Symmetry: Culture and Science

The Miura-ori opened out like a fan
The Japanese origami is well-known in the West, but most people consider it as just play for children with paper. Incidentally, there is one particular field where paper-folding has a great tradition in the West: it is the folding of table napkins, which topic has its literature and some authors call it an art (see, e.g., Josephine Ive's booklet *Table Napkin Folding: An Elegant Art*, Winchester, 1983; Melbourne, 1989).

Origami is, however, much more than just play for children, although the latter is also an important aspect. Some dictionaries translate the word as the art of paper-folding. This interpretation is much better, but still narrow: origami has many scientific aspects and technical applications, too. It has its own culture with a very interesting history. Let us make a brief journey into the world of origami.

1 ORIGAMI-HISTORY

The Japanese word *origami* is written with two Chinese characters (kanji) and usually there is a decensional syllabogram (kana) between them that belongs to the first one. Note that the characters adapted from the Chinese have mostly two (or two sets) of readings in Japanese: a Chinese-related one and a native Japanese one. In the case of the two characters of origami, both of them should be read in the Japanese way. Thus the expression *origami* is not an adaptation from the Chinese language, but an independently developed Japanese concept. On the other hand paper itself and some sort of paper-folding art was brought to Japan from China. Let us see very briefly the origins of both characters in the word *origami*:

折り紙
the first character should be read as or - it represents a cut tree and an axe, but later this original meaning was extended to the idea of breaking and folding;
- this is followed by a declensional syllabogram ri - it completes the reading of the first character ori;
- the second character is kami (which becomes gami) - the left-hand side represents a thread, while the other part refers to thin and flat objects, thus the entire character means paper.
It is interesting to see that the idea of cut is not so far from origami as is believed. Later we will return to this question.

Of course paper-folding was invented not only in China and Japan, but in almost all cultures where paper was in use. The Japanese, however, went further than other nations and developed it into a complex form of art and science. The history of this process includes many spiritual aspects. Indeed, in ancient Japan paper was very expensive, therefore it was used for ceremonies and rituals, not for play. In the native Japanese religion Shinto origami has a special importance. Moreover the oldest known origami-related activity is the katashiro, a symbolic representation of a deity made of paper. Folded paper is also used in case of various Shinto purification ceremonies. There is a special shrine paper for such purposes (jingga yoshi).

The origami step by step reached other fields of culture, from the tricks of magicians to the everyday life. The kimono dressing has many analogies with origami, because the textile should be folded in a special way. There is, however, a more direct connection: the satai, a folded paper case attached to the kimono. Noh theater costumes also frequently use origami attachments. Another field where the origami has a special importance is the etiquette of giving gifts. For example, sake bottles should be decorated by male and female butterflies. We may see in this practice an interesting connection with observing nature, an attitude which is much stronger in Japan than in the West. Indeed, creating animal-shapes by origami has a great tradition in general. To make a distinction between male and female butterflies reflects, however, a further idea of traditional Japanese thinking: the philosophy of yin-yang based on male and female principles. Note that the circular yin-yang symbol has a symmetric structure, based on 2-fold rotation antisymmetry (black-and-white symmetry), which represents the unity of the two opposite forces. Thus, a good drink, such as sake, should also satisfy this 'symmetry'. Returning to origami animals, we may mention another interesting aspect. A famous piece of art in this field is the frog that can jump if we press it on a spot at the backside. Thus, not only the similarity of form is important, but there is an attempt to show some functional similarity, too. (The jumping frog is very popular and it is included in many books on the field.) Origami has a connection even with love! In old times the love letters were specially folded making a regular pentagon by tying a knot on a strip of paper and tightening it flat. Note that the pentagon also plays an important role in traditional thinking: the system of five basic elements - earth, water, fire, metal, and wood - and their connections are represented by a regular pentagon and an
inscribed pentagram (five pointed star). Apropos, letters: the Japanese aerograms also refer to origami. These sheets, widely available in post offices, show carefully where we should make the first fold and the second one. Until very recently the aerogram sheet was decorated with two birds: one on the printed stamp and another one symbolizing the airmail with an origami crane. (Very recently, when the aerogram fee was changed, a new type was released without this origami bird).

Finally, let us discuss the question of animals versus technical objects in origami. There is no doubt that Japanese origami has more emphasize on animals, while the typical Western paper-folding for children has more interest in technical objects, such as flying machines and ships. More recently, however, these two sides influenced each other: Japanese have more interest in technical objects and the Westerners make more animals. Just one example: a group of Japanese origamists constructed a very large set of American aircraft.

