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Origami, 1

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(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

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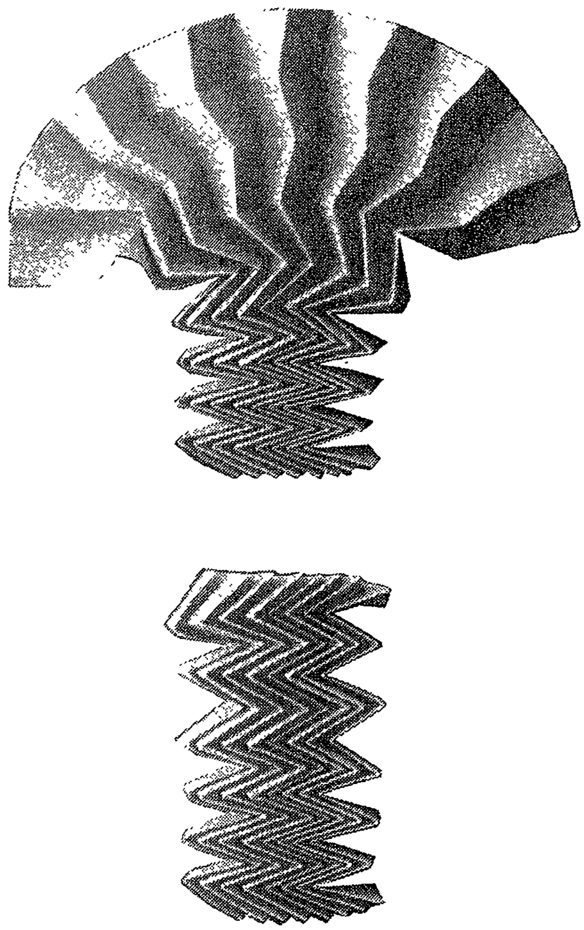

The *Miura-ori*
opened out like a fan

INTERNATIONAL SOCIETY FOR THE INTERDISCIPLINARY STUDY OF SYMMETRY (ISIS-SYMMETRY)

continued inside back cover

SYMMETRY
CULTURE & SCIENCE

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SPECIAL ISSUE: ORIGAMI, 1

Edited by
Dénes Nagy and György Darvas

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EDIT-ORI-*AL*
(INTRODUCTORY ESSAY)

SYMMET-ORIGAMI (SYMMETRY AND ORIGAMI) IN ART, SCIENCE, AND TECHNOLOGY

Dedicated to *V*(*D*)

The Japanese origami is well-known in the West, but most people consider it as just play for children with paper. Incidentally, there is one particular field where paper-folding has a great tradition in the West: it is the folding of table napkins, which topic has its literature. and some authors call it an art (see, e.g., Josephine Ive's booklet *Table Napkin Folding: An Elegant Art*, Winchester, 1983; Melbourne, 1989).

Origami is, however, much more than just play for children, although the latter is also an important aspect. Some dictionaries translate the word as the art of paper-folding. This interpretation is much better, but still narrow: origami has many scientific aspects and technical applications, too. It has its own culture with a very interesting history. Let us make a brief journey into the world of origami.

1 ORIGAMI-HISTORY

The Japanese word *origami* is written with two Chinese characters (*kanji*) and usually there is a declensional syllabogram (*kana*) between them that belongs to the first one. Note that the characters adapted from the Chinese have mostly two (or two sets) of readings in Japanese: a Chinese-related one and a native Japanese one. In the case of the two characters of origami, both of them should be read in the Japanese way. Thus the expression *origami* is not an adaptation from the Chinese language, but an independently developed Japanese concept. On the other hand paper itself and some sort of paper-folding art was brought to Japan from China. Let us see very briefly the origins of both characters in the word *origami*:

折り紙

– the first character should be read as *o* – it represents a cut tree and an axe, but later this original meaning was extended to the idea of breaking and folding;
– this is followed by a declensional syllabogram *ri* – it completes the reading of the first character *ori*;
– the second character is *kami* (which becomes *gami*) – the left-hand side represents a thread, while the other part refers to thin and flat objects, thus the entire character means paper.
It is interesting to see that the idea of cut is not so far from origami as is believed. Later we will return to this question.

Of course paper-folding was invented not only in China and Japan, but in almost all cultures where paper was in use. The Japanese, however, went further than other nations and developed it into a complex form of art and science. The history of this process includes many spiritual aspects. Indeed, in ancient Japan paper was very expensive, therefore it was used for ceremonies and rituals, not for play. In the native Japanese religion *Shintô* origami has a special importance. Moreover the oldest known origami-related activity is the *katashiro*, a symbolic representation of a deity made of paper. Folded paper is also used in case of various *Shintô* purification ceremonies. There is a special shrine paper for such purposes (*jingû yôshi*).

The origami step by step reached other fields of culture, from the tricks of magicians to the everyday life. The *kimono* dressing has many analogies with origami, because the textile should be folded in a special way. There is, however, a more direct connection: the *tatô*, a folded paper case attached to the kimono. *Nô* theater costumes also frequently use origami attachments. Another field where the origami has a special importance is the etiquette of giving gifts. For example, *sake* bottles should be decorated by male and female butterflies. We may see in this practice an interesting connection with observing nature, an attitude which is much stronger in Japan than in the West. Indeed, creating animal-shapes by origami has a great tradition in general. To make a distinction between male and female butterflies reflects, however, a further idea of traditional Japanese thinking: the philosophy of *yin-yang* based on male and female principles. Note that the circular *yin-yang* symbol has a symmetric structure, based on 2-fold rotation antisymmetry (black-and-white symmetry), which represents the unity of the two opposite forces. Thus, a good drink, such as *sake*, should also satisfy this 'symmetry'. Returning to origami animals, we may mention another interesting aspect. A famous piece of art in this field is the frog that can jump if we press it on a spot at the backside. Thus, not only the similarity of form is important, but there is an attempt to show some functional similarity, too. (The jumping frog is very popular and it is included in many books on the field.) Origami has a connection even with love! In old times the love letters were specially folded making a regular pentagon by tieing a knot on a strip of paper and tightening it flat. Note that the pentagon also plays an important role in traditional thinking: the system of five basic elements – earth, water, fire, metal, and wood – and their connections are represented by a regular pentagon and an inscribed pentagram (five pointed star). Apropos, letters: the Japanese aerograms also refer to origami. These sheets, widely available in post offices, show carefully where we should make the first fold and the second one. Until very recently the aerogram sheet was decorated with two birds: one on the printed stamp and another one symbolizing the airmail with an origami crane. (Very recently, when the aerogram fee was changed, a new type was released without this origami bird). Finally, let us discuss the question of animals versus technical objects in origami. There is no doubt that Japanese origami has more emphasize on animals, while the typical Western paper-folding for children has more interest in technical objects, such as flying machines and ships. More recently, however, these two sides influenced each other: Japanese have more interest in technical objects and the Westerners make more animals. Just one example: a group of Japanese origamists constructed a very large set of American aircraft.

We do not discuss here the detailed history of origami, but just return to the question of using cuts. As we mentioned earlier, the first character in the Japanese word *origami* clearly refers to cut, thus this operation was not excluded in the beginning. Indeed, origami representing the deity in *Shintô* ceremonies is made partly with cuts. Origami, however, also became an entertainment probably in the 9th-11th centuries, and gradually various schools and styles appeared. Origami without cuts was first developed in the Muromachi-period (1333-1568). Interested readers who would like to see a brief, but very informative survey on origami and its major types, may refer to Akira Yoshizawa's article "Origami" in the *Kodansha Encyclopedia of Japan* (Vol. 6, Tokyo, 1983), which is available in the major libraries in the West.

Let us see the beginnings of the Western interest in origami. The new edition of the *Oxford English Dictionary* (Vol. 10, Oxford, 1989, p. 933) gives credit to Robert Harbin's book *Paper Magic: The Art of Paper Folding* (London, 1956, p. 14) as containing the first usage of the word in an English context. There is also a reference to a 1922 article by F. Starr, published in San Francisco, where a Japanese book on origami is cited. We should note, however, that origami did not reach the Western world via the English-speaking countries. Probably Spain was the first Western country where origami was practised. Some origami-related materials reached them, supposedly via the Moors, in the 8th century. It was not definitely Japanese origami, but some paper-folding arts that originated in Asian countries, including Japan. The Spanish philosopher, writer, and poet Miguel de Unamono y Jugo (1864-1936), or, as the children called him, "Don Miguel", significantly contributed to the spread of paper-folding in art education. The birds and other animals made by him were so realistic, that, according to a famous story, some children believed that they could 'speak'. Paper-folding, following a Spanish influence, also became popular in Southern America. The list of famous paper-folders includes, among others, Leonardo da Vinci, the poet Shelley, the mathematician and writer Lewis Carroll. Their activities were, however, more or less independent from the

Japanese traditions, and they did not generate public interest. Paper-folding remained in the Western world play for children and for a few interested adults until the beginning of the stronger Japanese influence.

Probably Japanese magicians represented the first wave of introduction of Japanese origami to the Western world. Their quickly made paper objects and the transformations of these surprized the audience. Incidentally, the famous magician Houdini (1874-1926) also had an interest in paper-folding, and the previously cited Robert Harbin, the author of the book of 1956 where the word *origami* appears in English context, is also a noted magician. Another important wave of Japanese influence is due to public exhibitions. Akira Yoshizawa (b. 1911) made in 1955 an exhibition of origami art, including both figurative and abstract works, at the City Museum in Amsterdam. It was followed by a group exhibition in 1959 at the Cooper Union Museum in New York. The broad interest of the American people in origami lead to the establishment of the Origami Center of America in New York by Lillian Oppenheimer in the same year of 1959. In some sense Yoshizawa is the 'Kodály' or 'Laban' of origami: while Kodály developed the relative solmization (or solfaing) system in music education and Laban worked out the symbols for dance notation, both spreading very quickly internationally, Yoshizawa developed the symbols to describe folding processes. Thus the origamists also have an 'international language' for communication.

2 SYMMETRY AND ORIGAMI IN ART, SCIENCE, AND TECHNOLOGY

The expression in the title of this article "symmet-origami" is not only our play on words, but the conjunction of these two expressions makes some sense: symmetry + paper-folding. Indeed, the process of making origami objects often includes foldings according to symmetry axes. In addition to this, origami is very useful in making symmetric objects, including periodic patterns. Note that the term "symmetry" was adopted into Japanese as *shinmetorii*. Thus the above play on words is understandable even in Japanese, moreover there are some similar conjunctions, e.g., a special folding introduced by Koryo Miura is called *Miura-ori*.

2.1 Origami in mathematics and science and education

There is an interesting book by T. Sundara Row entitled *Geometric Exercises in Paper Folding* (Madras, 1893). Originally this book was published in India, but European mathematicians also recognized its importance. Thus, Felix Klein refers to it in his book *Famous Problems of Elementary Geom*etry. The main purpose of Sundara Row was to demonstrate how to make various regular polygons and how to approach conic sections by paper-folding. It seems that his book is independent from the Japanese traditions. Interestingly, a revised edition of Sundara Row's book was co-edited by the noted American historian of mathematics, David Eugene Smith, who was also interested in Japanese mathematics. Note, however, that Smith and Mikami's book, entitled *A History of Japanese Mathematics* (Chicago, 1914), does not refer to origami or to Sundara Row's cited book.

Another important step in the field was made in 1957 by the National Council of Teachers of Mathematics in the United States when they released a booklet by Donovan A. Johnson about paper folding in mathematics. It was followed by a brief article written by Martin Gardner in his series "Mathematical Games" in *Scientific American* (July 1959). An important novelty of this article is the fact that Gardner clearly refers in his title not only to paper-folding, but specifically to Japanese origami. At that time the expression "origami" was so new that Gardner considered it as a proper name and capitalized it through the entire article. As with many other topics in mathematics, Gardner obviously deserves great credit for popularizing some mathematical aspects of origami. Later the above cited council of teachers in the United States published another book on the subject by Alton T. Olson (*Mathematics through Paper Folding*, Reston, 1975).

A new boom in the field is connected, however, with Japanese authors. In 1979 Kodi Husimi (Koji Hushimi), the President of the Science Council of Japan in that time, published, together with his wife, a comprehensive book about the geometry of origami (*Origami no kikagaku*, Tokyo, 1979). The authors refer not only to Japanese scholars, but also to a modern edition of Sundara Row's cited book and to the monograph by Cundy and Rollett, which includes a chapter on polyhedra. Following the great success of the book, they prepared an enlarged edition in 1984. This book had a great impact on the new generation of scholars, and initiated additional research in origami-mathematics. Unfortunately many of these works, including the book by the Husimis, are not yet available in English.

In the field of chemistry, we should refer to Shukichi Yamana, who regularly publishes short notes on models of polyhedra constructed with empty envelopes in the *Journal of Chemical Education*. Now Chinese and Western authors have also joined this 'movement' in the same journal. Moreover the buckminsterfullerene is also 'origamized' by authors in Canada and the U.S.A. (see the papers by J. J. Vittal and by J. M. Beaton, April 1989 and August 1992, respectively). Perhaps it is useful to discuss here the idea of folding polyhedra, which is mostly independent from the classical Japanese traditions.

2.2 Folding polyhedra

There is a great record in history of folding polyhedra using a paper net. The roots go back to the Renaissance artists who had a special interest in polyhedra. It was obviously connected with the fact that they rediscovered Plato's ancient works where polyhedra, the so-called Platonic solids, are also discussed. Albrecht Dürer, the German artist, published a book in 1525 where he gives the nets of some regular and semiregular polyhedra. This book written in German has a modern facsimile edition and a parallel English translation (*The Painter's Manual: A Manual of Measurement of Lines, Areas, and Solids by Means of Compass and Ruler*, New York, 1977). The Italian artist, Daniel Barbaro, also published drawings of nets of polyhedra in his book *La pratica della perspettiva* (Venetia, 1569). This activity later became popular in both teaching of mathematics and crystallography.

Interestingly, a medical doctor with a strong interest in crystallography made a relevant contribution to this field in the mid 19th century. John Gorham developed a method of making 'plaited models' of crystal polyhedra. He constructed various polyhedra by plaiting paper strips together. He demonstrated his method at the Royal Society in London, and about forty years later he wrote a monograph about his findings (*A System for the Construction of Crystal Models on the Type of an Ordinary Plait, Exemplified by the Forms Belonging to the Six Axial Systems in Crystallography*, London, 1888). This book was rediscovered by the British mathematician A. R. Pargeter in the 1950s, who developed the rather experimental method of Gorham into a systematic approach. In addition to this, he also focused on those polyhedra that are important in mathematics, but not in crystallography (Plaited polyhedra, *Mathematical Gazette*, Vol. 43, pp. 88-101, 1959). This method was popularized by the second edition of the book on mathematical models by Cundy and Rollett in 1961 and later by the cited Japanese book on the geometry of origami by the two Husimis in 1979. A new variation of plaiting was worked out by Jean J. Pedersen in California. While the earlier authors used asymmetric strips, Pedersen "braids" (note the different terminology!) the Platonic solids and some other polyhedra with congruent straight strips. Her method was initially reported by Martin Gardner (*Scientific American*, September 1971), and later it was described by Pedersen herself in various papers (see, e.g., in: *Shaping Space: A Polyhedral Approach*, edited by M. Senechal and G. Fleck, Boston, 1988, pp. 133-147). More recently the Japanese artist and designer, T. Betsumiya, invented a similar technique, probably independently from Pedersen. Of course not only plaiting and braiding is useful making polyhedra, but we still may use the classical method of folding them from the full net. The modern 'champion' of polyhedral models is the reverend-mathematician Magnus Wenninger, who published several picture-books on this subject.

A further, less obvious connection between symmetry and origami is the fact that we may fold the surface of an exciting three-mirror (trihedral) kaleidoscope from a square-shaped paper (after cutting a corner). If we use a paper with a reflecting surface, or if we fold the surface with normal paper and then cover it with reflecting tape, we have an exciting kaleidoscope that can represent two regular polyhedra, the icosahedron and the dodecahedron, and some further ones, with carefully selected objects. In an earlier paper in this quarterly we gave a detailed description of this kaleidoscope and presented its golden-section based layout discovered by Caspar Schwabe (Vol. 1, No. 2, p. 122).

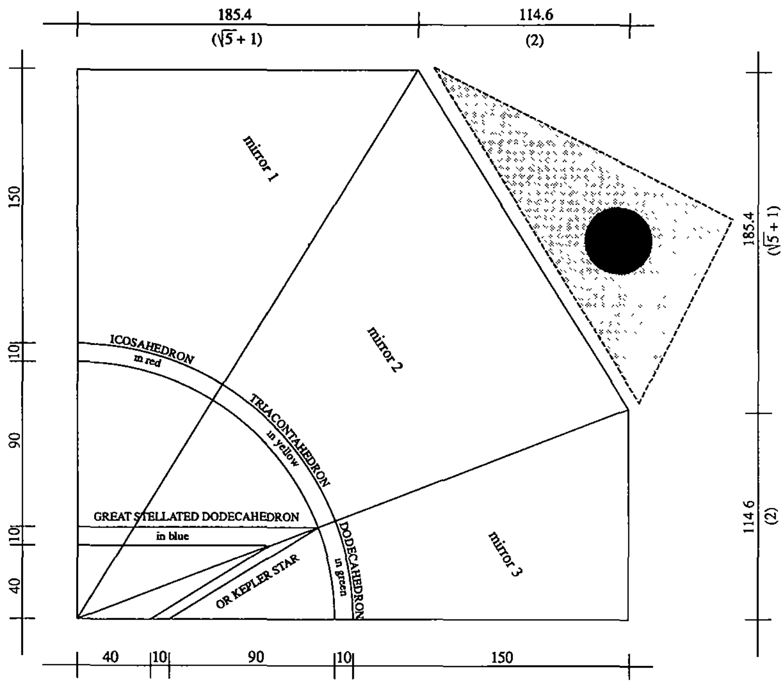


Figure: Golden section kaleidoscope (Caspar Schwabe, 1986).

2.3 Origami in art, design, and technology

Symmetry and origami plays an important role not only in mathematics and crystallography, but also in art education. We were glad to see in a very recent exhibition on the Bauhaus, commemorating the 75th year anniversary of its foundation, organized by the Kawasaki City Museum, Japan, some origami-related activity. Specifically, in 1930 Iwao Yamawaki, one of the first Japanese students of the Bauhaus, made origami works in the framework of her studies as a textile designer. Paper models demonstrating generalized symmetry operations were also used at the Hochschule für Gestaltung in Ulm, or using its English name, the Ulm School of Design, in the 1950s and 1960s (see Huff's booklets, *Symmetry: An Appreciation of its Presence in Man's Consciousness*, Parts 2 and 3, Pittsburgh, 1975 and 1977, pp. 2.15 and 3.9). This institution made a significant contribution to design education by continuing the Bauhaus traditions with the necessary modifications (the German name of the institution is identical with the secondary name of the Bauhaus at Dessau). Here symmetry became an organic part of the curriculum of basic design, a tradition that was continued by William S. Huff in the United States.

The Dutch graphic artist M. C. Escher is well-known by the most people who are interested in symmetry by his periodic drawings and representations of polyhedra and other scientific topics. It is less well known, however, that he was interested in constructing paper models. One of these models was exhibited at a recent exhibition in Nagasaki, Japan. The actual model is a double Möbius strip, and it is related to his woodcut *Knots* of 1965. According to the catalog of this exhibition (Nagasaki, 1991), Escher's studio was decorated with a number of 3-dimensional models made of paper, cardboard, and plastic.

We should also mention an interesting application of paper-folding and the theory of polyhedra in music education. Bernhard Ganter, a professor of mathematics in Darmstadt, Germany, designed a compound polyhedron, specifically a 'tower' of five octahedra, to demonstrate some basic concepts in musicology. His inspiration was a work by Möbius written in 1861. Ganter's compound polyhedron illustrates geometrically the following concepts and their connections:

– the vertices correspond to the notes of the chromatic scale,
– the edges correspond to the thirds and fifths, and
– the triangular faces correspond to the triads.

The net of this polyhedron was published for the Darmstadt Symposium and Exhibition *Symmetrie* in 1986. Perhaps musicologists should have more interest in polyhedra and paper-folding... Incidentally, towers of tetrahedra or octahedra are also popular in other fields. In 1989 Stan Tenen of California designed the *Tetra-Helix*, a tower of 11 tetrahedra resembling the structure of the DNA double helix, for representing not only a geometric metaphor of life, but also his research on the origin and nature of the Hebrew and other sacred alphabets. The triangular faces of the *Tetra-Helix* are decorated with Hebrew letters. Probably the most monumental tower of tetrahedra is the Art Tower in Mito, the capital of Ibaraki Prefecture, Japan, which was completed in 1990. This is one of the futuristic works by Arata Isozaki, the noted Japanese architect, whose name is also associated with such buildings as the Museum of Contemporary Art in Los Angeles, the Brooklyn Museum, the Phoenix Municipal Government Building in Arizona, the Museum of Contemporary Art in Stuttgart, and the Center Building of Tsukuba Science City in Japan.

We should return, however, to the Japanese origami traditions to see a very important development. A leading Japanese expert on spacecraft engineering, Koryo Miura, made a remarkable link between the old traditions of origami and the technology of the future. In the case of space flights, the transport of large objects is very expensive. Thus Miura decided to use origami-principles and to design various foldable structures for space researchers. His laboratory had great success in this field. In addition to this, the Tokyo Design Gallery invited him to make an exhibition there in 1990. When he retired not a long ago from his position as the Director of the Spacecraft Engineering Research Division of the Institute of Space and Astronautical Science, Kanagawa, near Tokyo, Japan, he was invited to become a professor at Seian University of Art and Design, Otsu, near Kyoto. His work is an exciting symbol of the connections between science and technology on one side and art and humanities on the other one. For more details see his article in this issue.

