order to facilitate the calculations, a database (DBase 111+) of the 240 artists and the average values for the parameters was created with the step t=3 yers.The resulting curve for "index of asymmetry" is shown on Figure 1.

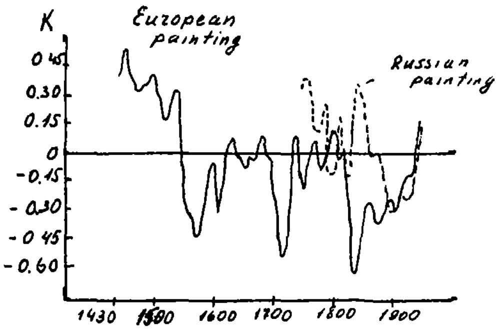

Fig.1 Cyclicity in painting evolution

Cyclic processes are typical in the evolution of art. The similar investigations was done for the evolution of music [2] and cyclicity was also obtained.

Mathematical description of this phenomenon can be based on the modification of equations for competitive interaction (Lottka-Volterra equations) as has been accomplished for the study of the interaction of three spheres of motion-picture culture: cinema, television and video [3], where cycles also take place.

References

1.Golitzin G.A., Georgiev M.N., and Petrov V.M. (1989) "Indicies of Hemispherical Asymmetry of Creativity of Painting" (in Russian). *Voprocy Psikhologii*. No.5, pp.148--154.

2.Danilova O.N. and Petrov V.M.. "Periodical Regularities in Musical Creativity". (1988) (in Russian). *Priroda*. No 10, pp.54--59

3.Ivanizky G.R. Rhythms in the Development of Complex Systems (1988)(in Russian). Moscow, Znanie.

4.S.Yu.Maslov. Asymmetry of Cognitive Processes. *Semiotica and Informatica*.(1983), (in Russian), No20, Moscow, Radio.

5.Petrov V.M. and Yevin I.A.. Emotional Unity of Informational Sphere: Sociological "Climate", Art and Economics. (1991). In: *Art and Emotions*. Editors: L.Ya.Dorfman, D.A.Leontiev. Perm, pp.160--169