We do not discuss here the detailed history of origami, but just return to the question of using cuts. As we mentioned earlier, the first character in the Japanese word origami clearly refers to cut, thus this operation was not excluded in the beginning. Indeed, origami representing the deity in Shinto ceremonies is made partly with cuts. Origami, however, also became an entertainment probably in the 9th-11th centuries, and gradually various schools and styles appeared. Origami without cuts was first developed in the Muromachi-period (1333-1568). Interested readers who would like to see a brief, but very informative survey on origami and its major types, may refer to Akira Yoshizawa's article “Origami” in the Kodansha Encyclopedia of Japan (Vol. 6, Tokyo, 1983), which is available in the major libraries in the West.

Let us see the beginnings of the Western interest in origami. The new edition of the Oxford English Dictionary (Vol. 10, Oxford, 1989, p. 933) gives credit to Robert Harbin's book Paper Magic: The Art of Paper Folding (London, 1956, p. 14) as containing the first usage of the word in an English context. There is also a reference to a 1922 article by F. Starr, published in San Francisco, where a Japanese book on origami is cited. We should note, however, that origami did not reach the Western world via the English-speaking countries. Probably Spain was the first Western country where origami was practised. Some origami-related materials reached them, supposedly via the Moors, in the 8th century. It was not definitely Japanese origami, but some paper-folding arts that originated in Asian countries, including Japan. The Spanish philosopher, writer, and poet Miguel de Unamuno y Jugo (1864-1936), or, as the children called him, “Don Miguel”, significantly contributed to the spread of paper-folding in art education. The birds and other animals made by him were so realistic, that, according to a famous story, some children believed that they could ‘speak’. Paper-folding, following a Spanish influence, also became popular in Southern America. The list of famous paper-folders includes, among others, Leonardo da Vinci, the poet Shelley, the mathematician and writer Lewis Carroll. Their activities were, however, more or less independent from the
Japanese traditions, and they did not generate public interest. Paper-folding remained in the Western world play for children and for a few interested adults until the beginning of the stronger Japanese influence.

Probably Japanese magicians represented the first wave of introduction of Japanese origami to the Western world. Their quickly made paper objects and the transformations of these surprised the audience. Incidentally, the famous magician Houdini (1874-1926) also had an interest in paper-folding, and the previously cited Robert Harbin, the author of the book of 1956 where the word origami appears in English context, is also a noted magician. Another important wave of Japanese influence is due to public exhibitions. Akira Yoshizawa (b. 1911) made in 1955 an exhibition of origami art, including both figurative and abstract works, at the City Museum in Amsterdam. It was followed by a group exhibition in 1959 at the Cooper Union Museum in New York. The broad interest of the American people in origami lead to the establishment of the Origami Center of America in New York by Lillian Oppenheimer in the same year of 1959. In some sense Yoshizawa is the ‘Kodály’ or ‘Laban’ of origami: while Kodály developed the relative solmization (or solfaing) system in music education and Laban worked out the symbols for dance notation, both spreading very quickly internationally, Yoshizawa developed the symbols to describe folding processes. Thus the origamists also have an ‘international language’ for communication.

2 SYMMETRY AND ORIGAMI IN ART, SCIENCE, AND TECHNOLOGY

The expression in the title of this article “symmetry-origami” is not only our play on words, but the conjunction of these two expressions makes some sense: symmetry + paper-folding. Indeed, the process of making origami objects often includes foldings according to symmetry axes. In addition to this, origami is very useful in making symmetric objects, including periodic patterns. Note that the term “symmetry” was adopted into Japanese as shinmetry. Thus the above play on words is understandable even in Japanese, moreover there are some similar conjunctions, e.g., a special folding introduced by Koryo Miura is called Miura-ori.

2.1 Origami in mathematics and science and education

There is an interesting book by T. Sundara Row entitled Geometric Exercises in Paper Folding (Madras, 1893). Originally this book was published in India, but European mathematicians also recognized its importance. Thus, Felix Klein refers to it in his book Famous Problems of Elementary Geometry. The main purpose of Sundara Row was to demonstrate how to make various regular polygons and how to approach conic sections by paper-folding. It seems that his book is independent from the Japanese traditions. Interestingly, a revised edition of Sundara Row's
book was co-edited by the noted American historian of mathematics, David Eugene Smith, who was also interested in Japanese mathematics. Note, however, that Smith and Mikami's book, entitled *A History of Japanese Mathematics* (Chicago, 1914), does not refer to origami or to Sundara Row's cited book.