Note that there is an international interest in making foldable structures. There are various independent achievements: from the artist Hiroshi Tomura to the mineralogist Konstantin Chepizhnyi, from the engineer Paul Schatz to the geometric artist Caspar Schwabe, or from the works by Hoberman Associates, Inc. in the U.S.A. to the Aleph Architects Ltd. in Japan. We plan to publish an article about "tomology", our term for Tomura's topological structures, and about the works by Katsushito Atake and the Aleph Architects; currently we only know Japanese publications about these. There is a well-illustrated survey article on Chuck Hoberman in the magazine *Discovery*, March 1992. The late Paul Schatz is unfortunately almost unknown outside the German speaking countries, because he did not publish many works and his comprehensive monograph on the study of rhythms and technology is available only in German (*Rhythmusforschung und Technik*, Stuttgart, 1975). In Switzerland, where he spent most of his life, and Germany there is, however, a Paul Schatz Society. One of his remarkable discoveries is the *Würfelgürtel* (i.e., girdle of the cube) in 1929. By cutting out from the cube two equal 'tripod-shape' figures at opposite corners, the remaining part forms a chain of tetrahedral units, the girdle, that can be endlessly turned "rhythmically" through its central hole. This discovery led Schatz to the invention of very effective mixing machines. A similar chain of tetrahedra was rediscovered, probably independently, by Schattschneider and Walker in the 1970s, see their book *M. C. Escher Kaleidocycles* (New York, 1977), which is now available in more than 15 languages. In connection with Caspar Schwabe we may refer to this quarterly: see his nets of flexing polyhedra in Vol. 3, No. 2, pp. 213-221, including the *quadricorn*, discovered by Schwabe himself. Last, but not least we suggest seeing the rich set of paper models of various flexible polyhedra made by the late Chepizhnyi of Moscow. Unfortunately, there are no proper publications about them, with the exception of a 1991 book on quartz, published in Kirgizstan (Kyrgyzstan). Chepizhnyi had thousands of models to demonstrate his "homological mineralogy", and we also plan to publish an article about this unique person.

3 MEETINGS AND SYMPOSIA IN THE FIELD OF ORIGAMI SCIENCE AND TECHNOLOGY

The First Interdisciplinary Symmetry Symposium and Exhibition of ISIS-Symmetry entitled *Symmetry of Structure* (Budapest, August 13-19, 1989) was attended by both Koryo Miura, the Japanese engineer cited above, and Humiaki Huzita, a Japanese physicist working in Italy. It was shortly before the *First International Meeting of Origami Science and Technology* (Ferrara, Italy, December 6-7, 1989) organized by Huzita. The presence of these two scholars in Budapest made a special impact on the symposium. Indeed, Miura's workshop was so successful that many people requested a repeat of it, and it gave a new field of research to a German scholar of architecture and bionics, Biruta Kresling of Paris; see her paper in this issue.

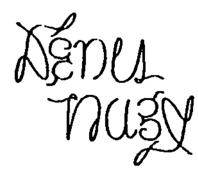
This interest in origami made it almost natural to host an "Origami festival" during the Second Interdisciplinary Symmetry Symposium and Exhibition of ISIS-Symmetry entitled *Symmetry of Pattern* (Hiroshima, August 17-23, 1992). The presence of Kodi Husimi and Humiaki Huzita, as well as the new generation of origami artists-scientists, made this event very special. Another excitement was provided by Caspar Schwabe who demonstrated his flexible polyhedra. The extended abstracts of that symposium are available in this quarterly, Vol. 3, No. 1 and No. 2 (see there the abstracts by Huzita, Kresling, as well as Kawasaki, Maekawa, and Schwabe in the issues No. 1 and No. 2, respectively).

We are honored by the kind invitation of Koryo Miura to become a participating organization of the forthcoming *Second International Meeting of Origami Science and Scientific Origami*, Otsu (near Kyoto), November 29-December 2, 1994.

This journal's interest in origami was declared in the very first issue (see Vol. 1, No. 1, p. 106, cf., "Aims and scope"). In the case of the present special issues (this one and the next one), we invited contributors from Japan, the U.S.A., and some European countries. We also plan to continue the origami-related meetings in the framework of ISIS-Symmetry events. Thus we plan a further origami workshop during the Third Interdisciplinary Symmetry Congress and Exhibition of ISIS-Symmetry entitled *Symmetry: Natural and Artificial*, Washington, D.C., August 14-20, 1995. That meeting should focus on origami science, or as a Japanese researcher called it, "origamics", in East and West.

We thank the kind help and cooperation to all contributors of this issue and to John Hosack and Jan Tent.

The detailed references of this article are available in the bibliography "Origami, paper-folding, and related topics in mathematics and science education", see the section *Symmetro-graphy*.



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For further details see Vol. 1, No. 1, pp. 107-108.

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FOLDS – THE BASIS OF ORIGAMI

Koryo Miura

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Abstract: *The physical and mathematical aspects of folds, which characterize the shape of artifacts made of a shin sheet of paper, are studied. The present aim is to clarify the reason why shapes of origami works are generally characterized by ragged surfaces consisting of convex and concave surfaces divided by sharp edges. The following conclusions are obtained: first, the ragged surface consisting of complex sharp folds is the natural characteristic deformation of a thin sheet of paper; second, the area of the spherical image for a complex of folds, which is the integral curvature of the domain, must be vanished; third, the above condition is satisfied only by compensation of positive and negative areas of spherical images corresponding to convex and concave areas on the origami surfaces.*

INTRODUCTION

The purpose of this paper is to study physical and mathematical aspects of a fold and of a complex of folds. Folds are found in abundance in natural creatures as in artifacts around us. However, the origin of this common geometric feature may be different from one thing to another. In this paper, the theoretical basis of folds for artifacts, which are fabricated basically through inextensional deformation from a thin flat sheet, is sought. Origami is, of course, the most typical one in this category (Miura, 1989a).

OBSERVATION OF A CRUSHED SHEET OF PAPER

Figure 1 is a photograph of a sheet of paper crushed arbitrarily by hand. It represents the characteristic shape of surfaces deformed from a sheet of paper. The most remarkable feature observed is its ragged shape consisting of many sharp ridges (folds). It can never be a smooth surface. Why is the shape so ragged?

The second feature of the surface is that it can be divided into convex and concave regions. And these two regions are usually divided by sharp ridges.

Although the present example cannot be an origami work, it represents a common and natural feature of an artifact made of a sheet of paper. The present author is especially interested in the shape around the vicinity of folds, which constitutes basic elements of origami. In the following section, its physical aspect (formation of folds) and mathematical aspect (geometry at the vicinity of folds) are investigated.

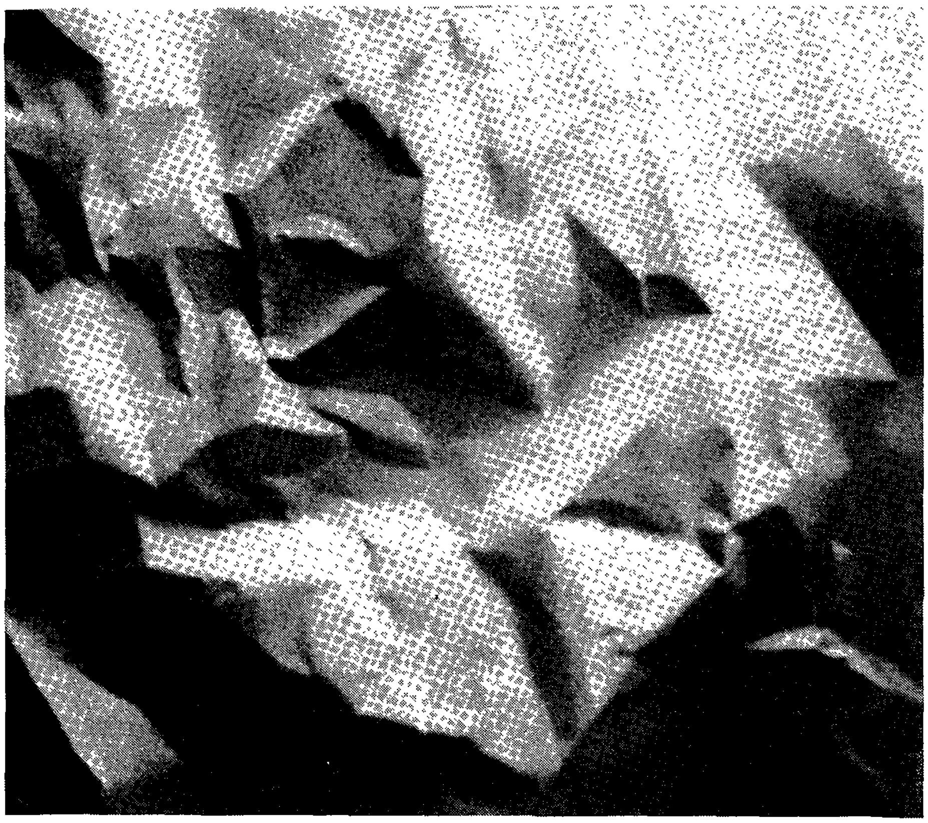

Figure 1: A crushed sheet of paper.

PHYSICAL ASPECTS OF FOLDS

Paper is not an abstract surface, it is a real material having physical properties such as elastic, plastic, and other behaviors.

A conventional method for describing deformations of a thin sheet type of material is to divide them into bending (out-of-plane of the sheet), stretching/contracting (in-plane), and shear (in-plane) deformations. Because of the thinness of sheets, other components of deformation are generally negligible. If the thickness decreases further, the strain energy of out-of-plane deformation decreases much faster than that of in-plane deformation. According to the minimum principle of strain energy, the sheet deforms in a way in which the total strain energy is minimized. Therefore, for a very thin sheet, the deformation should be dominated by bending deformation. This is the general view of physicists about a thin sheet of material.

The planar deformation of a piece of piano wire subject to terminal compression is considered now. Piano wire is a very thin elastic one-dimensional medium and thus, due to the strain energy theory, the deformation will be bending dominant and it is virtually inextensional along its axis. Thus it provides some information about the deformation of bending dominant media.

As a matter of fact, this problem of the elastic thin bar is famous, because this was one of the first examples of the variational principle which the great mathematician Leonhard Euler conceived.

Euler obtained the very beautiful and smooth curves as shown in Figure 2. Now these curves are called *Elastica*.

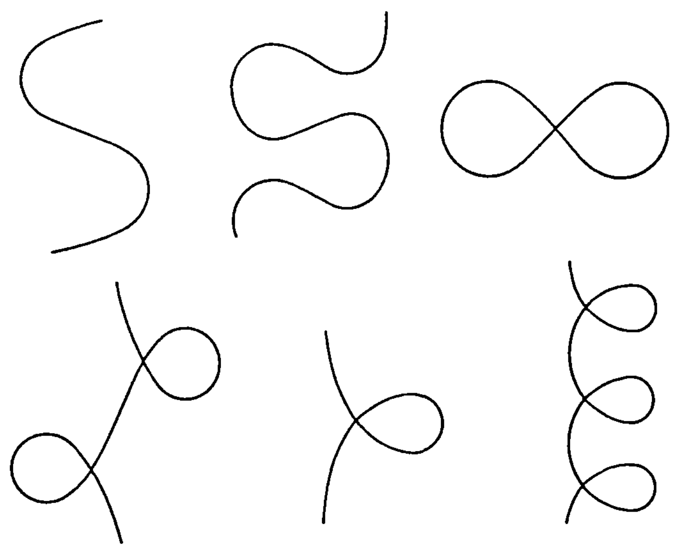

Figure 2: Euler's Elastica.

Since these smooth deformations are derived from the minimum principle, it can be said that these deformations are natural.

Now, the solution for a similar problem of Euler's for two-dimensional medium, that is, a thin sheet of paper is sought. This problem can be formulated as "to obtain the deformation for a thin sheet of material of an infinite extension subjected to uniform end shortening". Studies by Miura (1970) and Tanizawa-Miura (1978a) have revealed the solution. The ten numerical solutions for the problem are shown in Figure 3. The figures represent the out-of-plane deformations for a fundamental region by contour line maps. In these figures, the numerals represent the heights of deformations.

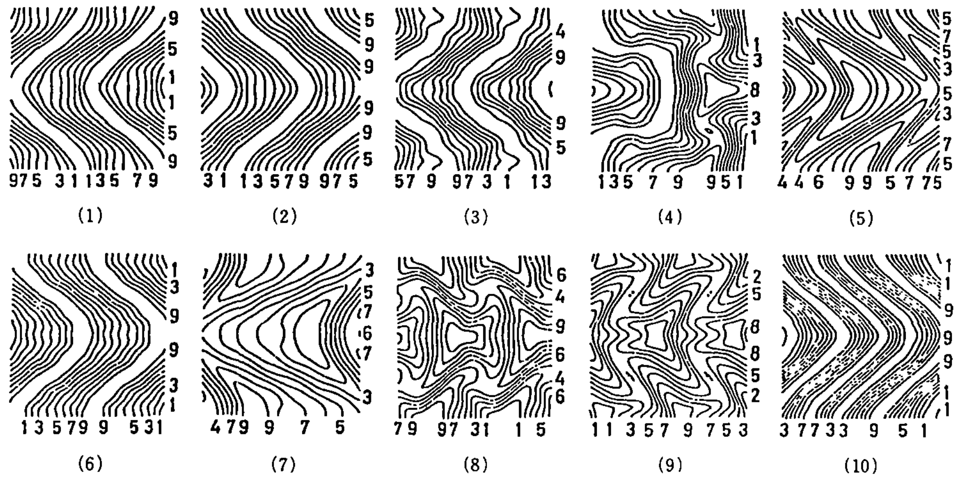

Figure 3: Out-of-plane deformations for thin plate subject to uniform end shortening; contour lines.

It can be observed that these deformations are characterized by ragged surfaces having sharp ridges (fold), which makes a sharp contrast with the smooth curves obtained in the one-dimensional medium.

When the least energy solution is sought among these solutions, and the thickness of the sheet is decreased to infinitesimal small, a polyhedral surface having a beautiful symmetry is obtained. It is composed of repetition of a fundamental region, which is further composed of four congruent parallelograms as shown in Figure 4.

As is well-known, the particular properties of this surface was applied to origami and maps by Miura (1978b and 1989b). Since this 'origami' is formed by natural forces and not by hand, it can be called 'natural origami'. It is interesting to note that this is the simplest origami of all.

Since this problem is an extension of a one-dimensional medium to a two-dimensional medium, the solution is by analogy considered to be *Plate Elastica*. It is surprising that the result actually obtained is quite different from our imagination of Elastica which is a symbol of 'smoothness'. The ragged surface consisting of complex sharp folds is the inherent and natural characteristic deformation for a thin sheet of paper.

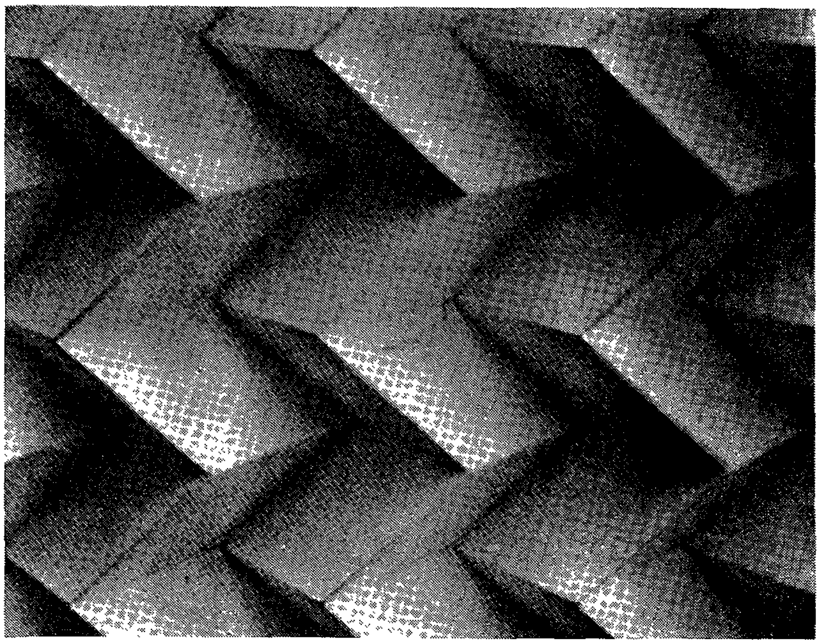

Figure 4: Polyhedral surface, *Plate Elastica*

THE BRIDGE BETWEEN PHYSICAL VS. MATHEMATICAL ASPECTS

The above study on folds indicates that, for a very thin sheet, the strain is concentrated in the vicinity of folds and there are small amounts of in-plane stretching and shear strains.

For the extremum case of vanishing thickness, in-plane strains vanish. This means that the middle plane of the sheet is absolutely inextensional. In other words, the resulting deformation is an isometric transfer of the initially flat surface of zero Gaussian curvature (the product of the principal curvatures). Because the Gaussian curvature is invariant under the isometric transfer, the zero Gaussian curvature is kept throughout the deformation.

$$K = 0 \qquad (1)$$

Since a sheet of paper, which we used for origami works, is initially flat (K=0), and almost inextensional within its plane, origami works can be described mathematically as the isometric transformations of the initially flat ($K=0$) surface. In this sense, surfaces of origami works belong to a group of developable surfaces.

MATHEMATICAL EXPRESSION OF FOLDS

Now, the mathematical expression is sought over a domain enclosing a complex of folds, for example, a domain somewhere on the surface of Figure 1. Miura (1989c) applied the method of spherical images (Gaussian representation) successfully to express the complex. Therefore, a similar but slightly different approach is used in this study.

The integral curvature $G(U)$ is obtained by integrating the Gaussian curvature K, multiplied by the element of area dA over a domain U of the surface S, of the region (Fig. 5).

$$G(U) = \iint_U K dA. \tag{2}$$

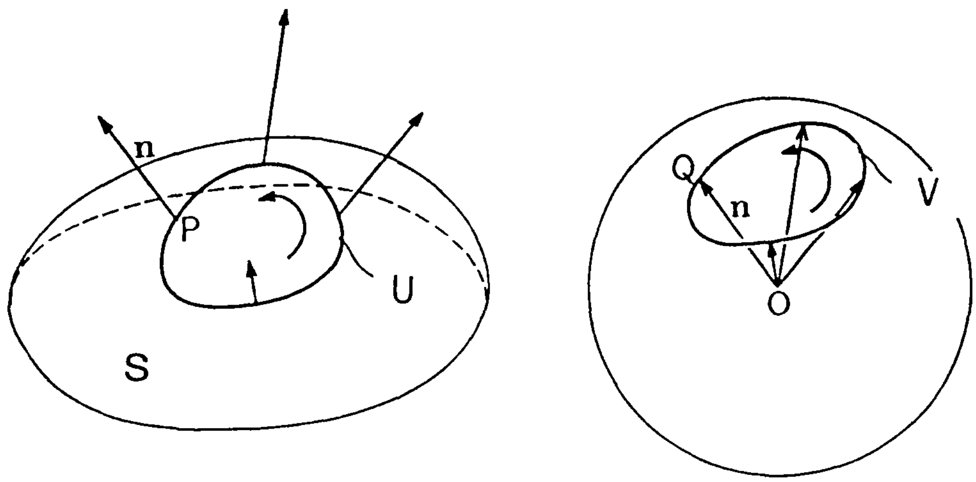

Figure 5: Spherical image of a surface.

One obtains an intuitive interpretation of the integral curvature G, and therefore of the Gaussian curvature K, by investigating the spherical image of a domain U in the surface S. Drawing the normal unit vector **n** of a point P of U from a fixed point, say the origin O, one obtains the spherical image. The ends of these vectors then describe a domain V in the unit sphere, which is the spherical image of U. Apart from the sign the area of the spherical image is then equal to the integral curvature G of U. Intuitively, it is obvious that this area is the larger the more sharply the surface S curves.

Because the surface of origami is generally ragged, as discussed before, the area of the spherical image of origami should be large. However, since $K = 0$ everywhere on the surface of origami, the integral curvature G should also be zero.

$$G(U) = \iint_U K dA = 0 \tag{3}$$

One has to explain this apparent contradiction. Also, there is another difficulty concerning the mathematical singularity at the fold. Because the above mathematical procedure is valid only for a smooth surface, one cannot define the spherical image for a fold.

Figure 6 shows a single-folded sheet of paper. The spherical image for arbitrary curves on the planes P and Q are shown in Figure 6 as p' and q'. Then consider an arbitrary curve PQ which connects P and Q by crossing the fold at R, which is apparently the singular point (angular point). It is natural to think that a fold can be taken as the infinitesimal limit of a bend with a finite curvature. The same can be said about their spherical images. Thus, we can make a reasonable definition of the spherical image of a fold as follows:

> "the spherical image of a fold R dividing a pair of flat surfaces P and Q is the great circular arc r' connecting the images p' and q' of these surfaces".

The definition also holds for a point on a curved fold.

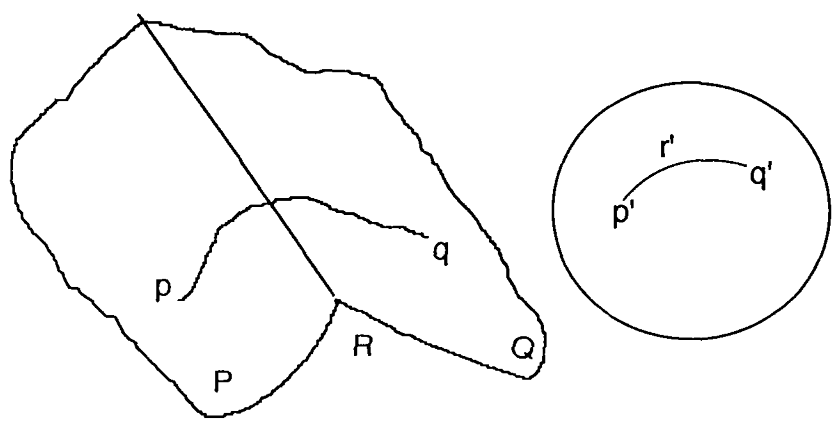

Figure 6: Spherical image of a fold.

SPHERICAL IMAGE OF ORIGAMI

Now we have a method to investigate the geometric properties of a fold and a complex of folds. In the following, this method is applied to study a vicinity of a node where three mountain folds and one valley fold meet. This is the basic and the simplest complex of folds in origami (Fig. 7).

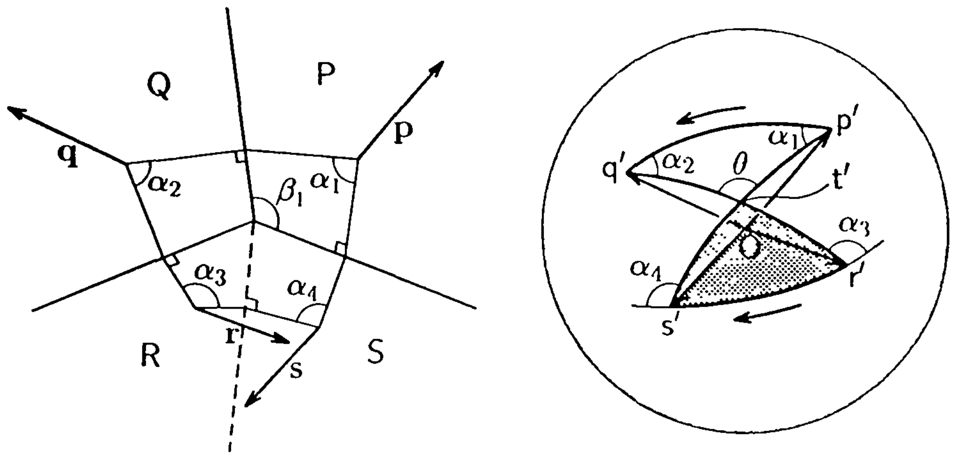

Figure 7: Spherical image of a complex of folds.

The complex of folds is described as:
– it consists of flat surfaces, P, Q, R and S,
– it consists of three mountain fold and one valley fold,
– P and Q form a convex region, R and S form a concave region, or it is divided into a convex and a concave region by qr – sp mountain folds.

Here, the convexity of a small domain of a surface is defined so that the line segment joining any two points within the domain does not come out of the surface. It should be noted that this definition is slightly different from the conventional definition of the convexity of a surface.

The spherical image for the complex is described as:
– surfaces P, Q, R, and S are mapped onto points p, q, r, and s, respectively,
– folds pq, qr, rs, and sp are mapped onto great circular arcs pq, qr, rs, and sp,
– the spherical image is an 8-shaped pattern,
– the P, Q convex region is mapped onto the positive triangle tpq, while the R, S concave region is mapped onto the negative triangle trs, (based on the conventional mathematical premise).

It is observed that, for an arbitrary closed curve around the node, the convex region is mapped to the positive spherical triangle, while the concave region is mapped to the negative spherical triangle. Through mathematical manipulation, it is proved that both triangles have the identical absolute area. Therefore the total area of the spherical image vanishes, that is, Equation (3) is satisfied. This logic can be extended to more general cases including multiple folds and curved folds as well.

The following conclusion is obtained. For a complex of folds, the zero integral curvature G is only satisfied by compensation of positive and negative spherical images, which correspond to convex and concave surfaces of origami. This is why the zero of K and G is satisfied even for the ragged surfaces consisting of positive and negative surfaces divided by sharp edges.

CONCLUDING REMARKS

The aim of this study is to clarify the reason why the shapes of origami works are generally characterized by ragged surfaces consisting of convex and concave surfaces divided by sharp edges. The following conclusions are obtained.
(1) The ragged surface consisting of complex sharp folds is the natural characteristic deformation of a thin sheet of paper.
(2) The area of the spherical image of a complex of folds, which is the integral curvature of the domain, must vanished.
(3) The above condition is satisfied only by compensation of positive and negative areas of spherical images corresponding to convex and concave areas on the surface of origami. A more rigorous mathematical approach is necessary to extend the present study.

ACKNOWLEDGMENT

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HOMMAGE À MIURA

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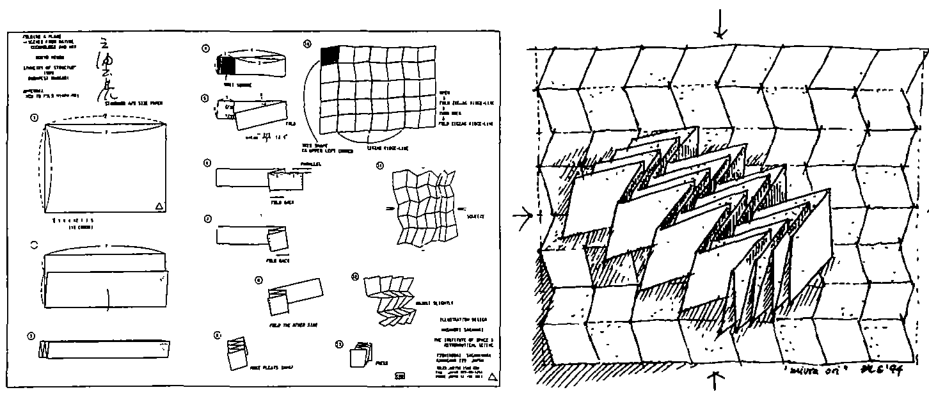
According to an old tradition in my native country of Germany, every young lad had to spend several months or even years of his life as a *Fahrender Geselle*, or 'travelling apprentice', going from one master to the other in order to gain knowledge and perfection in his profession. My favourite Grimm's fairy tale, lengthily entitled in German *"Tischlein-deck-dich und Knüppel-aus-dem-Sack"* (Table, cover thyself and stick, out of the sack) tells of one such fellow who worked in the shop of a cabinet-maker. Once he had finished his apprenticeship, his master gave him the farewell gift of two masterpieces: a table, that looked quite ordinary, but that was covered with the nicest and tastiest dishes imaginable at the simple command of its new owner, and a club that occasionally flung out of its sack to punish violently, but with great justice, any rascal, who pierced the secret of the table and tried to take it for himself.

Nowadays is there any sense in looking for a master? Haven't we got the quickest and most efficient techniques in history? Techniques, that give us access to a huge mass of information? Why look for a man whose knowledge and craftsmanship must be transmitted through personal instruction? One may argue that information appears in an incoherent and random order. To discriminate, we need a guide and a master. If that is true, how may it come about? It happened to me when I met Koryo Miura.

Strangely enough, when I met Miura, whom I now consider as my master, I was unaware of my apprenticeship. It happened in August 1989, at a congress in Budapest, organised by the ISIS-Symmetry Society. Miura, at that time professor and director of the Department of Air & Spacecraft Engineering at the University of Tokyo and director of the Laboratory on spacecraft engineering research at the ISAS (The Institute of Space and Astronautical Science) in Sagamihara, gave a 30-minutes lecture and, in the evening of the very same day, a short workshop that lasted another 15 or 20 minutes, on the theme of Folding a plane – Scenes from nature, technology and art (Miura, 1989).

A WORKSHOP ON *MIURA-ORI**

Miura presented his very impressive invention of a folding technique he called *Miura-ori*, whose mechanical principle he could demonstrate with a simple sheet of paper. At the very first sight of it, exposed on a table under a poster in the congress hall, the folded paper intrigued me. Throughout the conference it became more and more astonishing and during the workshop that I attended in the evening, simply marvellous. Miura proposes using this folding technique for lightweight structures (the sandwich panel 'zetacore'), for guided folding devices such as the solar energy collectors of space platforms, and for so-called 'solar sails' driven in space by means of solar wind, i.e. by the pressure of the Sunlight (Figs. 1). In all cases, a simple and reliable device had to be found. The paper folding, made from a single sheet, that opened by diagonal stretching without jamming and that closed again so easily, without causing fatigue to the hinges, contained all the secret of Miura's art. Like the marvellous table in the Grimm tale, this sheet of paper revealed to its new owner not only the physical properties of a folded solar sail but the secret of the whole treasure of eastern origami and the culture surrounding it.



Figures 1: (a) The original folding exercise of the *Miura-ori*; (b) A folding obtained by squeezing (bilateral compression).

In order to practice the folding of his *Miura-ori* during the short workshop, Miura first gave us an A4 size copy of a map of Budapest, which we were to fold using the city's points of reference. Then, when we were sure to do it correctly, he gave us a yellow, A3 size sheet of paper with the instructions printed directly on it. When I showed my final 'apprentice piece' to the 'master', he signed it kindly with his

* Note: *ori-gami* means Folding-the-Paper (*ori*: folding and *kami*: paper), hence *Miura-ori* means The folding of Miura Nature's folding techniques.

Japanese signature and gave me the authorization to teach *Miura-ori* to my own students. At the end of the workshop, I promised Mr. Miura to send him all the scientific papers I could find on folding of insect wings.

NATURE'S FOLDING TECHNICS

The foldable insect wing is a fascinating topic. For several years I regularly spent all my summer holidays on the Mediterranean coast in the South of France, where I could directly observe flying insects and study their wings and the patterns of the venation with a magnifying glass and draw them. I was especially interested in the folding techniques that allow *coleoptera* to fold and unfold their delicately pleated membranes during take-off or landing (Figs. 2).

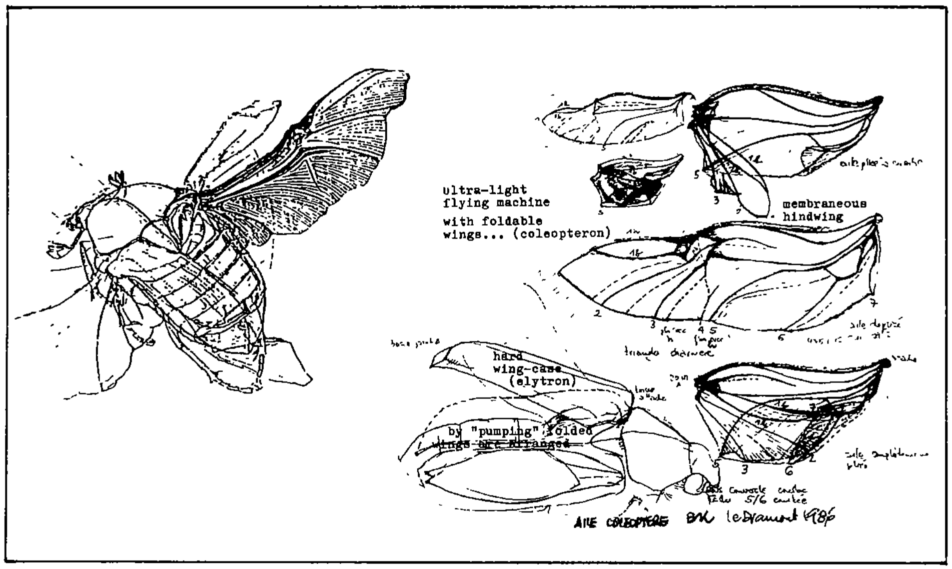


Figure 2: (a) Spread wings, ready for flight; (b) Folding mechanism in the membranous hindwing of a beetle (original drawings by B. Kresling).

For many years I also had the opportunity of following the research carried out under Werner Nachtigall at the Zoological Institute at the University of Saarbrücken, Germany. He is the author of *Insects in Flight*, a book that compiles information on this topic (Nachtigall, 1974). There, in the laboratories of Möhl, Dreher and Wisser, I observed the flight of the migratory locust (*Locusta migratoria*) and of the blue bottle-flies (*Calliphora erythrocephala*), using either stroboscopic flashing and videotapes, or oscillographs, or pictures from smoke-wakes.

Back in Paris, extremely happy at having Miura's instructions on the method and the algorithm of his folding, I began to introduce the topic in my own courses. But I soon realised, that I had promised Miura too much. I didn't understand anything yet about the folding of *coleoptera* wings and decided to learn more about biological *origami* and to show Miura only once I fully understood it myself.

Together with the students in courses on 'bionics' and 'dynamic folding' (designers, computer graphists and architects) we began exploring the particular mechanical behaviour of the *Miura-ori* and tried to find analogous mechanical principles in natural structures:

MIURA-ORI AND THE BUDDING LEAF

When pulled in one direction, the *Miura-ori*, though an elastic structure, doesn't shrink, but on the contrary also stretches out perpendicularly. We discovered that such a 'negative Poisson's ratio' is also characteristic of leaf development: after the two lobes of a budding leaf of a tree are freed from the *involucra* and opened up, the 'V'-shaped pattern of the leaf's ribs constrains the folded membrane to stretch symmetrically outward from the central vein. Since growth largely depends on mechanical constraints, the 'V'-shaped pattern similar to a detail of the *Miura-ori* provokes elongation combined with symmetrical outstretching (Fig. 3).

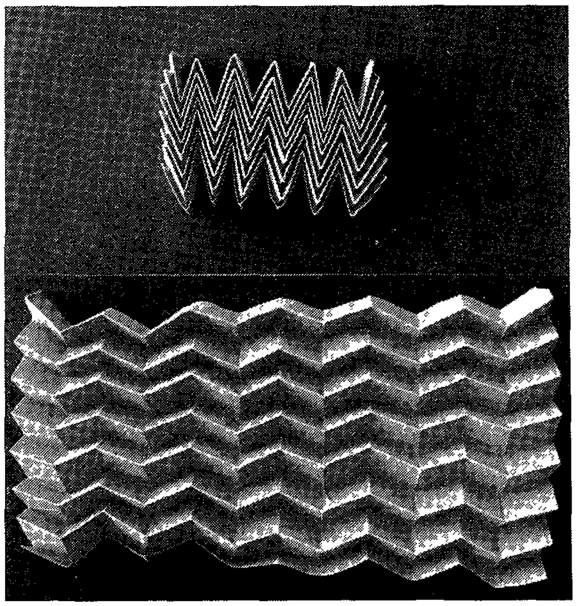


Figure 3: Negative Poisson's ratio of the *Miura-ori*: (a) Opening the 'V'- shaped angles equals (b) stretching out the whole surface.

MIURA-ORI AND ZIGZAGGED CENTRAL LEAF VEINS

Mechanically speaking, a single straight fold acts as a hinge. This means that pivoting neither creates shear stress nor deforms the neighbouring faces. But considered as a whole, the folding system of the *Miura-ori,* (which presents three series of zigzag-lined folds), transmits a deformation of any one part to any other part of the surface. The 'W'-shaped folds act in the same way as curved folds: when folded or stretched, convexity and concavity respectively decrease or increase. The folds relieve shear stress. The surface behaves as an elastic kinetic chain, and is synergetic.

Pivoting of any part provokes either (1) a general change of all dihedral angle values (while the faces remain flat), or (2) changes that are more geometrically and mechanically complex (while the faces are twisted). Whereas in the first case the released surface automatically finds its state of minimum energy expenditure, in the second case the surface is constrained and stores a great amount of energy.

The mechanical property of a simultaneous change in all dihedral angles type 1 is used for securing the middle-layer in Miura's sandwich panel 'zetacore' (Fig. 4a). In a bending-test, however, the axes of symmetry constitute failure lines, since the stability depends on the precision of the points of intersection. The resulting deformation is analogous to certain leaf-patterns (Fig. 4b).

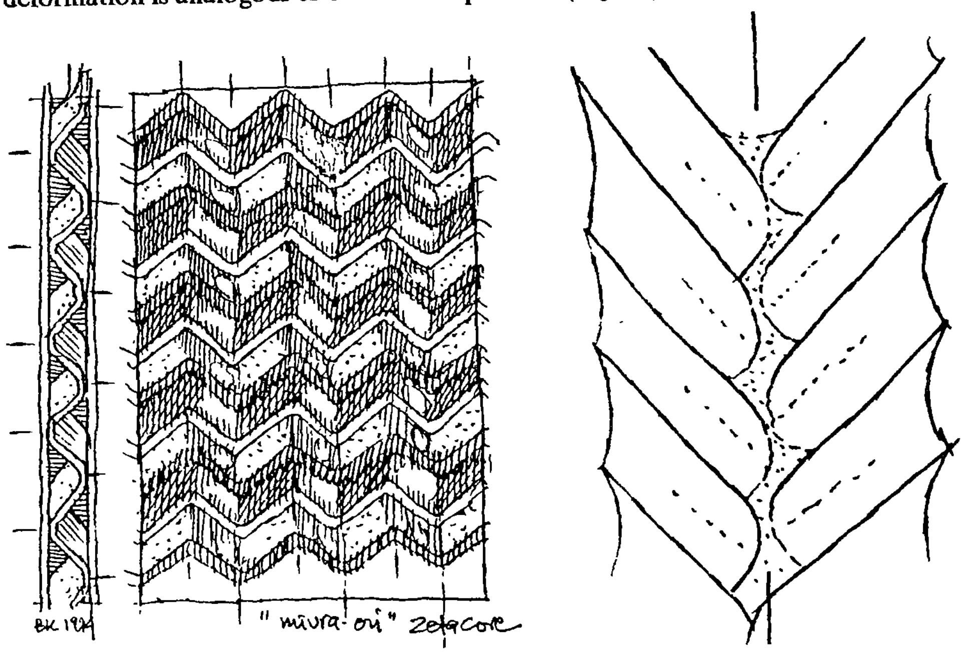


Figure 4: (a) The sandwich panel 'zetacore'. Used as a middle-layer between two flat faces the *Miura-ori* stiffens the panel. (b) The axes of symmetry constitute failure lines, analogous to certain leaf-patterns

The property of the energy-storing twisted faces type 2 may be used for prestressing a structure, now able to support heavier loads. In nature, several such 'prestressed' patterns help to 'control' the mechanical behaviour of a system:

– 'V' shaped or curved folds prevent a growing structure from reversing itself from convex to concave or collapsing (as in a budding leaf, Fig. 5)

– A *Lambda*-shaped pattern alternating with a 'V'-shaped pattern creates axes of shear stress and helps to form palmate leaves (Fig. 6) or lobate leaves (Fig. 7) (Kresling, in prep.).

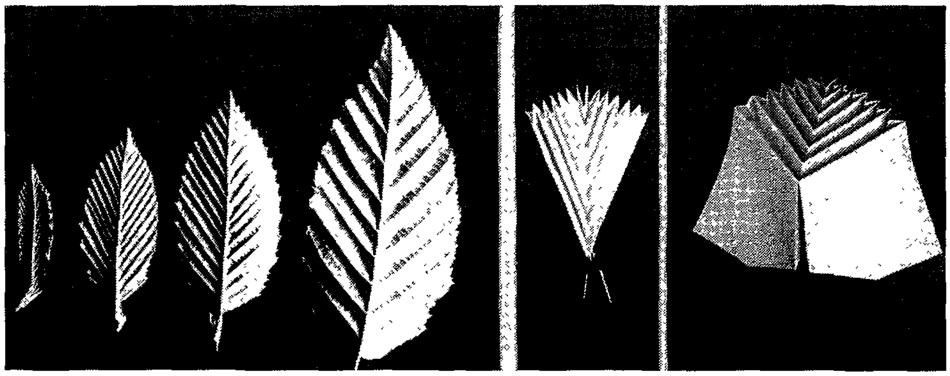


Figure 5: Simulation of the budding mechanism in a leaf: parallel secondary ribs (leaf of the hornbeam).

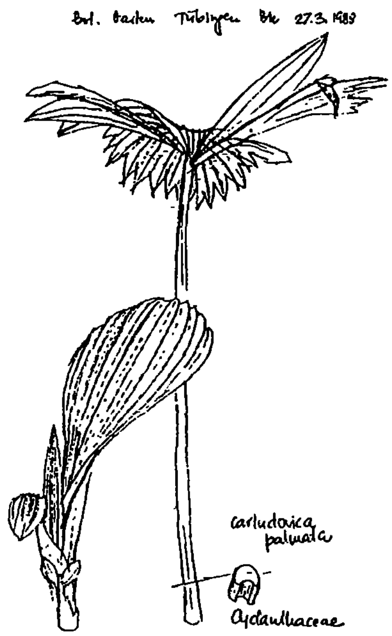


Figure 6: Development of a palmleaf: concentric primary ribs stiffen the folds. The membranes tear off at the periphery, thus bending the fan.

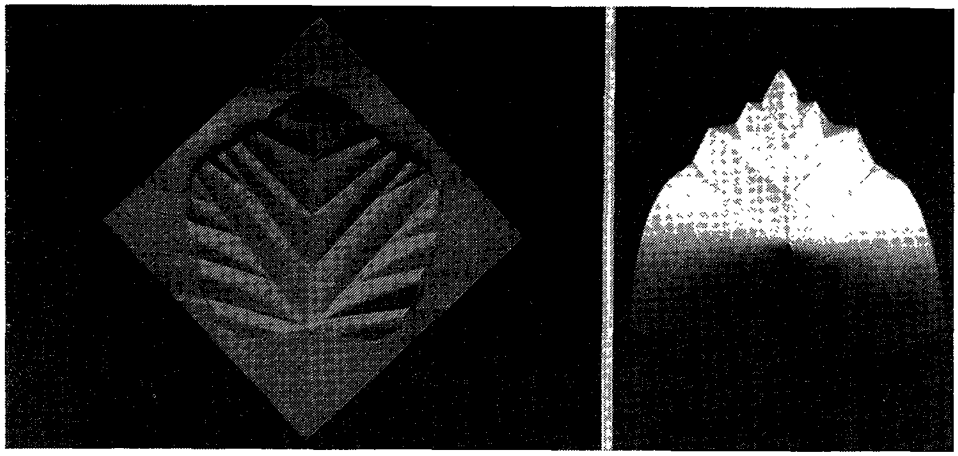


Figure 7: Simulation of the budding mechanism of a leaf (lime or maple-tree): convergent secondary ribs deform the lobes from concave to convex.

MIURA-ORI AND THE MICRO-PLEATING OF A BIOLOGICAL MEMBRANE

In October 1992, I attended a symposium in Stuttgart, Germany, on 'Natural Structures', organised by the SFB 230, a research-group with wide-spread interests on engineering, architecture and biology. After my conference on "Folded structures in nature – Lessons in design", Helmut W. Meyer from the medical department of the University of Jena, Germany, asked me for a model of the *Miura-ori*, that I had presented. He told then me, that he was studying the micro-texture of biological membranes by freezing and breaking the samples. In an early state (the so-called Pβ-phase) between fluidity and viscosity, when a biological membrane gets defined, certain double-layer lamellae form ripples, others have patterns similar to the papier-mâché egg cartons, but here were certain patterns whose texture was similar to the *Miura-ori* (Fig. 8). When he slipped the folded piece of paper into his pocket, he said that he was very happy at the coincidence, because for a biologist it was important to have a model that could explain the possible mechanism of such a pattern.

MIURA-ORI AND THE INSECTS WING FAN

The mechanical behaviour of a paper fan is of the second type (2). The bamboo sticks and the paper strips interact and deform one another. Such a structure is bistable: at two positional extremes – outstretched or folded – the system is at rest. In order to move the system from one extreme position to the other, energy is needed. We discovered that a similar effect is responsible for the opening of insect wing fans. More or less convergent curved folds provoke the opening of the folds near the thorax in to a stretching motion of the distal fan. The folding system of the insect wing also corresponds to the *Miura-ori*, but this time is used as a fan with convergent zigzag-folds (Fig. 9).