Another important step in the field was made in 1957 by the National Council of Teachers of Mathematics in the United States when they released a booklet by Donovan A. Johnson about paper folding in mathematics. It was followed by a brief article written by Martin Gardner in his series "Mathematical Games" in *Scientific American* (July 1959). An important novelty of this article is the fact that Gardner clearly refers in his title not only to paper-folding, but specifically to Japanese origami. At that time the expression "origami" was so new that Gardner considered it as a proper name and capitalized it through the entire article. As with many other topics in mathematics, Gardner obviously deserves great credit for popularizing some mathematical aspects of origami. Later the above cited council of teachers in the United States published another book on the subject by Alton T. Olson (*Mathematics through Paper Folding*, Reston, 1975).

A new boom in the field is connected, however, with Japanese authors. In 1979 Kodi Husimi (Koji Hushimi), the President of the Science Council of Japan in that time, published, together with his wife, a comprehensive book about the geometry of origami (*Origami no kikagaku*, Tokyo, 1979). The authors refer not only to Japanese scholars, but also to a modern edition of Sundara Row's cited book and to the monograph by Cundy and Rollett, which includes a chapter on polyhedra. Following the great success of the book, they prepared an enlarged edition in 1984. This book had a great impact on the new generation of scholars, and initiated additional research in origami-mathematics. Unfortunately many of these works, including the book by the Husimis, are not yet available in English.

In the field of chemistry, we should refer to Shukichi Yamana, who regularly publishes short notes on models of polyhedra constructed with empty envelopes in the *Journal of Chemical Education*. Now Chinese and Western authors have also joined this 'movement' in the same journal. Moreover the buckminsterfullerene is also 'origamized' by authors in Canada and the U.S.A. (see the papers by J. J. Vittal and by J. M. Beaton, April 1989 and August 1992, respectively). Perhaps it is useful to discuss here the idea of folding polyhedra, which is mostly independent from the classical Japanese traditions.

### 2.2 Folding polyhedra

There is a great record in history of folding polyhedra using a paper net. The roots go back to the Renaissance artists who had a special interest in polyhedra. It was obviously connected with the fact that they rediscovered Plato's ancient works where polyhedra, the so-called Platonic solids, are also discussed. Albrecht Dürer, the German artist, published a book in 1525 where he gives the nets of some regu-

Interestingly, a medical doctor with a strong interest in crystallography made a relevant contribution to this field in the mid 19th century. John Gorham developed a method of making 'plaited models' of crystal polyhedra. He constructed various polyhedra by plaiting paper strips together. He demonstrated his method at the Royal Society in London, and about forty years later he wrote a monograph about his findings (A System for the Construction of Crystal Models on the Type of an Ordinary Plait, Exemplified by the Forms Belonging to the Six Axial Systems in Crystallography, London, 1888). This book was rediscovered by the British mathematician A. R. Pargeter in the 1950s, who developed the rather experimental method of Gorham into a systematic approach. In addition to this, he also focused on those polyhedra that are important in mathematics, but not in crystallography (Plaited polyhedra, Mathematical Gazette, Vol. 43, pp. 88-101, 1959). This method was popularized by the second edition of the book on mathematical models by Cundy and Rollett in 1961 and later by the cited Japanese book on the geometry of origami by the two Husimis in 1979. A new variation of plaiting was worked out by Jean J. Pedersen in California. While the earlier authors used asymmetric strips, Pedersen “braids” (note the different terminology!) the Platonic solids and some other polyhedra with congruent straight strips. Her method was initially reported by Martin Gardner (Scientific American, September 1971), and later it was described by Pedersen herself in various papers (see, e.g., in: Shaping Space: A Polyhedral Approach, edited by M. Senechal and G. Fleck, Boston, 1988, pp. 133-147). More recently the Japanese artist and designer, T. Betsumiya, invented a similar technique, probably independently from Pedersen. Of course not only plaiting and braiding is useful making polyhedra, but we still may use the classical method of folding them from the full net. The modern ‘champion’ of polyhedral models is the reverend-mathematician Magnus Wenninger, who published several picture-books on this subject.