Figure 8: Miura-ori type micro pleating of a biological membrane (Lecithin in the Pβ'-phase), by courtesy of Helmut Werner Meyer, University of Jena.

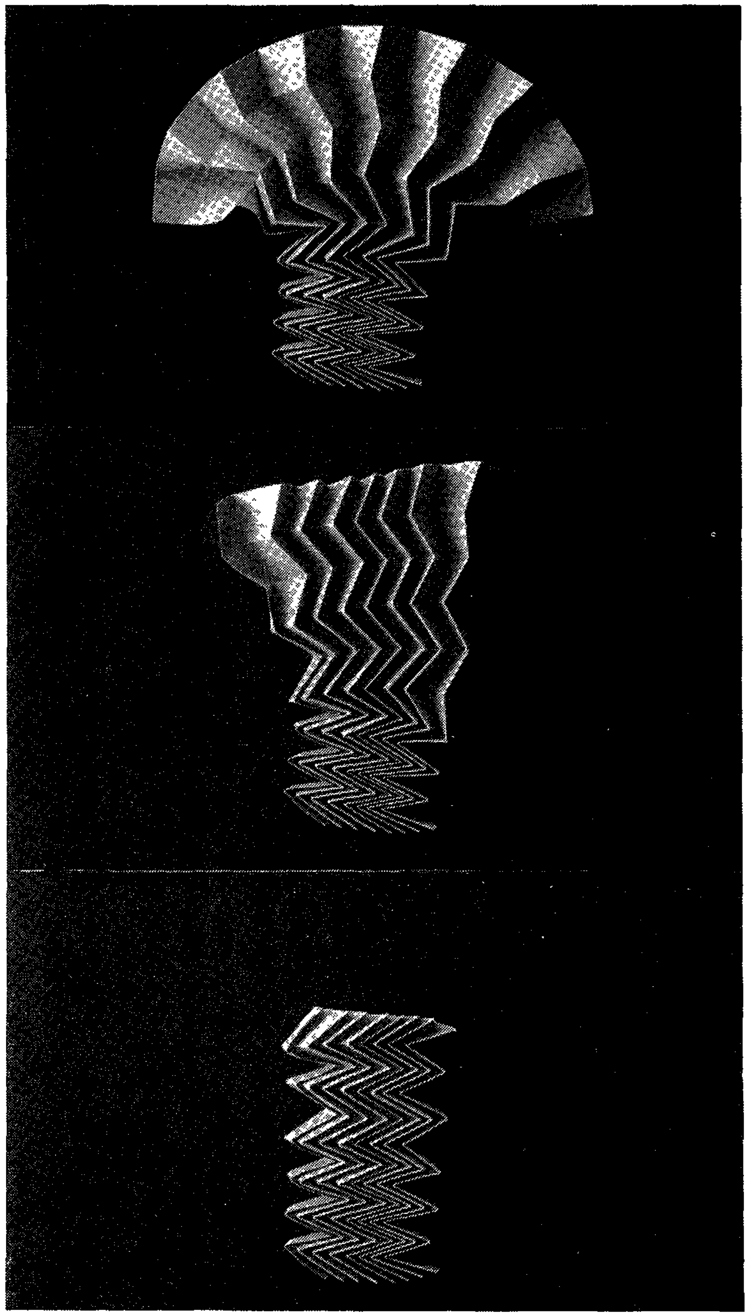


Figure 9: (a) The *Miura-ori* opened out like a fan, simulates the mechanism responsible for the outstretching of the beetle's membranous hindwing.

To complete the picture, we would like to point out that, as far as closing again, the folding mechanism is not fully automatic: it must be assisted. By using 'pumping' motion in its abdomen, the beetle fits its wings around the protuberant main hinges in the hard wing-cases (the elytra), thereby folding its membranous wing like a parachutist folds his parachute.

We have applied the mechanical principles we analysed in *Miura-ori* and in natural structures to industrial design. For instance, in order to stabilize folded lightweight structures and to avoid failure caused by weak lines or weak points, we used a new technique: a surface with curved folds which is (a) constrained and (b) combined with its own system before deformation. This technique suits tubular structures especially well. Current research is shown in Figure 10.

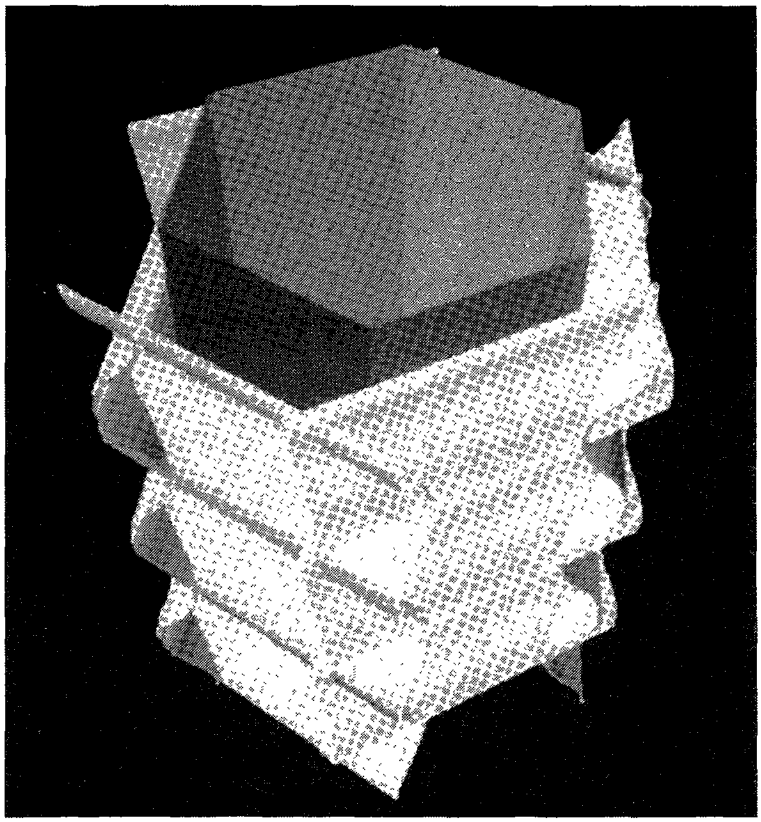


Figure 10: Computer simulation by N. Maillard (student in courses on 'dynamic folding' at Supinfocom Valenciennes, 1992) of the stabilization technique of a tubular structure. Curved fold lines equal interlocked corrugated surfaces. The risk of failure due to deformation of the elastic prestressed structure (increased shear stress of the curved folds) is avoided by combining with a second straight-folded tube.

DESIGN BETWEEN STABILITY AND FAILURE

In August 1992 I made my first trip to Japan. On the way to the 2nd ISIS-Symmetry Congress at the Synergetics Institute in Hiroshima, I was happy to stop in Sagamihara and meet Koryo Miura at the ISAS-Institute. He was able to measure the amount of information transmitted by the single sheet folded in the *Miura-ori* technique when looking at the number of foldable structures that our European students had designed simply by analysing its mechanical principles. For us Europeans *origami* meant making simulations with the sensitive material of paper – "showing constraints directly, it's a material that does not lie" – and working out our design projects on the limit between stability and failure (Figs. 11-22).

While a great part of Japanese origami enhances the elegance of algorithms and creates charming pieces out of neatly folded parcels, we actually crush structures in 'one-second-experiments' and observe their spatial development and stabilization, as well as its load-carrying capacity. At the ISAS-Institute, I discovered, that our students had unknowingly developed several tubular structures quite similar to the Yoshimura-patterned designs that Miura had proposed as models for under-water shelters and for supersonic fuselages.

Over the last two years, ever inspired by the lessons of Miura's masterpiece, we have worked out other sandwich panels and stable tubular structures. We have also turned to another extreme of technical origami, provoking failure: controllable destruction and recycling of space-taking design.

In spring 1995, the German biologist Fabian Haas plans to come to Paris for research on the mechanics of the folding systems in *coleopteran* wings and its biological evolution, and to exchange ideas on folding with us. Eventually, it will have taken not less than six years to keep our promise: to send to Miura all we know about insect wing folding ...

Acknowledgement: I thank Letitia Farris-Toussaint for kindly revising the English version.

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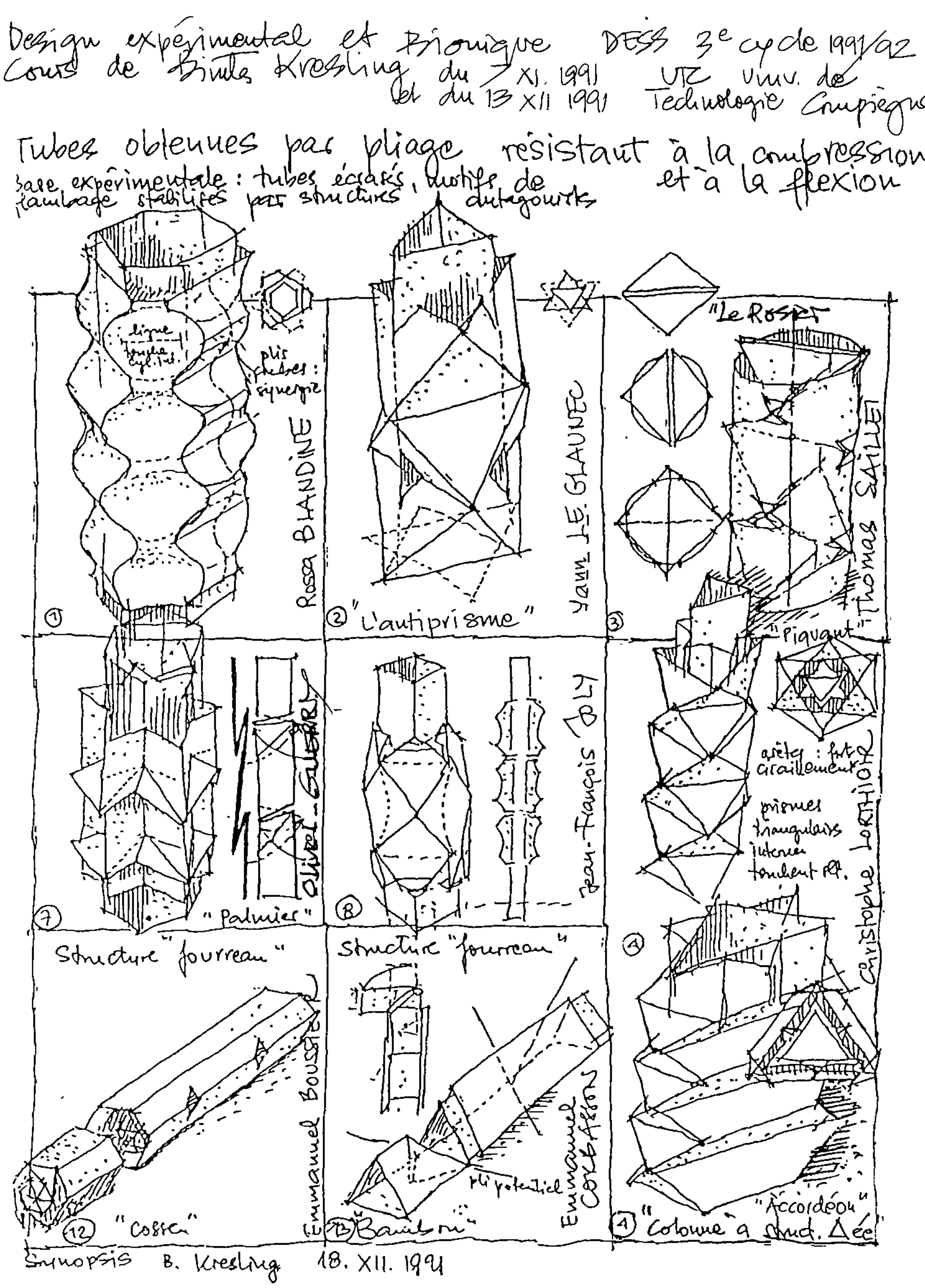
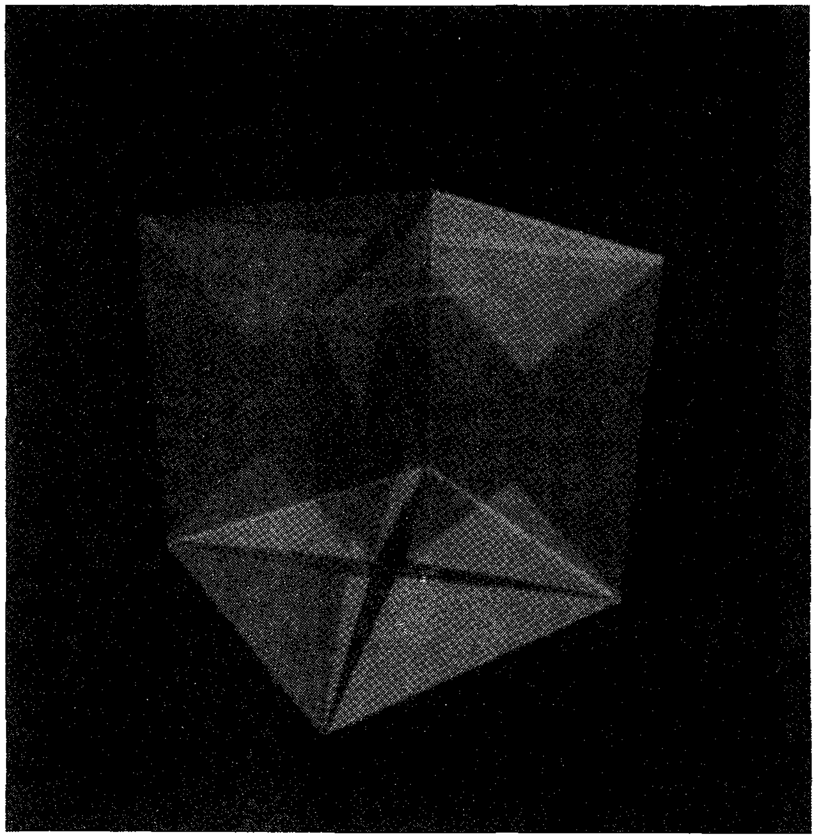
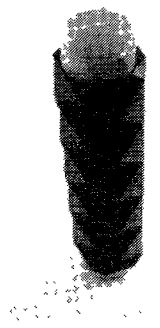
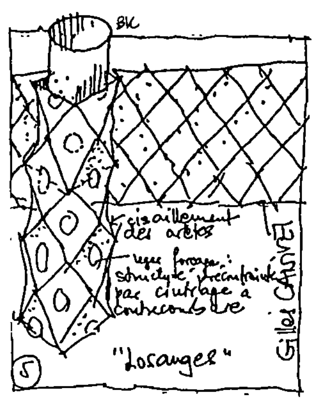
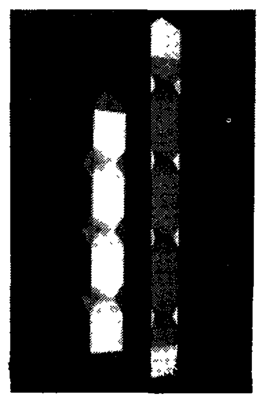


Figures 11-18: Foldable, but stable tubular structures: R. Blandine, E. Boussier, E. Corbasson, O. Guerry, Y. Le Glaunec, Ch. Lorthioir, Th. Saillet, students on bionics and experimental design at the University of Technology, Compiègne (1991-1992), (Synopsis B. Kresling).









Figures 19-22: Foldable, but stable structures: Multi-purpose packaging *Oméga-cube* (Patented 1990), D. Bourlet, B. Kresling and SupInfoCom, Valenciennes, computer simulation by N. Maillard. Tubes showing Yoshimura-pattern: G. Canivet, UTC Compiègne, N.N. ISD, Valenciennes (1991-1992), students on dynamic folding.

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BREAKING SYMMETRY: ORIGAMI, ARCHITECTURE, AND THE FORMS OF NATURE

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Abstract: *Most origami figures that attempt to replicate the forms of nature exhibit bilateral or rotational symmetry like the mammals, birds, and flowers they take as their model. In this paper I will show how I have attempted to capture some of nature's* asymmetric *forms such as leaves, mountains, and a coiled rattlesnake. Drawing analogies from nature and the formation of human settlements, I will examine different processes of growth and show how I have utilized those processes to 'break symmetry', transforming patterns that are symmetric into ones that are asymmetric. I will conclude by offering some subjective opinions on the philosophy and aesthetics of asymmetry.*

1. INTRODUCTION: THE SYMMETRIES OF THE SQUARE

It is easy to see why paperfolders are drawn to natural subjects such as birds, mammals, and simple flowers. The square, with its many symmetries, lends itself to capturing forms like these because they have bilateral or rotational symmetry. My own origami work for many years was devoted to discovering the seemingly limitless potential contained within the square's geometry. Inspired by the investigations of a handful of scientists and artists into the mathematical structure of nature (most notably by Loeb, 1971; Mandelbrot, 1982; Smith, 1981; Stevens, 1974; Thompson, 1917; and, more recently, Hargittai, 1994), I applied symmetry operations such as reflection, rotation, translation, change of scale, and the grafting of one pattern onto another to generate complex forms from simple ones. Some examples are shown in Figures 1 and 2. Such models of mine as a sailboat, motorcar, tortoise, reindeer, tiger, and elephant demonstrate bilateral mirror symmetry. A squid, scorpion, lighthouse, hot-air balloon, sun, and star employ rotational symmetry. A kangaroo, octopus, alligator, and butterfly use grafting and changes of scale. Three examples of symmetry operations taken from my first origami book (Engel, 1989) show how complicated forms like these were generated (Fig. 3).

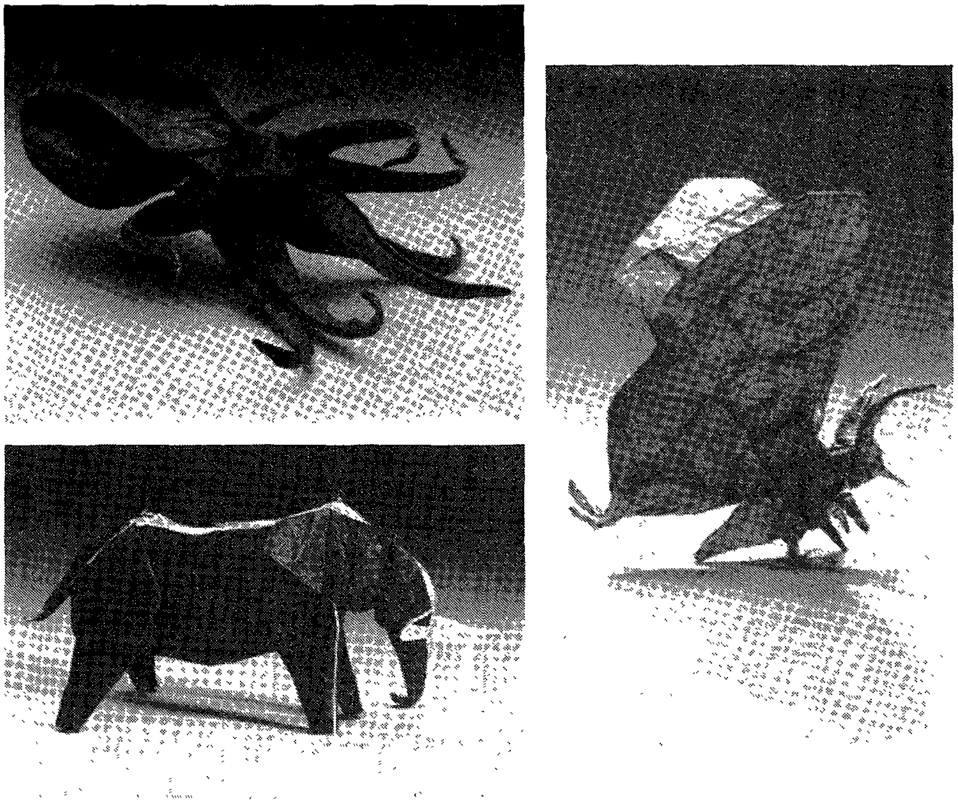


Figure 1: Some of the author's original models exhibit a high degree of symmetry. As befits an octopod, the author's octopus has eight-fold rotational symmetry as well as mirror symmetry. Its form is generated through a process of grafting. An elephant has only bilateral symmetry while a butterfly begins with four-fold rotational symmetry but in its final stage exhibits only bilateral symmetry, a process that may mirror the cellular development of the organism itself.

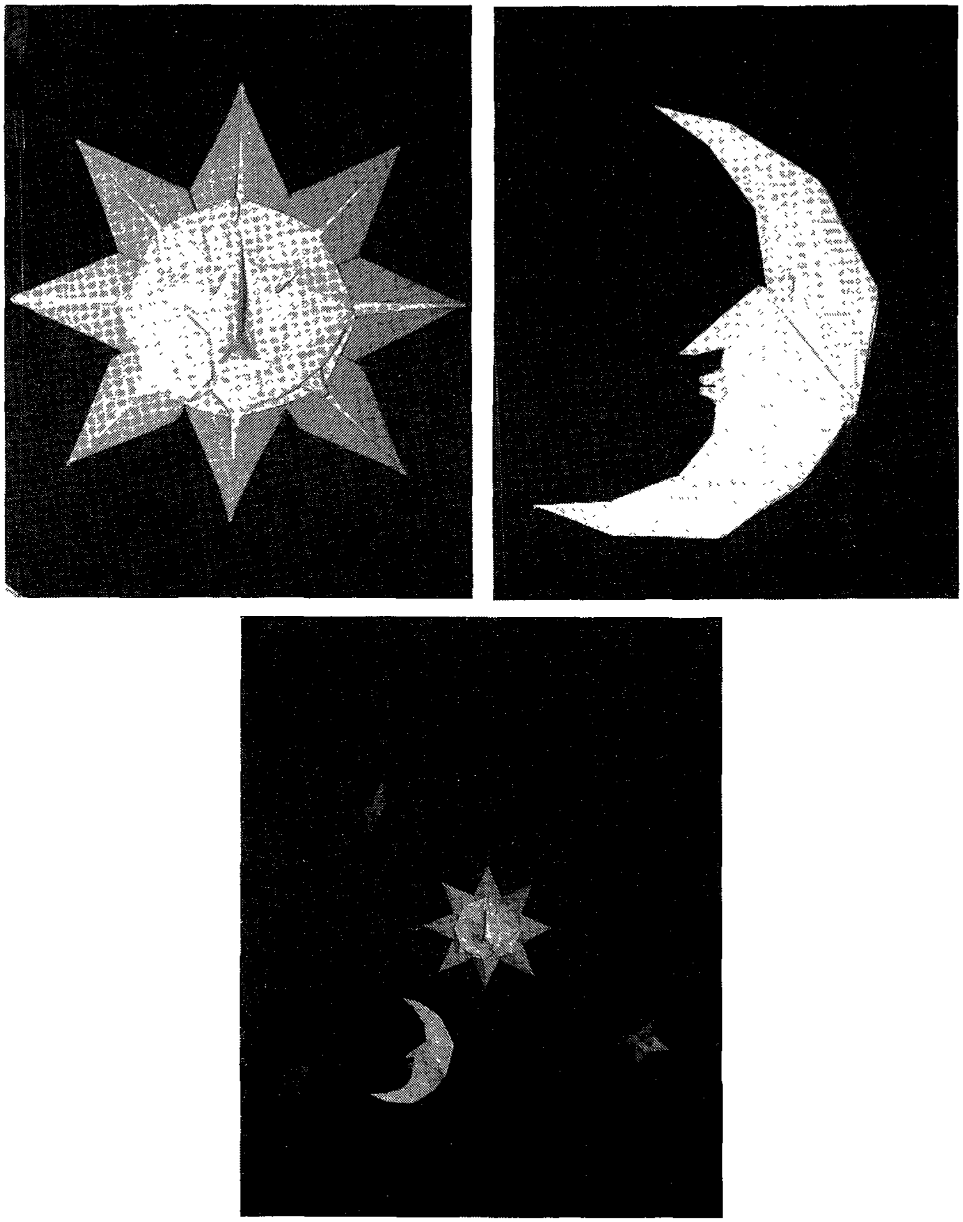
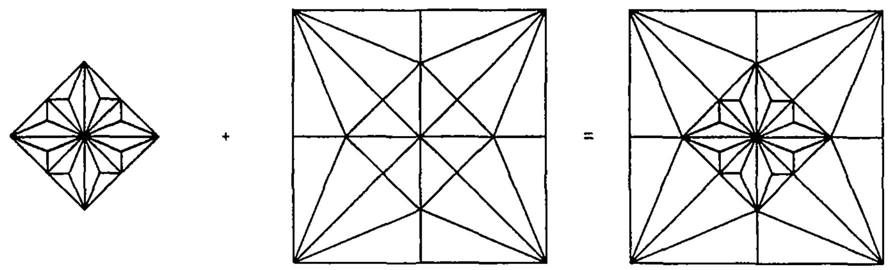
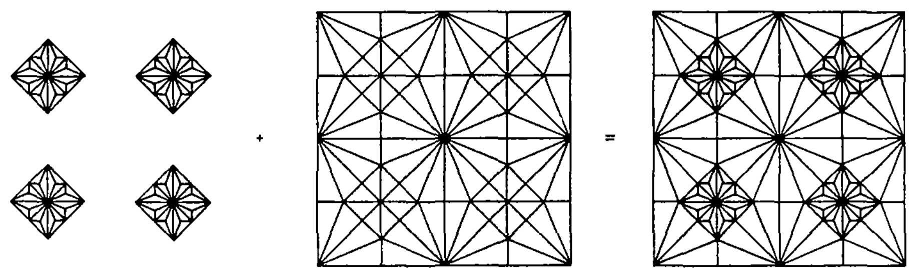


Figure 2: Mobile of a sun, moon, and stars reveals varying types of symmetry and asymmetry. The rays of the origami sun have eight-fold rotational symmetry and many axes of mirror symmetry, although the face is only bilaterally symmetric. In order to capture the twinkling of a real star, the origami star has rotational but no mirror symmetry, which tends to make an object appear static. Only the moon is asymmetric; although the real moon is a sphere, when it is lit by the sun from one side, as shown here, it takes on the asymmetry of a human face seen in profile.

Grafting a frog base onto a bird base.



Grafting four frog bases onto a blintzed frog base.



Replication of a hybrid module.

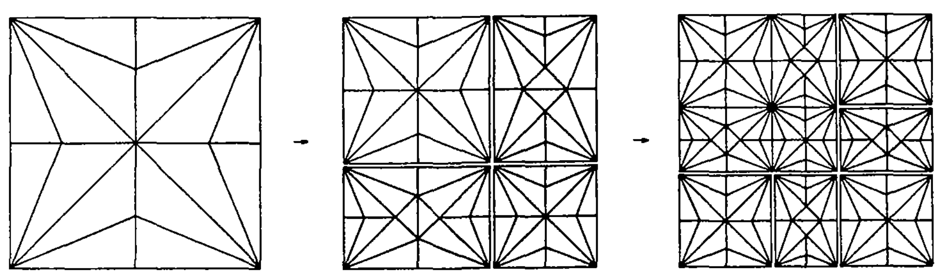


Figure 3: Three diagrams reveal the generation of complicated forms from simple ones by the application of symmetry operations. These diagrams, called folding patterns, reveal the network of creases in a finished model when is opened out to the original square.

In these investigations I have not been alone. Other authors such as Kasahara and Takahama (1985); Maekawa and Kasahara (1983); and Montroll and Lang (1990) were simultaneously exploring the symmetries of the square and venturing into areas unknown to me. The more recent accomplishments of paperfolders such as Shuzo Fujimoto, Toshikazu Kawasaki, Fumiaki Kawahata, Seiji Nishikawa, Chris Palmer, Jeremy Schafer, and Issei Yoshino, to mention just a few with whose work I am familiar, reveal the extraordinary riches that this geometrical approach has mined.

2. THE ASYMMETRIES OF NATURE

But while the unfolded square clearly offers a treasure trove of symmetric patterns, the forms of nature rarely live up to the purity and perfection of the square. Nature is, in fact, full of imperfections: twisted branches and vines, gnarled trunks, varied and ever-changing cloud formations, the mottled coloration of hair and fur, the ragged chasms and promontories of a mountain chain. The branching of trees and of river deltas, the dividing of cells and of soap bubbles, the multiplying of whorls in water and of puffs in a cloud, the cracking of mud and of eggshells, and the slow accretion of crystals and of chambers in a snail shell are all examples of forms that are coherent even though they may lack clear-cut symmetry. What all of these forms do possess, however, is great beauty, a beauty borne not of their perfection but of their imperfection – of the delicate balance they maintain between order and chaos.

As an example of how asymmetric patterns occur in nature, consider the cracking of a drying drop of oil. First we behold the fluid, formless drop. Outside, its black skin glistens; within, its molecules swim in a vast, slippery sea. Now the drop falls, it lands on a smooth, flat surface and slowly dries in the sun. As time passes, it contracts and cracks. At first, there may be only a single, large fracture that extends in a crooked arc across the surface of the shrinking oil. Then, smaller fissures occur. They meet the first fracture at angles that are close to perpendicular, but never exact. As the oil continues to constrict, dry, and flake, still smaller lines appear, bent and fragmentary, filling the empty gaps left by the earlier cracks (Fig. 4). The result is a complex and beautiful pattern – an asymmetric pattern – that is poised somewhere between order and chaos.

Take a second look at the pattern of cracks in the dried oil drop and imagine, for a moment, that you are in an airplane looking down at the ground. What do you see but the myriad patterns of human habitation – boulevards and avenues, streets and alleys, the entire circulation network of an ancient human settlement. The resemblance to an aerial photograph of a desert town in Iran (Fig. 5) is uncanny. What do these similarities tell us? Are they merely accidental, or do they reveal a larger truth about the forms created by nature and by man? As an architect as well as a paperfolder, I was truck by these similarities while I was conducting architectural

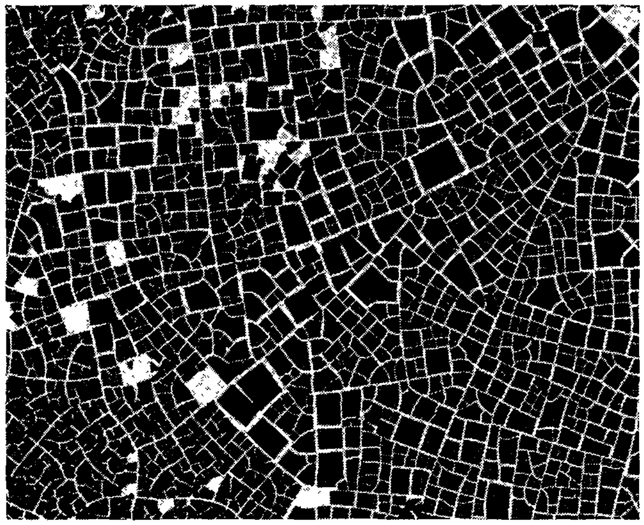


Figure 4: Typical cracking pattern in nature, the fissures in a gelatinous preparation of tin oil, has order but no symmetry. (Copyright by Manfred Kage/Peter Arnold Inc.)

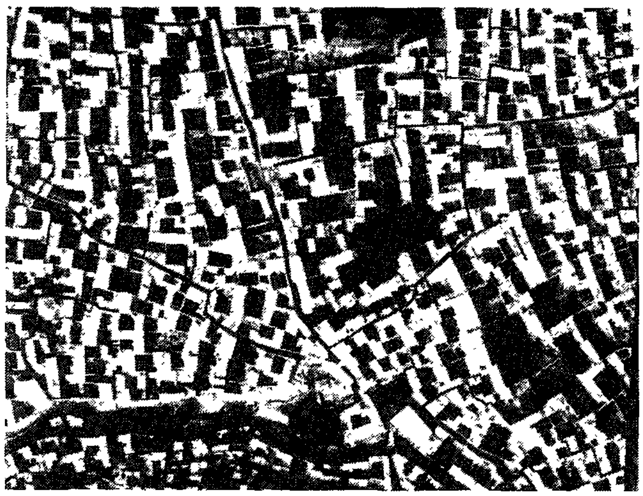


Figure 5: Aerial photograph of an Islamic settlement, the town of Hamadan, Iran, bears an uncanny resemblance to the cracking of a drop of oil.
(Copyright of the Oriental Institute of the University of Chicago.)

work for several years in India and Sri Lanka. In stark contrast to the rigid, highly geometric forms of most contemporary housing designs, the ancient human communities that I visited felt as natural as the actual products of nature. And because they were equal parts order and chaos, they were extraordinarily beautiful.

My investigations revealed that traditional human settlements come into being in much the same way as the forms of nature. While traditional settlements vary tremendously with region, culture, and climate, they are nonetheless the product of a few consistent environmental, technological, economic, and social rules. Like the cracks in the drying oil drop, a traditional community is formed incrementally, through many small modifications and interventions, over a prolonged period of time. A settlement may begin with only a single path, a path that connects two nearby towns or the center of town with a well or a watering hole. As the settlement grows, the path may be enlarged or paved to form a street, and a bazaar may form along its sides; new footpaths may be added that extend in opposite directions, connecting the street to a new temple, church, or mosque; additional homes may be built by individual families who are migrants to the town or the children of the original settlers. Over a period of time, families expand, families divide, the community expands, the community divides, and the buildings and streets multiply. Through this process of expansion and partition, the community takes the form of its maturity.

The organic quality we observe in nature and in traditional settlements is thus the result of *repetition* and *randomness*: repetition because a particular process has occurred repeatedly over the course of time, and randomness because it never occurs exactly the same way twice. The imperfections in the finished product demonstrate that the pattern was not hewn by a single creator or executed in a single master stroke. In the dried oil drop, for example, none of the areas of dried oil is square, but *almost* all have four sides. The cracks meet not at perfect right angles, but *nearly* so. Even that most repetitive of natural processes, the transcription of genetic information by DNA, is never perfect; the resulting mutations create the genetic variations that allow species to adapt and survive.

3 GENERATING ASYMMETRY IN ORIGAMI

If repetition and randomness are the blueprint for the forms of nature and the ancient settlements of man, then we should be able to employ them to capture the asymmetries of nature through origami. To do so requires paying close attention to the irregularities of nature, the erratic shapes and subtle curves that mark a form as organic and natural instead of machine-made and mechanical. But because we are working with a square of paper and not with living cells, crystals, or water droplets, our approach demands equally that we exploit the intrinsic properties of paper and square.

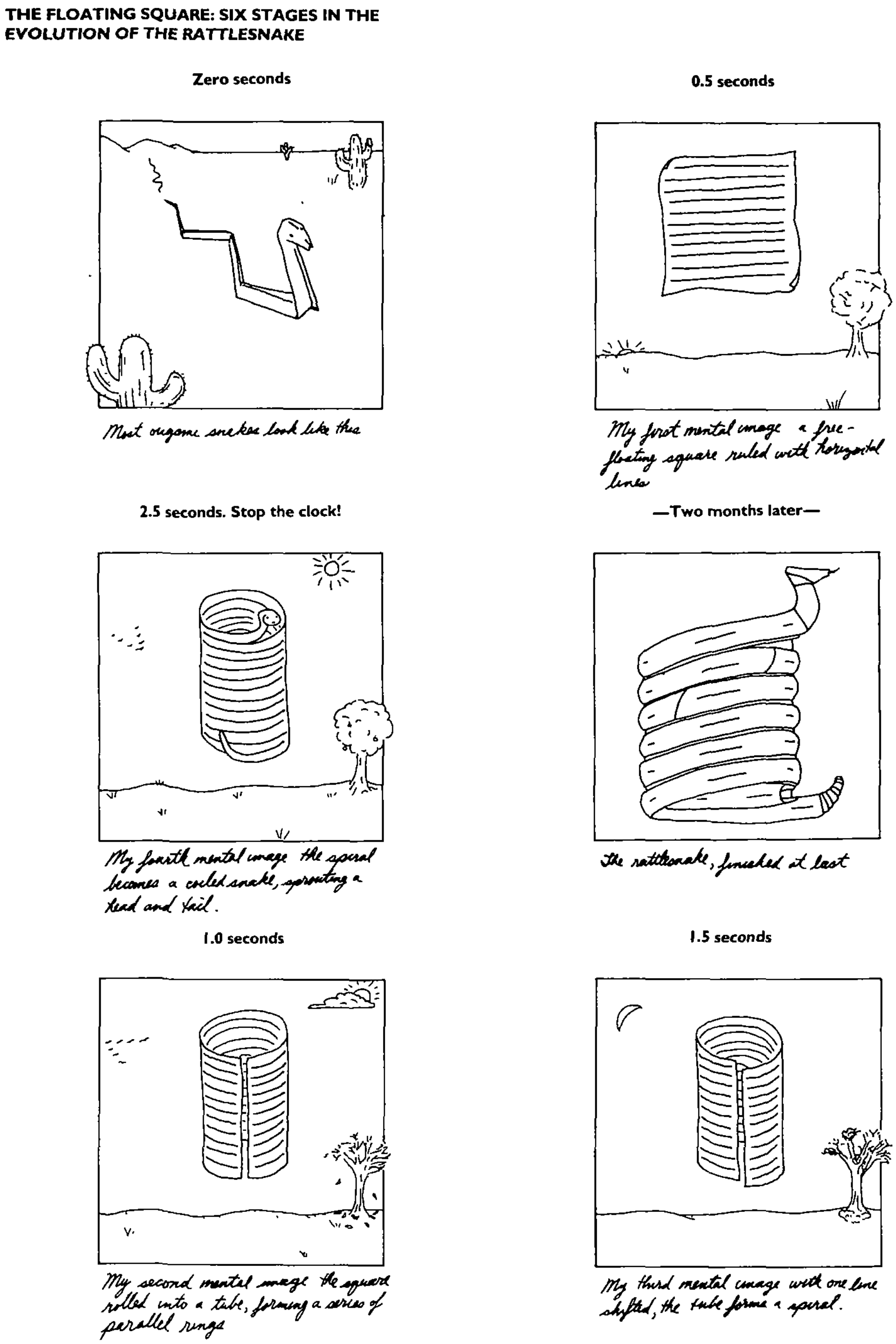


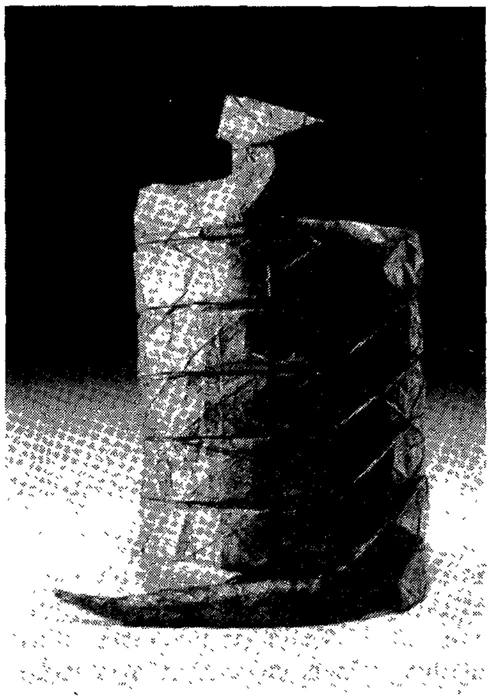
Figure 6: Conceptual diagrams illustrate the evolution of the author's origami rattlesnake.

My first example of breaking symmetry is one that I would consider an unsuccessful, early attempt in this direction. In my first origami book, I explained the process behind the formation of my model of a rattlesnake. As the conceptual diagrams I included to illustrate (Fig. 6), I wanted to create a snake that was different from all others in the origami repertoire. To make the longest possible snake from a square, other folders had lined up the body of the snake with the diagonal of the square and collapsed the two other corners accordion-style to narrow the body. That was it. Subtle variations in the position of the head and tail were the only clues to distinguish one model from another.

How was I to make my snake different? In order to begin, I conjured up images of snakes that I had seen and sought the traits that most stood out. What do snakes do that no other animal does? What aspects of their anatomy, their evolution, their movement define 'snakeness'? In short, what makes a snake a snake? Snakes slither, I thought, they undulate, they hang from trees, they strike – and they coil. I couldn't think of any other animal that coils. I made up my mind to invent a coiled snake.

At that moment, images flashed before me as my mind raced to find a solution. I saw a square of paper floating through space. On the square was a pattern of horizontal lines. I pictured the square rolled into a tube. The horizontal tubes turned into parallel rings running up and down the tube. Then, suddenly, in the conceptual breakthrough that was the decisive break with bilateral symmetry, I shifted the edges of the square by one line. Now, instead of rings, there was a spiral – one long coil running all the way around the tube, like the stripe on a barber pole. The pattern changed again; the edges sealed, and a head and a tail sprouted from each end of the spiral. I had my snake. The conceptual part was done. The rest of the execution required great ingenuity, but it never matched the simplicity and clarity of that first conceptual leap. To my mind, the resulting model (Fig. 7) is clever, but it is not beautiful. Because it remains bound by rigid geometry, it does not partake of the orderly chaos, the randomness within repetition, that marks the true forms of nature.

Before we leave the snake, it is worth examining the model's step-by-step diagrams to locate the exact moment in the folding process when the snake becomes bilaterally asymmetric. For readers familiar with the model, the decisive step is number 11 (Fig. 8). Up until this point, except for the tiny triangular flaps at the four corners of the square (which could, in fact, be turned in any direction), the model is bilaterally symmetric along the diagonal of the square. Step 11 asks the folder to swivel clockwise the long flaps that protrude from the top and bottom of the model. (These will be the head and rattle of the snake. If the flaps are swiveled counterclockwise, the snake will coil with the opposite handedness.) By layering rotational symmetry on top of bilateral symmetry, the bilateral symmetry is broken, never to be regained.



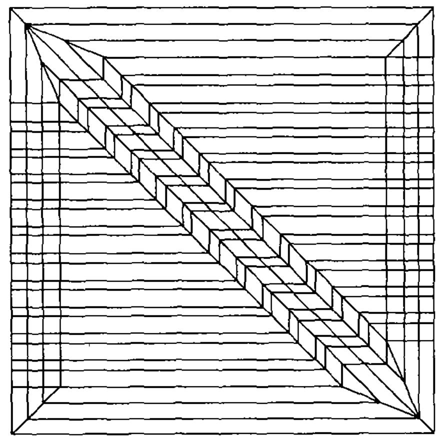


Figure 7: Completed rattlesnake is a helix that can coil clockwise or counterclockwise. Its folding pattern is similarly asymmetric.

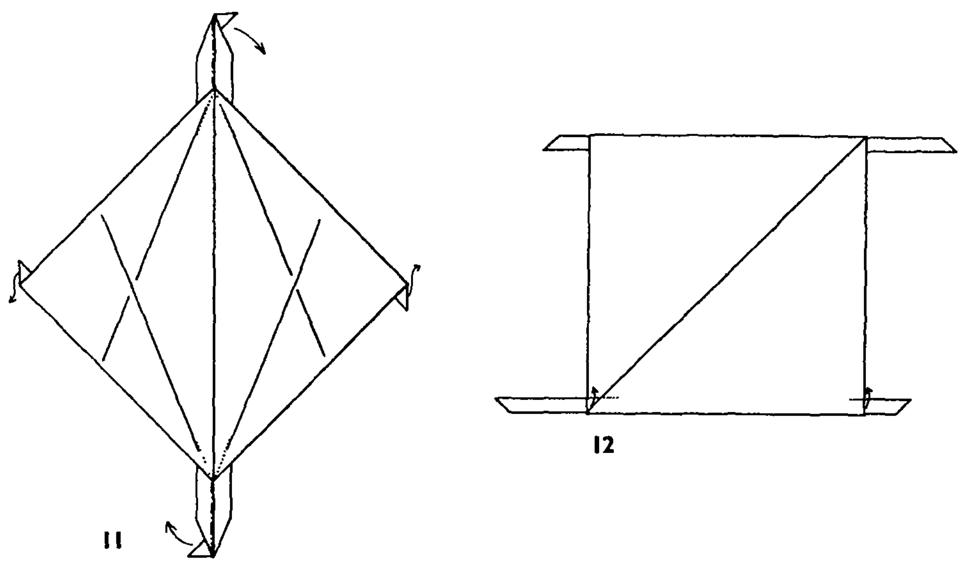


Figure 8: Between steps 11 and 12 of the rattlesnake, bilateral symmetry has been broken and only rotational symmetry remains.