A further, less obvious connection between symmetry and origami is the fact that we may fold the surface of an exciting three-mirror (trihedral) kaleidoscope from a square-shaped paper (after cutting a corner). If we use a paper with a reflecting surface, or if we fold the surface with normal paper and then cover it with reflecting tape, we have an exciting kaleidoscope that can represent two regular polyhedra, the icosahedron and the dodecahedron, and some further ones, with carefully selected objects. In an earlier paper in this quarterly we gave a detailed description of this kaleidoscope and presented its golden-section based layout discovered by Caspar Schwabe (Vol. 1, No. 2, p. 122).
2.3 Origami in art, design, and technology

Symmetry and origami play an important role not only in mathematics and crystallography, but also in art education. We were glad to see in a very recent exhibition on the Bauhaus, commemorating the 75th year anniversary of its foundation, organized by the Kawasaki City Museum, Japan, some origami-related activity. Specifically, in 1930 Iwao Yamawaki, one of the first Japanese students of the Bauhaus, made origami works in the framework of her studies as a textile designer. Paper models demonstrating generalized symmetry operations were also used at the Hochschule für Gestaltung in Ulm, or using its English name, the Ulm School of Design, in the 1950s and 1960s (see Huff's booklets, *Symmetry: An Appreciation of its Presence in Man's Consciousness*, Parts 2 and 3, Pittsburgh, 1975 and 1977, pp. 2.15 and 3.9). This institution made a significant contribution to design education by continuing the Bauhaus traditions with the necessary modifications (the German name of the institution is identical with the secondary name of the Bauhaus at Dessau). Here symmetry became an organic part of the curriculum of basic design, a tradition that was continued by William S. Huff in the United States.

The Dutch graphic artist M. C. Escher is well-known by the most people who are interested in symmetry by his periodic drawings and representations of polyhedra.
and other scientific topics. It is less well known, however, that he was interested in constructing paper models. One of these models was exhibited at a recent exhibition in Nagasaki, Japan. The actual model is a double Möbius strip, and it is related to his woodcut *Knots* of 1965. According to the catalog of this exhibition (Nagasaki, 1991), Escher's studio was decorated with a number of 3-dimensional models made of paper, cardboard, and plastic.

We should also mention an interesting application of paper-folding and the theory of polyhedra in music education. Bernhard Ganter, a professor of mathematics in Darmstadt, Germany, designed a compound polyhedron, specifically a ‘tower’ of five octahedra, to demonstrate some basic concepts in musicology. His inspiration was a work by Möbius written in 1861. Ganter's compound polyhedron illustrates geometrically the following concepts and their connections:

- the vertices correspond to the notes of the chromatic scale,
- the edges correspond to the thirds and fifths, and
- the triangular faces correspond to the triads.

The net of this polyhedron was published for the Darmstadt Symposium and Exhibition *Symmetrie* in 1986. Perhaps musicologists should have more interest in polyhedra and paper-folding. Incidentally, towers of tetrahedra or octahedra are also popular in other fields. In 1989 Stan Tenen of California designed the *Tetra-Helix*, a tower of 11 tetrahedra resembling the structure of the DNA double helix, for representing not only a geometric metaphor of life, but also his research on the origin and nature of the Hebrew and other sacred alphabets. The triangular faces of the *Tetra-Helix* are decorated with Hebrew letters. Probably the most monumental tower of tetrahedra is the Art Tower in Mito, the capital of Ibaraki Prefecture, Japan, which was completed in 1990. This is one of the futuristic works by Arata Isozaki, the noted Japanese architect, whose name is also associated with such buildings as the Museum of Contemporary Art in Los Angeles, the Brooklyn Museum, the Phoenix Municipal Government Building in Arizona, the Museum of Contemporary Art in Stuttgart, and the Center Building of Tsukuba Science City in Japan.

We should return, however, to the Japanese origami traditions to see a very important development. A leading Japanese expert on spacecraft engineering, Koryo Miura, made a remarkable link between the old traditions of origami and the technology of the future. In the case of space flights, the transport of large objects is very expensive. Thus Miura decided to use origami-principles and to design various foldable structures for space researchers. His laboratory had great success in this field. In addition to this, the Tokyo Design Gallery invited him to make an exhibition there in 1990. When he retired not a long ago from his position as the Director of the Spacecraft Engineering Research Division of the Institute of Space and Astronautical Science, Kanagawa, near Tokyo, Japan, he was invited to become a professor at Seian University of Art and Design, Otsu, near Kyoto. His work is an
exciting symbol of the connections between science and technology on one side and art and humanities on the other one. For more details see his article in this issue.