My second experiment in breaking symmetry was constructing a mountain range. Again, as in my investigation of the rattlesnake, I began by letting images spin through my mind. I knew that in order to capture the ragged and craggy quality of rock, I would have to avoid obvious symmetries – nothing could look more artificial than a rock that is identical on all sides – as well as strict horizontals and verticals. But if the resulting figure was too uneven – if, say, I were simply to crumple the paper – it would lack the aspect of self-similarity that characterizes a real mountain chain. (Self-similarity, or scaling, is a property of forms that possess fractal geometry: when it is enlarged, each part resembles the whole.)

I concluded, then, that the mountains would involve a repetitive geometric pattern but still strive to appear 'natural' and 'organic' when viewed in perspective. After much experimentation, I produced a mountain that is fundamentally a pyramid with three sides and a bottom. To shape the different sides, I devised a spiraling sequence of 'closed-sink' folds that avoids unnatural-looking horizontal or vertical creases. The folds are similar in shape, but because they rotate and reduce in size with each turn, no two faces of the mountain are the same, and the resulting origami model is markedly asymmetric. It is possible to assemble dozens of mountains of varying sizes (and opposite handedness) to make a mountain chain that ranges from tiny foothills to towering peaks. When seen from directly above (Fig. 9), the sinks reveal their exacting geometry and do not look at all natural (though they do resemble the logarithmic spirals of a sunflower or snail shell). When seen in perspective, however (Fig. 10), they capture something of the orderly chaos of nature.

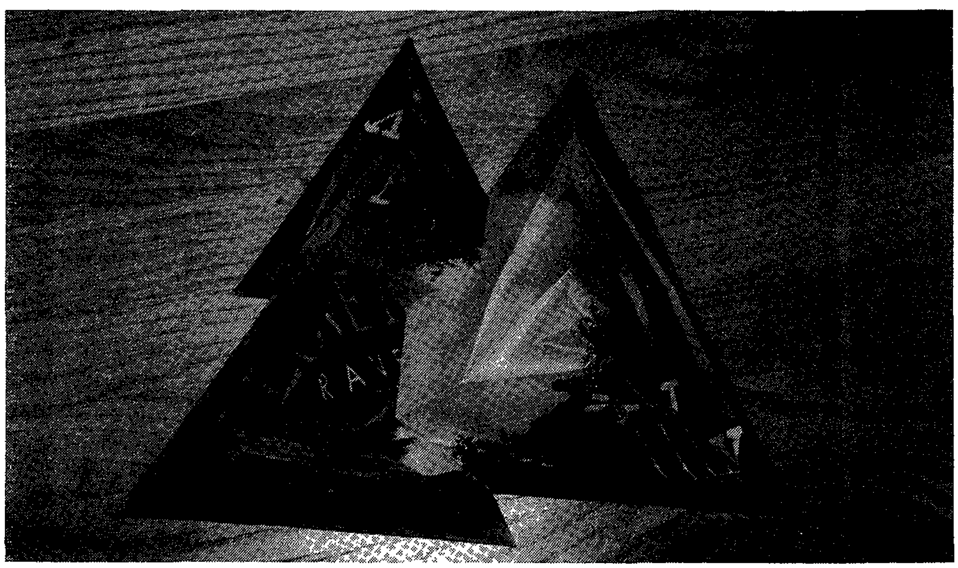


Figure 9: A view of the mountains from directly overhead reveals their exacting geometry. Note the spiraling pattern of closed sinks.

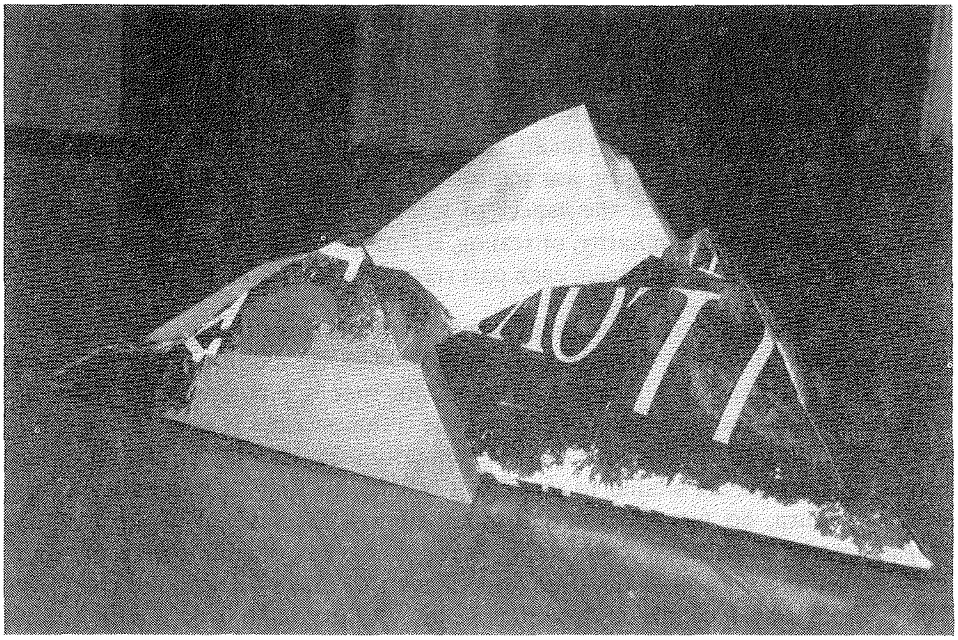


Figure 10: Seen in perspective, the origami mountains reveal the irregular crags and promontories, the 'orderly chaos', of their counterparts from nature.

The inspiration for my next investigation in breaking symmetry was the death, earlier this year, of my friend and fellow paperfolder, Mark Turner. Mark was an imaginative folder who furiously devoted his energies in the last few months of his life to creating new plant forms. Plants are a particularly challenging subject for origami. Their attenuated branches and stems, rounded leaves, shaggy fronds, and varied branching patterns constitute a poor fit for the taut geometries of the square. But in what appeared to be a single, sustained burst of invention, Mark originated a highly individual approach to folding and then, with the fastidiousness of a botanist, pursued its implications from family to family and species to species throughout the plant kingdom. His sensuous, curving plant forms breathed new life into the familiar technique of box-pleating, which to my mind once seemed destined to produce only replicas of matchboxes, cars, and modular furniture, the mechanical handiwork of man. Among Mark's accomplishments revealed in his as-yet-unpublished manuscript (Turner, 1993) are several models with bilateral asymmetry, such as the grasses and sunflower that possess an alternating branching pattern (Fig. 11).

As a tribute to Mark, I decided to undertake a model of a plant with great spiritual symbolism: a leaf from the tree *Ficus religiosa*, known throughout Sri Lanka and India as the *bodhi* or pipal tree (Fig. 12). 2500 years ago, Siddartha Gautama, the Buddha, achieved enlightenment while meditating beneath a *bodhi* tree. Since that time, the *bodhi* tree has been cultivated throughout the Buddhist and Hindu world as a symbol of *nirvana*, relief from suffering, which can only be attained by following the *dharma*, the code of thought and conduct laid out by the Buddha. The leaves of the tree have many variants, but are immediately recognizable by their elongated, tapering tip and asymmetrically branching veins.

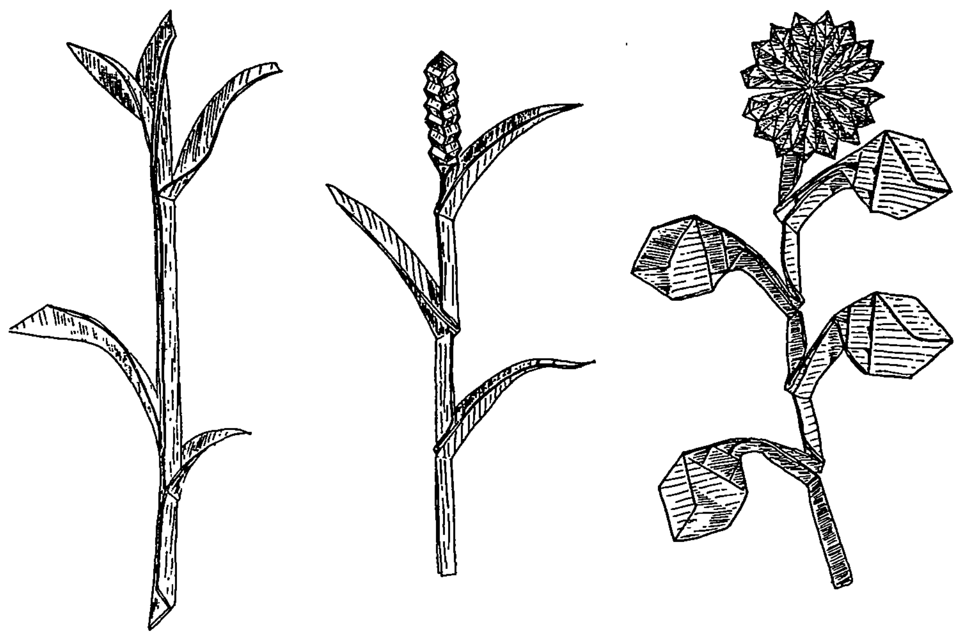


Figure 11: Mark Turner's grasses and sunflower have an alternating branching pattern.

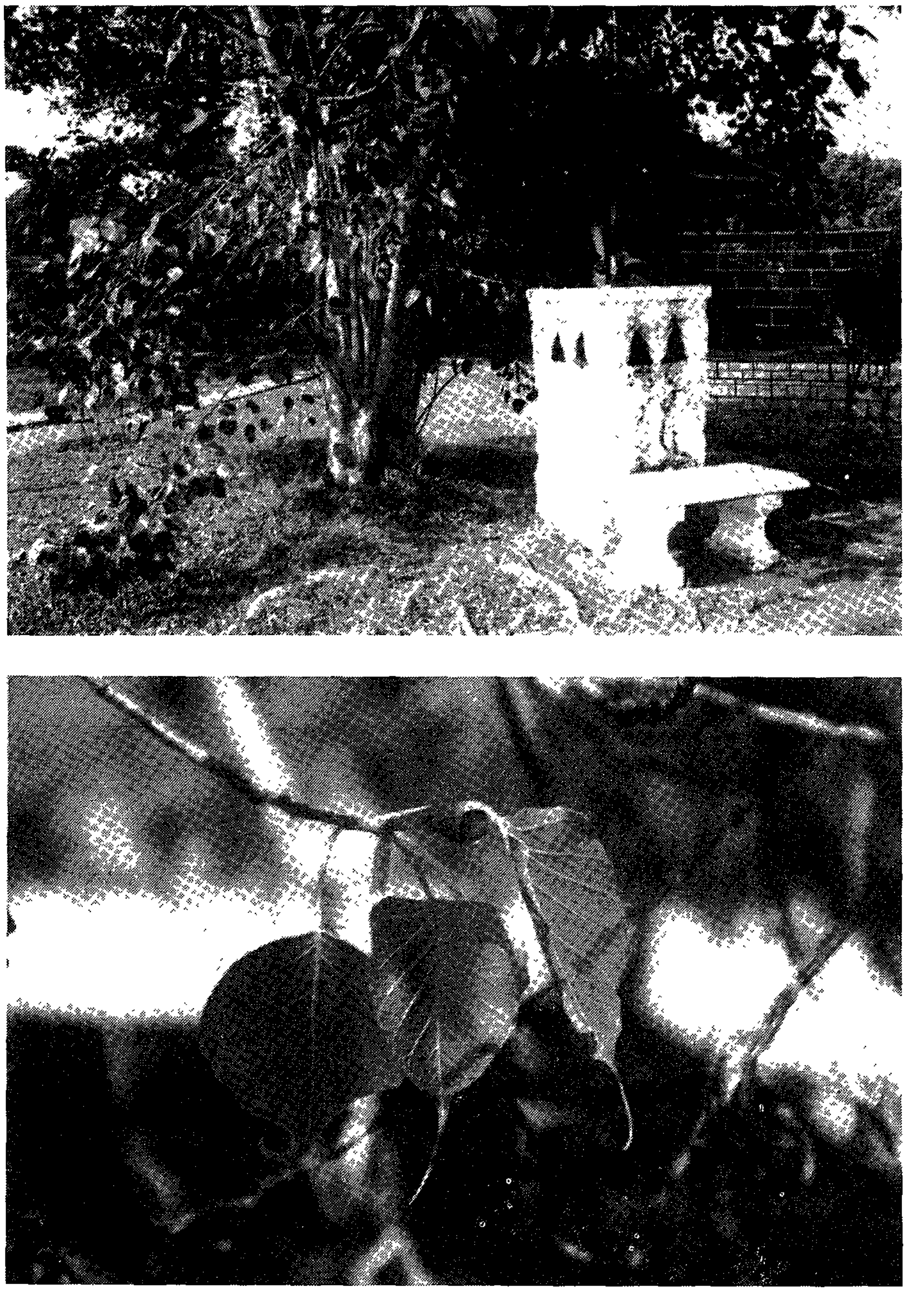
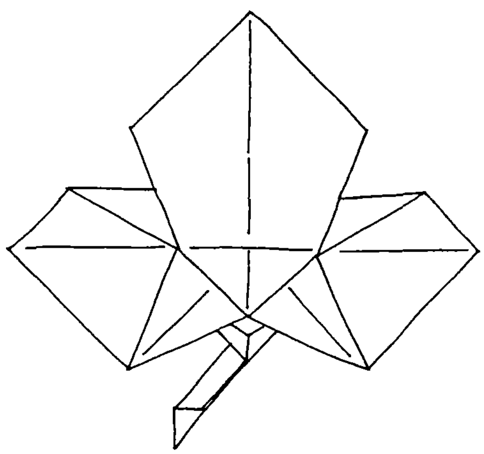


Figure 12: *Bodhi* tree with a statue of the *Buddha in samadhi*, or meditation, pose. Close-up of the leaves reveals their distinctive tapering point.

As with the rattlesnake and the mountain range, I began by sifting a series of images through my mind. Remembering the lessons I had learned from my study of natural forms and human settlements, I decided that the origami model would have to contain both systematic and random elements, and that it would begin with an orderly, symmetric structure and move gradually toward asymmetry. As I had previously designed a different form of leaf, a maple leaf (Fig. 13), I started from there. I eventually succeeded in devising a symmetric *bodhi* leaf (Fig. 14) and then set about uncovering a process by which to make it asymmetric.



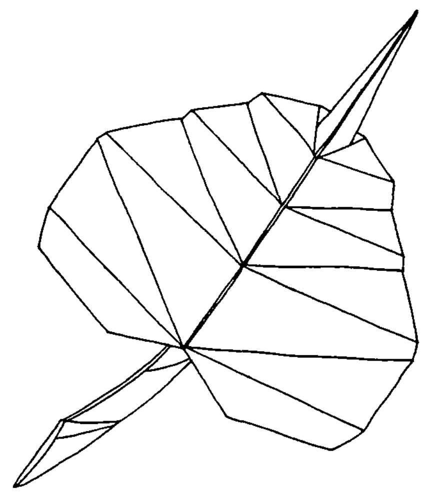


Figure 13: Maple leaf has bilateral symmetry.

Figure 14: First version of the *bodhi* leaf has bilateral symmetry.

But breaking symmetry did not prove easy! I gradually realized that to stagger the placement of the veins along the leaf's central spine required introducing a special mechanism, an operation with the paper that would shift the paper up on one side while shifting it down on the other. This mechanism turned out to correspond to a symmetry operation, introducing a small rotational symmetry within the larger bilateral symmetry of the leaf. In order to introduce rotational symmetry, I had to escape from the plane of the paper into the third dimension.

The operation is a kind of pinwheel. It begins by lying flat within the plane of the leaf, lifts out of the plane in order to rotate either clockwise or counterclockwise, and then collapses back into the plane when its work is finished. Just as the third dimension is required to turn a right-handed handprint into a left-handed one (or to lift a Flatlander out of his plane to turn him into his mirror image), the third dimension turns out to be a prerequisite for making the *bodhi* leaf asymmetric. In the completed leaf (Fig. 15), the operation is too small to be easily visible to an observer. But a sequence of photographs that I took of a larger prototype of the operation shows just how it works (Fig. 17). It was only in the preparation of this paper that I realized how much the operation has in common with the Iso-area folding theorem devised by Kawasaki and elaborated upon by Palmer and others. Clearly there is much fertile territory for exploration.

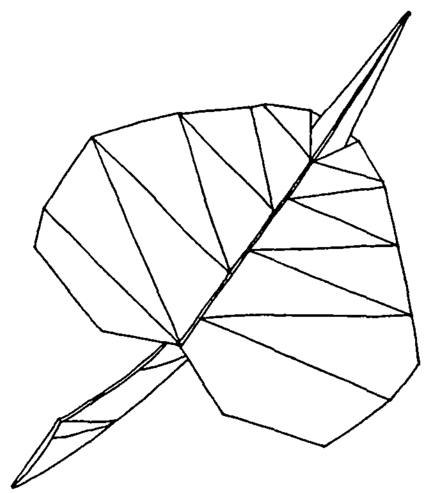


Figure 15: Final version of the *bodhi* leaf has an asymmetric branching pattern.

My last example is so new that the model is not yet completed! Seeking a completely different type of asymmetry to capture, I am now endeavoring to create a model of a leaf from the *begonia* plant. The *begonia* leaf (Fig. 16) at first appears so odd and disproportionate that it is hard to locate even the vestiges of symmetry. And yet the luminous patterns and varied textures of different *begonia* leaves clearly exhibit both repetition and randomness. A few minutes of scrutiny reveal the nature of this order (Fig. 18). At the point where the stem joins the leaf (this point is called the *hilus*), the two lobes are bilaterally symmetric. Similarly, the very tip of the leaf also exhibits bilateral symmetry. And yet these two axes of symmetry are rotated with respect to each other by as much as 90 degrees. When I examined dozens of leaves from different species of *begonia* plants, including tiny leaves just in the process of forming, I found this pattern of broken symmetry repeated over and over. How could it have come into existence?

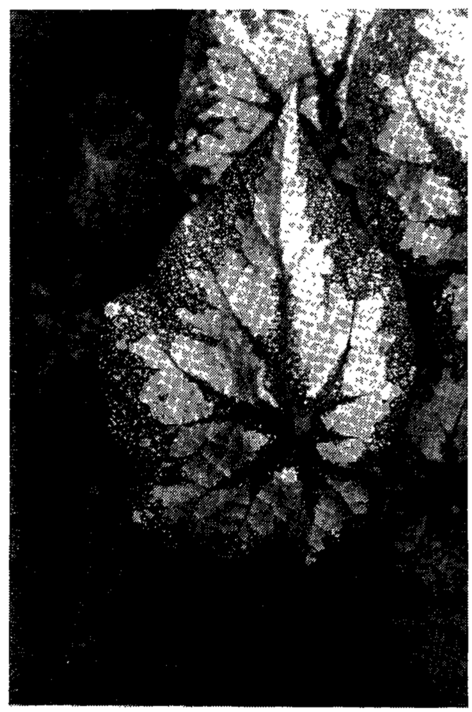


Figure 16: Two photographs of *begonia* leaves reveal their characteristic asymmetry.

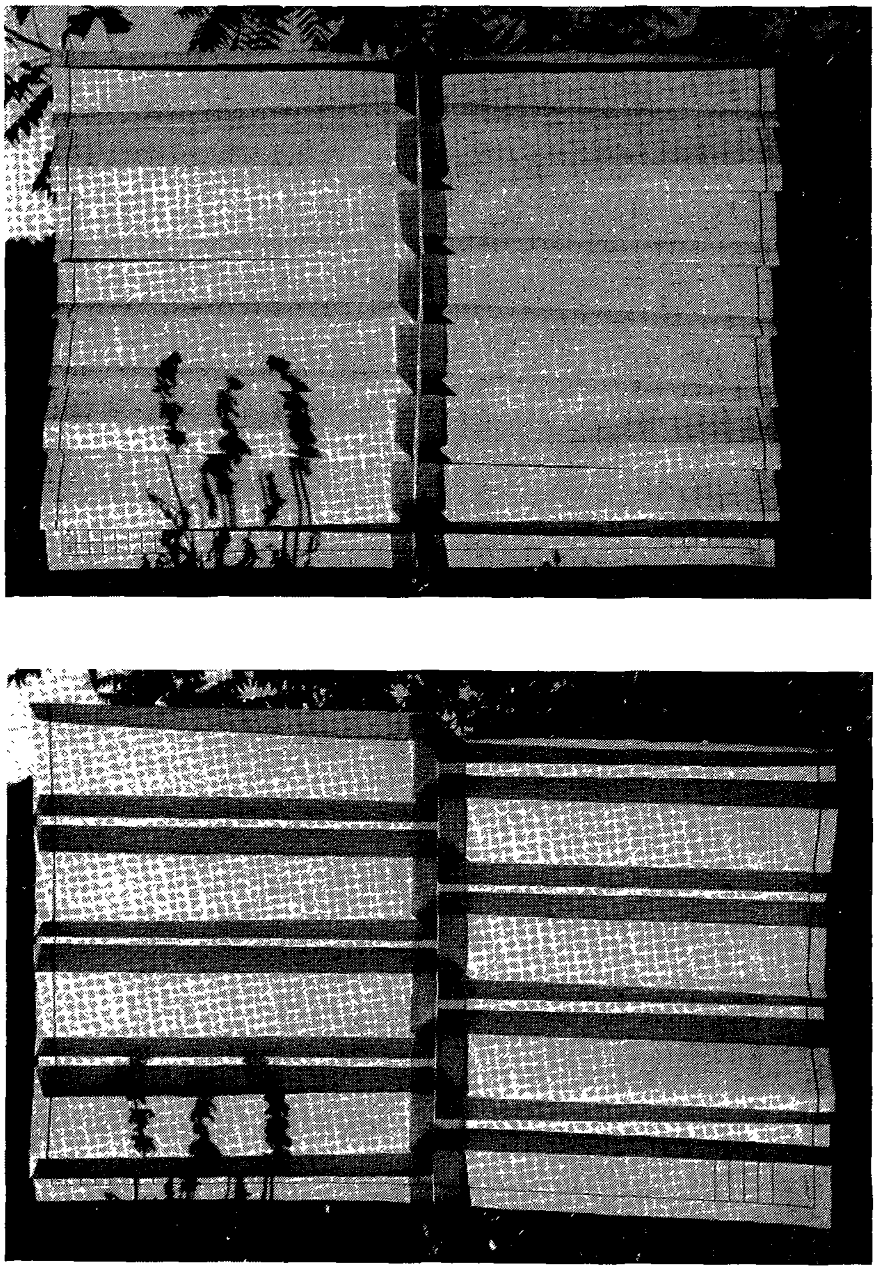
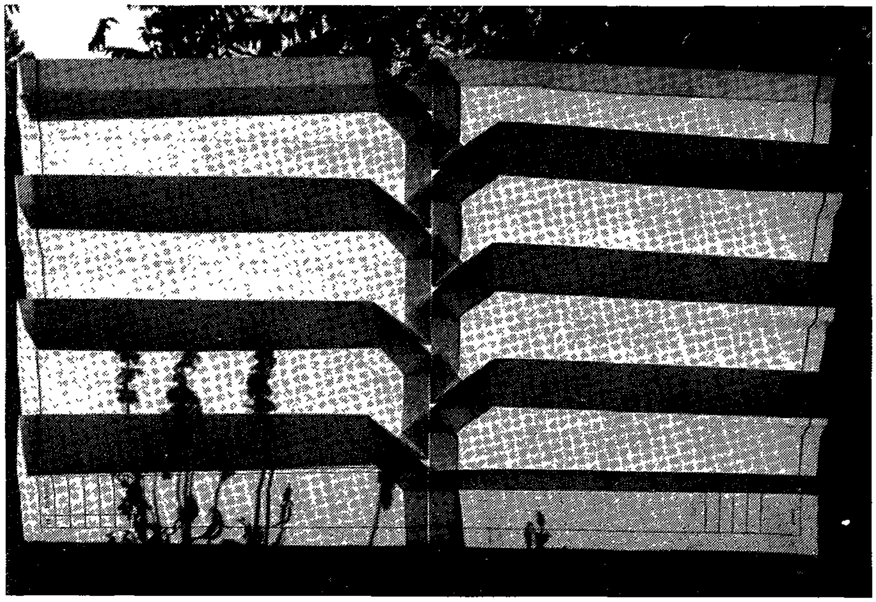


Figure 17: Four photographs show the evolution of a prototype model from symmetry to asymmetry. The key is a pinwheel mechanism that lifts the center of the paper into the third dimension, then lays it back down asymmetrically (1) and (2).