Note that there is an international interest in making foldable structures. There are various independent achievements: from the artist Hiroshi Tomura to the mineralogist Konstantin Chepizhnyi, from the engineer Paul Schatz to the geometric artist Caspar Schwabe, or from the works by Hoberman Associates, Inc. in the U.S.A. to the Aleph Architects Ltd. in Japan. We plan to publish an article about "tomology", our term for Tomura's topological structures, and about the works by Katsushito Atake and the Aleph Architects; currently we only know Japanese publications about these. There is a well-illustrated survey article on Chuck Hoberman in the magazine Discovery, March 1992. The late Paul Schatz is unfortunately almost unknown outside the German speaking countries, because he did not publish many works and his comprehensive monograph on the study of rhythms and technology is available only in German (Rhythmusforschung und Technik, Stuttgart, 1975). In Switzerland, where he spent most of his life, and Germany there is, however, a Paul Schatz Society. One of his remarkable discoveries is the Würfelgärtel (i.e., girdle of the cube) in 1929. By cutting out from the cube two equal 'tripod-shape' figures at opposite corners, the remaining part forms a chain of tetrahedral units, the girdle, that can be endlessly turned "rhythmically" through its central hole. This discovery led Schatz to the invention of very effective mixing machines. A similar chain of tetrahedra was rediscovered, probably independently, by Schattschneider and Walker in the 1970s, see their book M. C. Escher Kaleidocycles (New York, 1977), which is now available in more than 15 languages. In connection with Caspar Schwabe we may refer to this quarterly: see his nets of flexing polyhedra in Vol. 3, No. 2, pp. 213-221, including the quadricorn, discovered by Schwabe himself. Last, but not least we suggest seeing the rich set of paper models of various flexible polyhedra made by the late Chepizhnyi of Moscow. Unfortunately, there are no proper publications about them, with the exception of a 1991 book on quartz, published in Kirgizstan (Kyrgyzstan). Chepizhnyi had thousands of models to demonstrate his "homological mineralogy", and we also plan to publish an article about this unique person.

3 MEETINGS AND SYMPOSIA IN THE FIELD OF ORIGAMI SCIENCE AND TECHNOLOGY

The First Interdisciplinary Symmetry Symposium and Exhibition of ISIS-Symmetry entitled Symmetry of Structure (Budapest, August 13-19, 1989) was attended by both Koryo Miura, the Japanese engineer cited above, and Humiaki Huzita, a Japanese physicist working in Italy. It was shortly before the First International Meeting of Origami Science and Technology (Ferrara, Italy, December 6-7, 1989) organized by Huzita. The presence of these two scholars in Budapest made a special impact on the symposium. Indeed, Miura's workshop was so successful that many people
requested a repeat of it, and it gave a new field of research to a German scholar of architecture and bionics, Biruta Kresling of Paris; see her paper in this issue.

This interest in origami made it almost natural to host an “Origami festival” during the Second Interdisciplinary Symmetry Symposium and Exhibition of ISIS-Symmetry entitled *Symmetry of Pattern* (Hiroshima, August 17-23, 1992). The presence of Kodi Husimi and Humiaki Huzita, as well as the new generation of origami artists-scientists, made this event very special. Another excitement was provided by Caspar Schwabe who demonstrated his flexible polyhedra. The extended abstracts of that symposium are available in this quarterly, Vol. 3, No. 1 and No. 2 (see there the abstracts by Huzita, Kresling, as well as Kawasaki, Maekawa, and Schwabe in the issues No. 1 and No. 2, respectively).

We are honored by the kind invitation of Koryo Miura to become a participating organization of the forthcoming *Second International Meeting of Origami Science and Scientific Origami*, Otsu (near Kyoto), November 29-December 2, 1994.

This journal’s interest in origami was declared in the very first issue (see Vol. 1, No. 1, p. 106, cf., “Aims and scope”). In the case of the present special issues (this one and the next one), we invited contributors from Japan, the U.S.A., and some European countries. We also plan to continue the origami-related meetings in the framework of ISIS-Symmetry events. Thus we plan a further origami workshop during the Third Interdisciplinary Symmetry Congress and Exhibition of ISIS-Symmetry entitled *Symmetry: Natural and Artificial*, Washington, D.C., August 14-20, 1995. That meeting should focus on origami science, or as a Japanese researcher called it, “origamics”, in East and West.

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The detailed references of this article are available in the bibliography “Origami, paper-folding, and related topics in mathematics and science education”, see the section *Symmetro-graphy*.

This “signature” is Douglas R. Hofstadter’s ambigram. Please rotate it by 180 degrees. For further details see Vol. 1, No. 1, pp. 107-108.