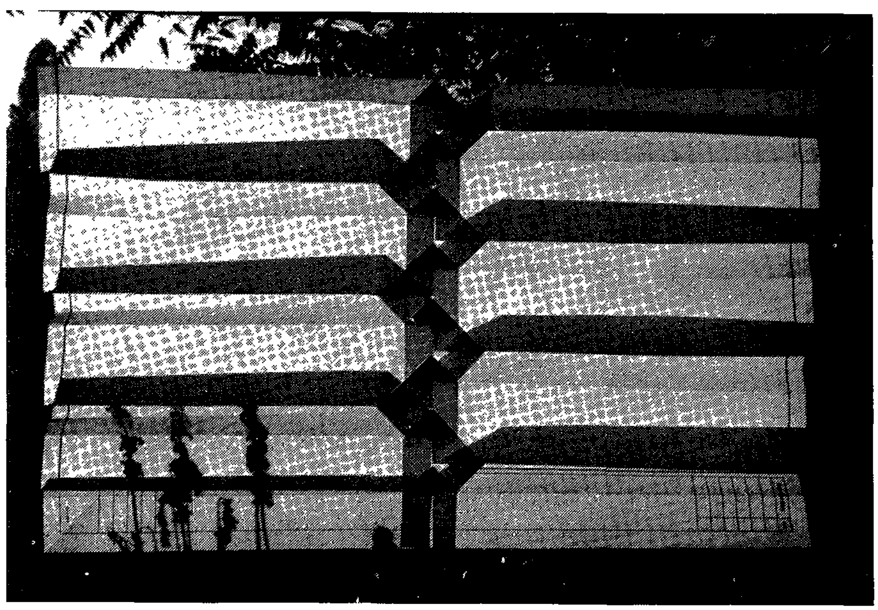


Figure 17: Four photographs show the evolution of a prototype model from symmetry to asymmetry. The key is a pinwheel mechanism that lifts the center of the paper into the third dimension, then lays it back down asymmetrically (3) and (4).

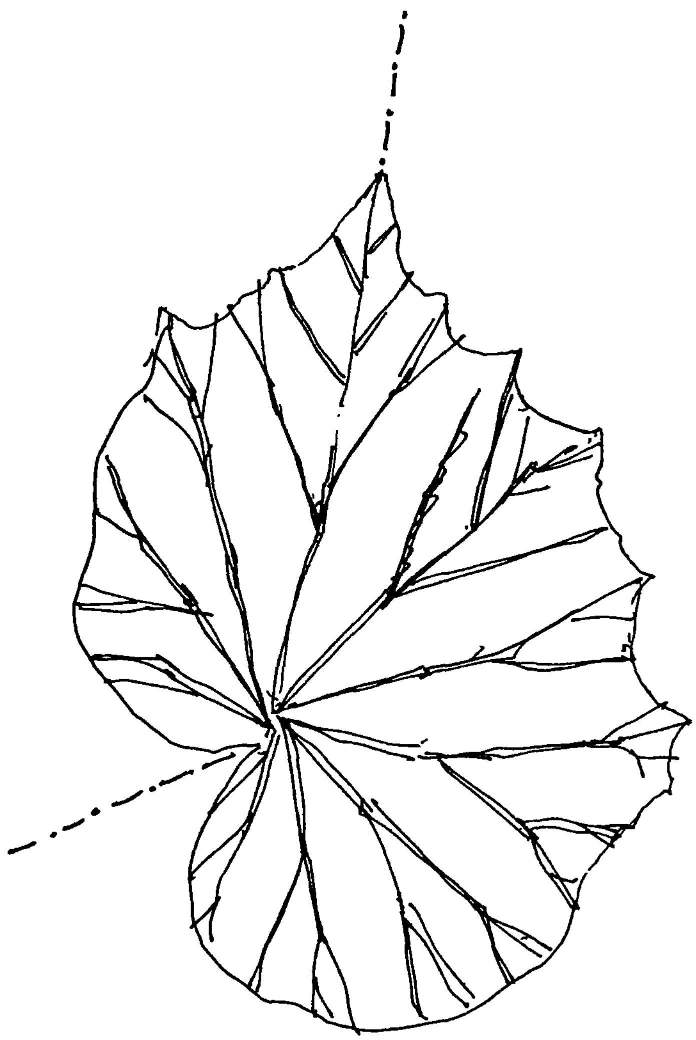
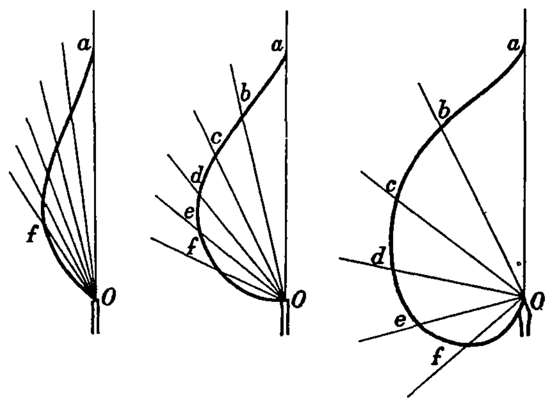


Figure 18: A page from the author's sketchbooks illustrates the *begonia* leaf's broken axis of symmetry.

I have concluded that in the original phylogenetic blueprint for the leaf, these two axes aligned. If the two sides were then to grow at vastly different rates, one half of the leaf would rotate relative to the other, and the result would be the shape that we see before us. But is this too simple an explanation for such a complicated form? Several weeks after I had begun working on an origami design of the *begonia* leaf, I found unexpected confirmation of my theory in the work of the British mathematician and biologist D'Arcy Thompson. In Thompson's analysis of bilaterally symmetric leaves, he had attributed the variation in their shapes to their differential rates of growth. Presuming a point of no growth at the *hilus*, Thompson drew vectors from the *hilus* to the edges of the leaf and analyzed those vectors in terms of their radial and tangential rates of growth. If the leaves of three different species of plants have identical tangential velocities but varying radial velocities of growth, the results will be the lanceolate, ovate, and cordiform shapes shown here (Fig. 19).



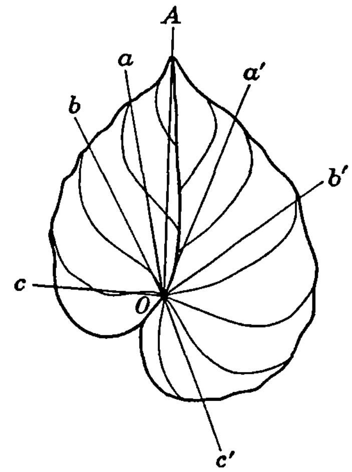


Figure 19: D'Arcy Thompson's analysis of the growth of symmetric leaves.

Figure 20: The same analysis, extended to include the asymmetric *begonia* leaf, reveals that despite the leaf's eccentric shape, orderly forces are still at work.

Extending his analysis to the *begonia* leaf (Fig. 20), Thompson measured vectors on both sides of the leaf and showed consistent but different rates of growth for the two sides. Based on my own and Thompson's mathematical analysis, I have been working to make an origami *begonia* leaf by starting with a form containing a single axis of bilateral symmetry and then varying the rates of tangential and radial growth to produce the broken symmetry revealed in my sketch. While the results are not ready to be made public, they point the way to new and unforeseen challenges in capturing the asymmetries of nature (Fig. 21).

4 CONCLUSION

From the four examples cited here it is clear that there are many approaches to breaking symmetry. It can be done in a single, decisive step, as in the rattlesnake and the *bodhi* leaf, or by a series of small alterations, as in the mountain range and the *begonia* leaf. However it is accomplished, breaking symmetry does not mean dispensing with order. Far from it! The process of breaking symmetry merely introduces a new type of order, a layering on of several types of symmetry, giving richness and diversity to phenomena that to our eyes would otherwise have been static and uninteresting.

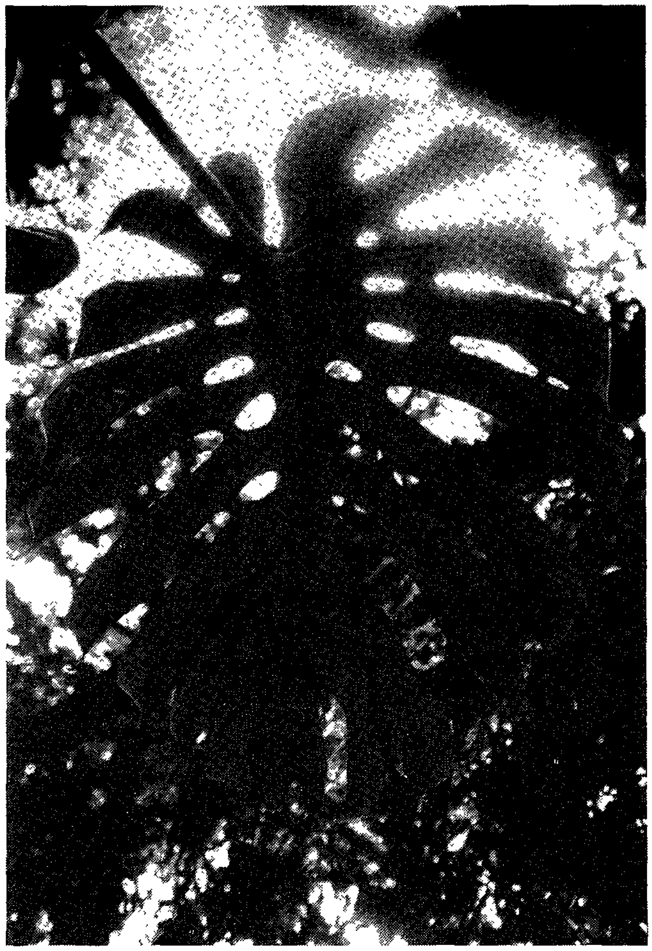


Figure 21: Nature has devised asymmetric plant forms, like this perforated *philodendron* leaf, that almost defy imagination. Can paperfolders meet the challenge of folding them from a single piece of paper?

The beauty of asymmetry, whether it is a cracking pattern in mud, the winding streets of an ancient settlement, or the alternating veins in a *bodhi* leaf, is different from the beauty of symmetry. While symmetry speaks of perfection, of idealized forms and objects that exist in a realm where they are untouched by time, asymmetry revels in the flaws and imperfections of the world, a world bound by the inexorable process of birth, growth, development, aging, and death. Like any transforming work of art, an asymmetric origami model reflects back on the life and forms that inspired it. If it reopens our eyes to the wonders of nature and sends us scurrying back to the original animals, leaves, and mountains to see them anew, the artist has certainly succeeded in his quest.

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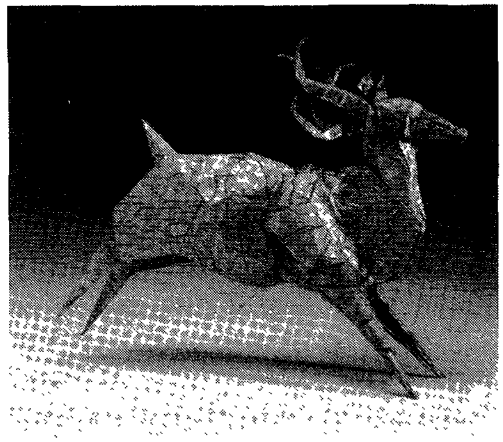
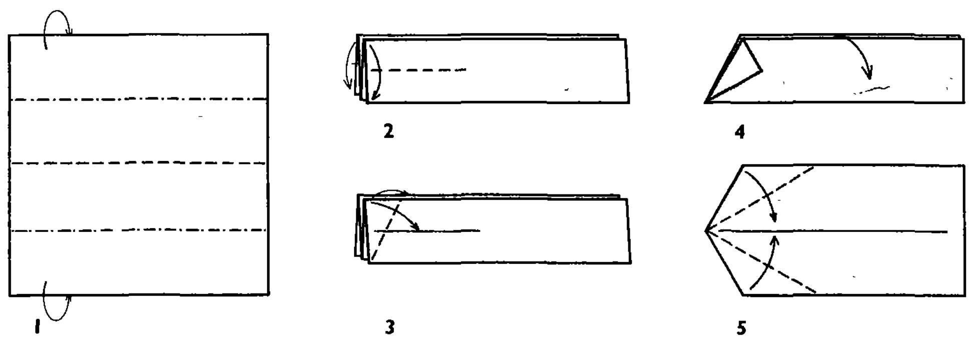
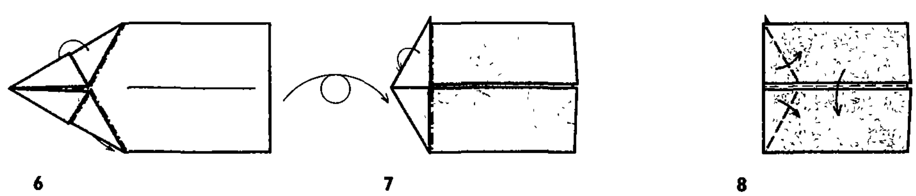
REINDEER

Use a sheet of paper colored on one side. A 10-inch square will produce a model 3 inches long. For your first attempt, use a square measuring at least 18 inches to a side.



1 Divide the square horizontally into quarters. Pleat like an accordion.
2 Crease lightly edge to edge. Repeat behind.
3 Valley-fold the corner so it meets the crease. Repeat behind.
4 Swing the front face down.
5 Valley-fold the left-hand edges to the centerline
6 Following the edges of the existing flaps, rabbit's-ear behind. Turn the model over.
7 Following the edge of the existing flap, mountain-fold behind.
8 Following the edges of the hidden flaps, inside reverse–fold and close the model.





9 Following the edge of the hidden flap, valley-fold the lower-left-hand corner to the upper right. Repeat behind.

10 Valley-fold the right-hand edge to the left. The crease should lie on top of the former lower-left-hand corner. Crease firmly and unfold to step 9.

11 Following the existing creases, crimp the entire model symmetrically.

12 Unfold the entire square.

13 Following the existing creases, refold the square. In the middle of the paper are two vertices where many lines meet. These vertices will plunge downward as the left and right sides of the paper swing upward and toward each other.

14 Inside reverse-fold the two flaps projecting from the top. Valley-fold the two side flaps down and to the *right*.

15 Squash one flap.

16 Lift the loose paper upward, and close the flap.

17 Valley-fold two flaps up and to the left, returning them to their position in step 14.

18 Following the edge of the hidden flap, crease and unfold. Then swivel two flaps up and to the left. When you are done, the model will not lie flat.

19 The model is now three-dimensional. The exposed white portion shows paper that is seen almost directly on edge. Squash the shaded flap.

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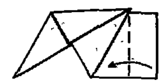

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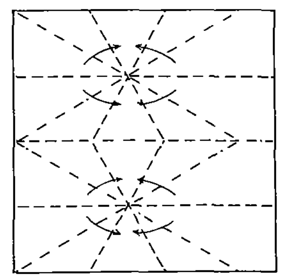

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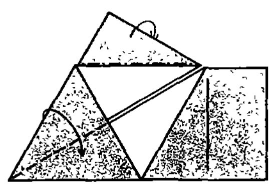

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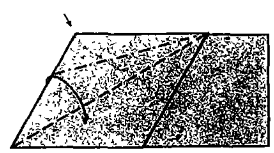

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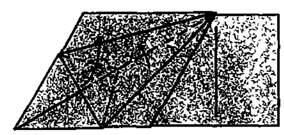

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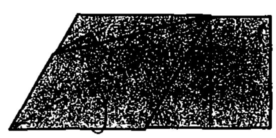

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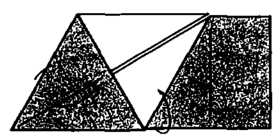

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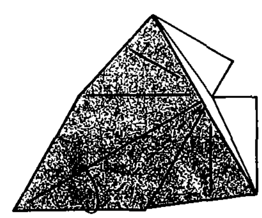

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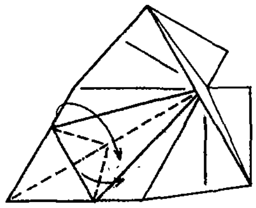

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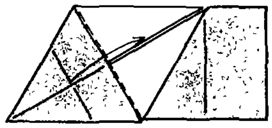

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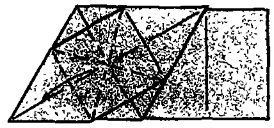

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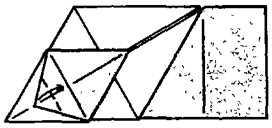

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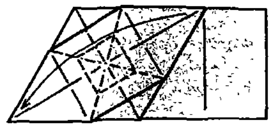

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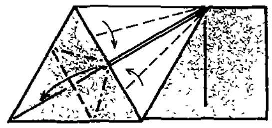

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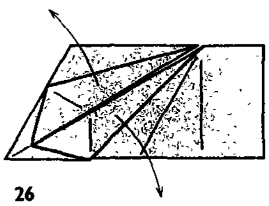

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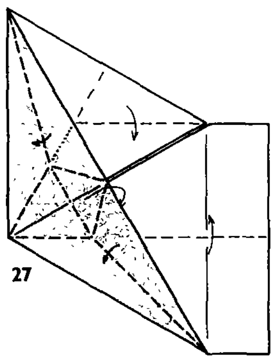

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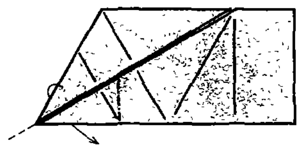

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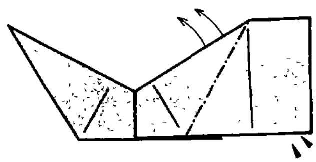
20 Lift the loose paper upward, as in step 16, and swing two flaps down and to the right.
21 Valley-fold the lower-left-hand corner up and to the right as far as it will go.
22 Push with your finger from behind to form a little pyramid of the shaded square. The square will pop forward and flatten.

23 Valley-fold the tip halfway, and unfold to step 22.
24 Following the existing creases, open double-sink and close the flap in the same motion.
25 This is a form of petal fold. Repeat behind.

26 Open up and spread the loose paper.
27 Following the existing creases, tuck the small triangle into the pocket behind and close the model in the same motion. Use tweezers. Repeat behind.
28 Swivel the flap at the left of the slot counterclockwise. The model will crimp symmetrically.



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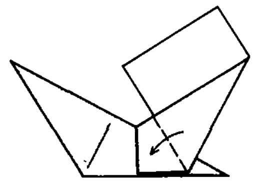

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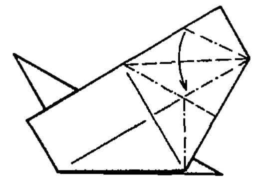

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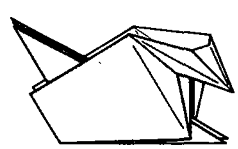

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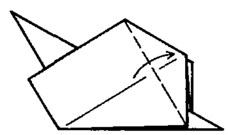

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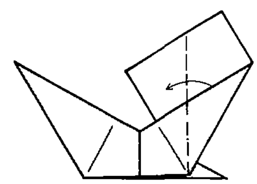

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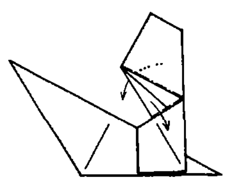

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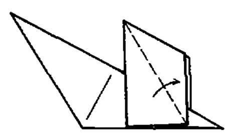

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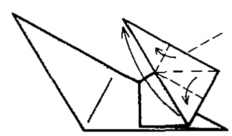

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29 Open the model slightly and inside reverse-fold the entire assembly at the right. (This procedure could be called a "closed" inside reverse fold, because it pops directly from one position to another.) The central flap turns inside out in the process.

30 Valley-fold the white flap down and to the left. Repeat behind.

31, 32 Make individual creases as shown. Then, in a single motion, push in at the front of the model and collapse it into a three-dimensional rabbit's ear Massage the creases into place. The result is symmetrical.

33 Valley-fold the shaded flap up and to the right. Repeat behind.

34 Inside reverse-fold through the shaded portion. Repeat behind.

35 Swing the white triangle down and to the right. The shaded portion of the flap will automatically swing down and to the left. Repeat behind.

36 Valley-fold the shaded flap up and to the right. Repeat behind.

37 Bring the three corners of the white triangle together, and collapse the loose paper like a fan. Repeat behind.

38 The flaps pointing up and to the left are the rear legs. Mountain-fold them, leaving the central tail flap in place. Valley-fold the white flap down and to the right. Inside reverse–fold the shaded tip. Repeat behind.

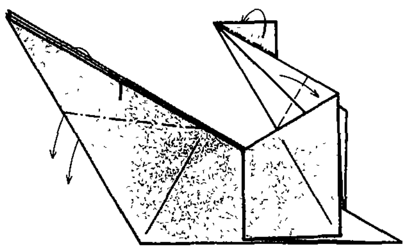
39 Crimp the legs. Swing the tail all the way underneath and forward. Narrow the white flap with valley folds, and swing it up and to the right. The shaded flaps pointing down and to the right are the front legs. Creasing lightly, lift up the flap obscuring the front leg. Repeat behind.

40 The position of the drawing has been rotated slightly. Inside reverse–fold the tail and swing it toward the rear. (This crease is hidden from view). Then, in a single motion, crimp the front legs and rotate the head and neck assembly clockwise. Narrow the hip with a mountain fold. Repeat behind.

41 Narrow the belly with mountain folds, and tuck the loose paper into the adjacent pockets formed by the tail. Narrow the front leg, valley-folding the double thickness. Swing one white flap and one shaded flap over to the left, and tuck the excess paper into the body. Repeat all folds behind.

42 Inside reverse–fold the hind leg. Without making any new creases, slide the top layer off the front leg, and tuck it into the pocket beneath. Cut-away view: Squash. Repeat all folds behind.

43 Narrow the hind leg symmetrically with valley folds, and tuck the loose paper inside with mountain folds. (*The mountain folds will pinch the back of the hips* slightly.) Slide another layer off the front leg. Turn the valley fold into a mountain fold, and tuck the layer into the pocket beneath. Cut-away view: Closed-sink the big flap. Squash the little flap at the top. Repeat all folds behind.



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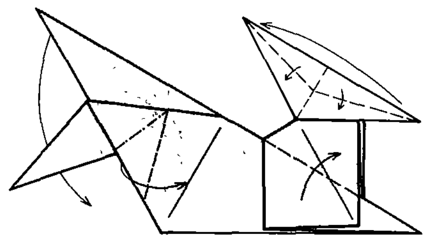

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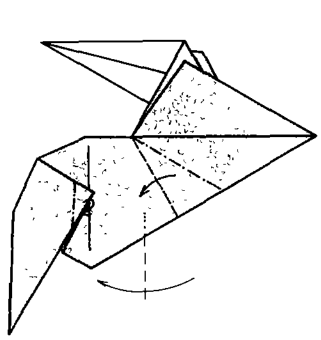

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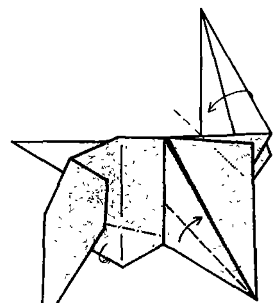

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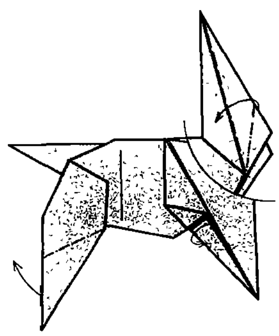

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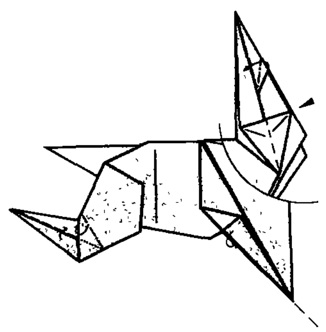

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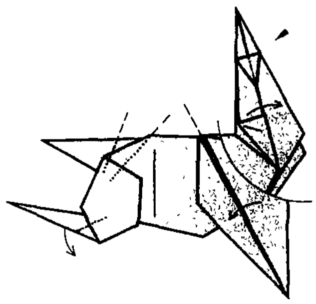

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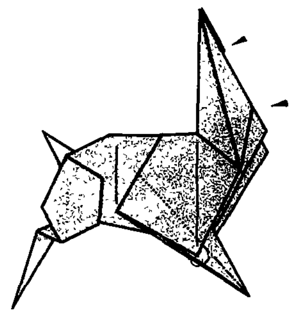

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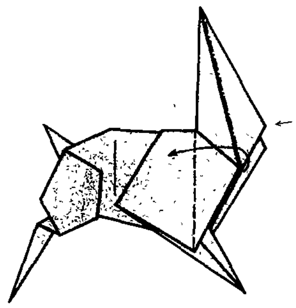

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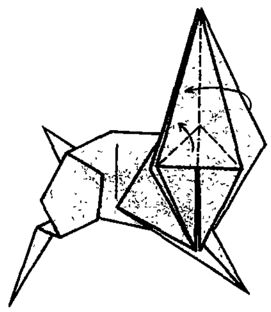

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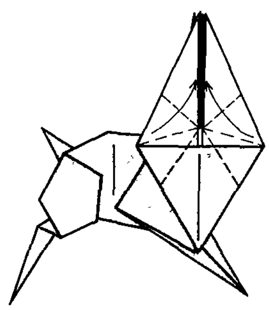

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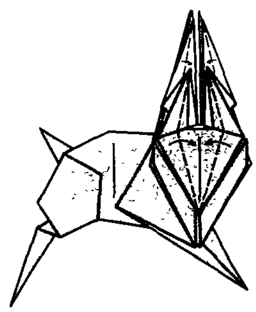

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44 Inside reverse–fold the hind leg. Close the front leg with a valley fold. Crimp the tail symmetrically. Cut-away view: Closed-sink the little flap. Swing the big flap to the right. Repeat all folds behind.

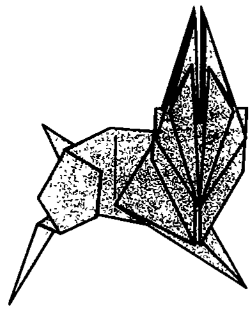
45 Closed-sink two head flaps. Mountain-fold the top layer of the front leg. (Part of this crease is hidden from view.) Repeat both folds behind.

46 Squash the next head flap, and swing it to the rear. Repeat behind.

47 Lift the tiny triangle inside the squash fold, and collapse it upward. The white flap at the center of the model contains the head and the ears. Swing it into view.

48 Mountain- and valley-fold the head assembly, and collapse it upward. Flatten it. Then pull the front flap down slightly to expose the inside.

49 Open the head assembly slightly, and narrow all the flaps with valley folds. Flatten again.



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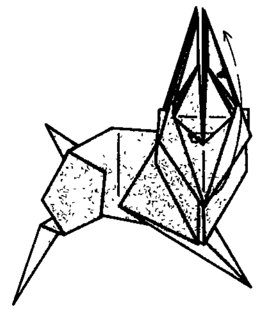

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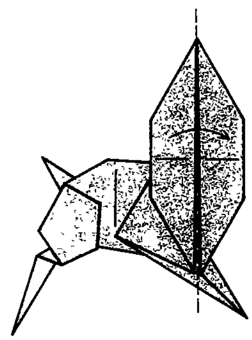

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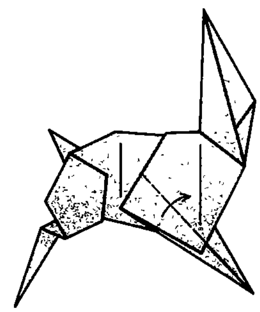

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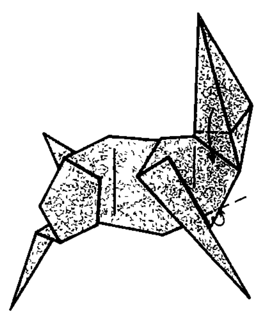

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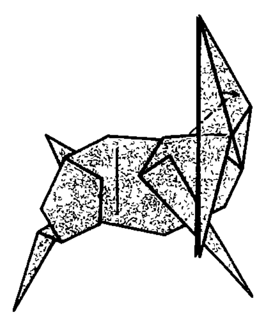


55

50 Open the head assembly slightly, and pull out the loose paper. Following the existing creases, flatten it into a petal fold.

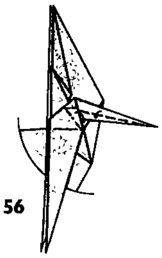
51 Mountain-fold the tip of the single-ply triangle. This will be the eyes. Following the existing crease, swing the entire head assembly upward.

52 Valley-fold the head assembly to the right.

53 The model is now entirely symmetrical. Narrow the front leg with a valley fold. Repeat behind.

54 Narrow the belly with a mountain fold. Swing down the near antler. Repeat both folds behind.

55 Inside reverse-fold the adjacent antler. Repeat behind.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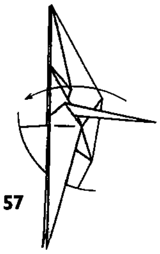
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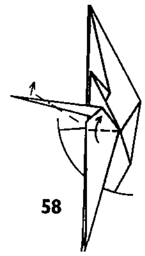
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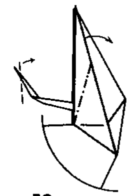
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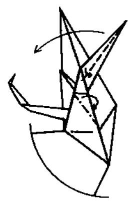
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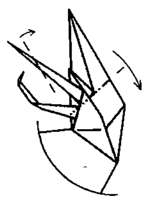
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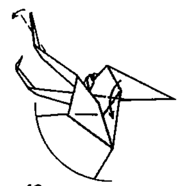
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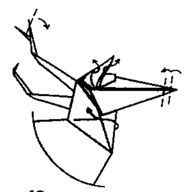
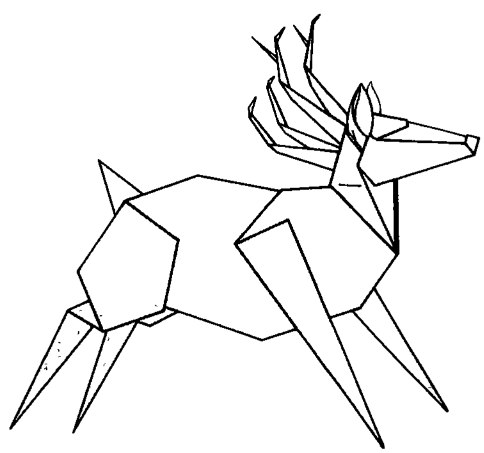

63

56 Here through step 63 are details of the head. Narrow the projecting antler with valley folds. Repeat behind.
57 Swing the projecting antler to the rear. Repeat behind.
58 Inside reverse–fold the rear antler. Swing up the front antler. Repeat behind.
59 Outside reverse–fold the rear antler to form the tine. Inside reverse–fold the front antler. Repeat behind.

60 Narrow the front antler with valley folds, and swing it to the rear. Repeat behind.
61 Inside reverse–fold the upper rear antler. Repeat behind. Inside reverse–fold the head through the base of the ears.
62 Separate the tines of the upper rear antler. Repeat behind. Turn the head flap inside out with valley folds on either side. The ears will pop up.
63 Outside reverse–fold the upper front tines. Spread the ears. Pull out the loose paper from either side of the neck to enlarge the jaw. Roll the tip of the head to form the nose.



The completed REINDEER.

(1976–78)

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DRAWING THE REGULAR HEPTAGON AND THE REGULAR NONAGON BY ORIGAMI (PAPER FOLDING)

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Abstract: *The regular heptagon and the regular nonagon are good examples for showing paper folding ability, since neither can be made by Euclidean methods (using ruler and compass). For the convenience, starting with a square diagram, the concentric polygons are demonstrated here. As you see, this does not disturb the generality. Here the simplest method is described first, and then the explanation follows.*

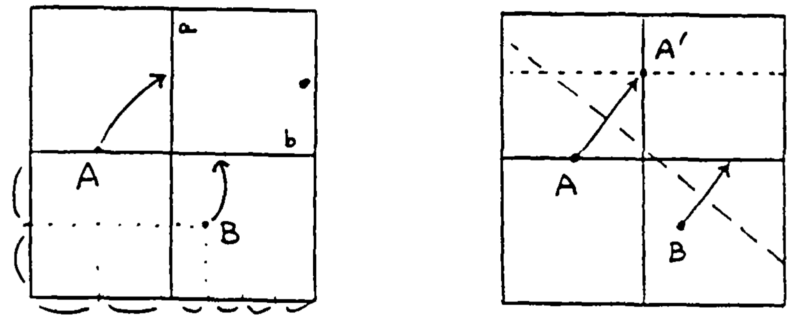
1 HEPTAGON

1.1 How to *draw* a heptagon using origami.

(1) Make two medians, *a* vertical and *b* horizontal. Take two points *A* and *B* of coordinates (1/4, 1/2) and (5/8, 1/4) respectively, using a rectangular system with (0, 0) at lower left corner and (1, 1) at upper right corner. Subsegments of lengths 1/2, 1/4, 1/8 . . ., etc., are easily made by folding, since each folding step makes a half. Now *fold* such that point *A* comes onto line *a* exactly and point *B* comes onto line *b* exactly at the same time.

(2) The realisation is not unique: three are obtained. The three displacement positions of *A* on line *a*, say *A'*, *A"* and *A'"* correspond to shoulder height, hip height and foot respectively.

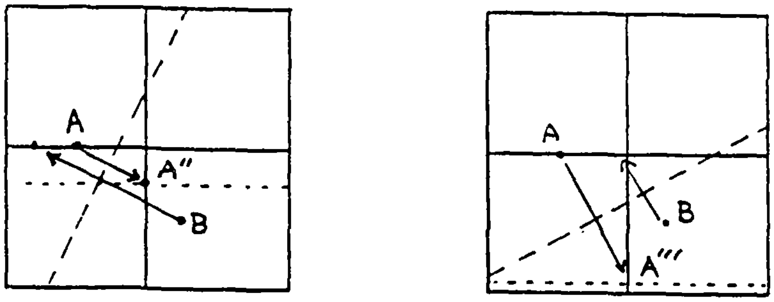
(3) Considering the upper-most point of line *a* in the square (the point with coordinate (1/2, 1)) as the head and the center, or (the point with coordinates (1/2, 1/2)) as the center of heptagon, the completion is easily made.



Folds

Realization I

Figure 1



Realization II

Realization III

Figure 2

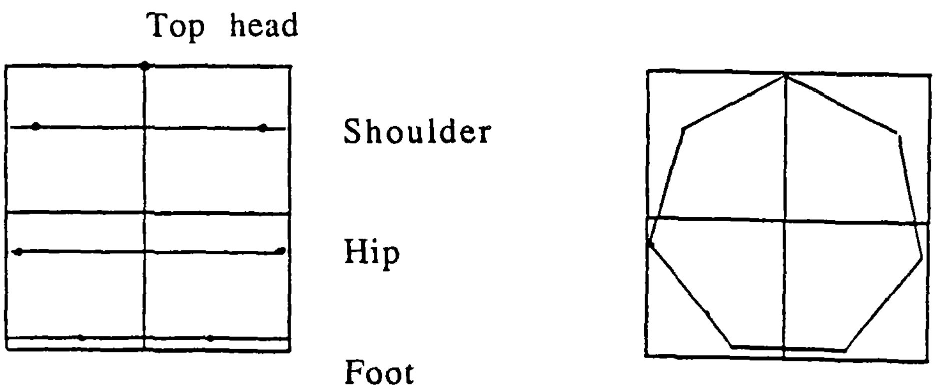


Figure 3

1.2 Explanation of the Method

Setting the origin of coordinates in the center of the heptagon, the seven points of the diagram can be represented in the complex plane,

$$1,\quad e^{(2\pi/7)i},\quad e^{2(2\pi/7)i},\quad e^{3(2\pi/7)i},\quad e^{4(2\pi/7)i},\quad e^{5(2\pi/7)i},\quad e^{6(2\pi/7)i}$$

For simplicity, replacing $e^{(2\pi/7)i}$ by A, the problem is now to solve the equation:

$$A^6 + A^5 + A^4 + A^3 + A^2 + A + 1 = 0$$

Knowing the fact:

$$A^6 = 1/A,\quad A^5 = 1/A^2,\quad A^4 = 1/A^3,\quad \text{etc.}$$

the equation can be rewritten as,

$$1/A + 1/A^2 + 1/A^3 + A^3 + A^2 + A + 1 = 0$$

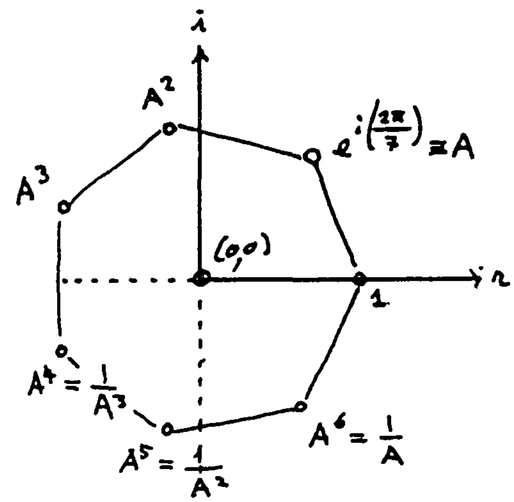


Figure 4

By simple relations:

$$(A + 1/A)^2 = A^2 + 2 + 1/A^2 \qquad \text{then} \qquad A^2 + 1/A^2 = (A + 1/A)^2,$$

and

$$(A + 1/A)^3 = A^3 + 3A + 3(1/A) + 1/A^3$$

then

$$A^3 + 1/A^3 = (A + 1/A)^3 - 3(A + 1/A)$$

replacing $A + 1/A$ by Z, the equation is reduced to a third order:

$$Z^3 + Z^2 - 2Z - 1 = 0.$$

The graphical meaning of Z is as shown in Figure 5.

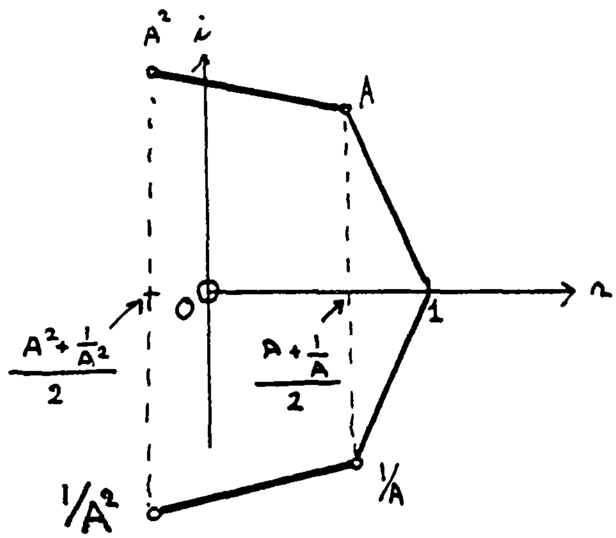


Figure 5

To solve this, let us remember the classic geometric representation for finding a zero of a polynomial. To solve an equation

$$ax^3 + bx^2 + cx + d = 0$$

first we have to make a grid of coefficients using the following rule: from the starting point O make a segment OA to the right horizontally of length a, then turn to the right at a right angle if coefficients a and b are of same sign, if they are not of same sign, then turn left. Similarly draw c and d making points B, C and D. Let us call these straight lines a, b, c and d. Now we start from the initial point and hit a billiard ball so that, when the ball hits a new line (wall), it bounces or deviates from its path at right angles towards the next line, finally hitting the target, in this case point D.

If the target is hit, then it is easy to understand the initial slope of the trajectory is an actual solution. All the triangle formed by the trajectory and the grid lines are similar and the following relations can be obtained:

$$x/a = y/(x + b) = z/(y + c)$$

Therefore

$$y = (x/a)(x + b)$$

then,

$$(x/a)[(x/a)(x + b) + c] = z = -d.$$

That is,

$$a(x/A)^3 + b(x/A)^2 + c(x/A) + d = 0$$

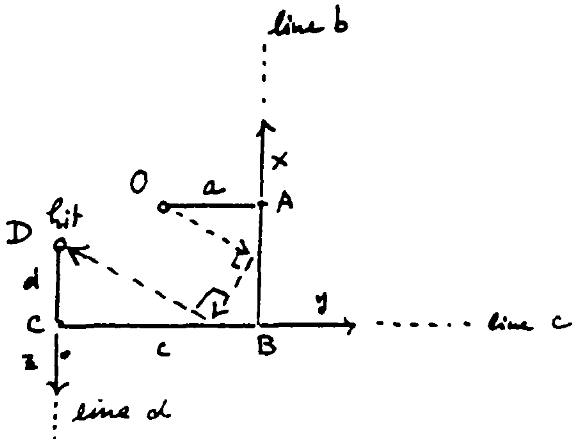


Figure 6

The complexity of coefficient signs can be reduced by changing the sign of definition of d to:

$$ax^3 + bx^2 + cx = d$$

Then the diagram becomes

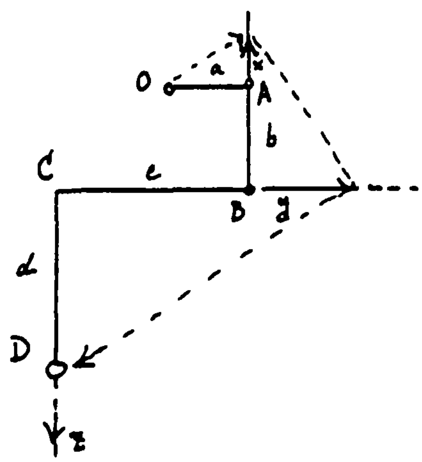


Figure 7

The problem is now how to hit the target and how to decide on the initial condition (that is, the slope of the direction of the ball destined to hit the target D). The origami solution to solve an equation of third order is easy following the Beloch method. Make a line parallel to line b at an equal distance from segment a but on the opposite side, call it line u. Now make a line parallel to line c at an equal distance from segment d but on the opposite side, call it line v.

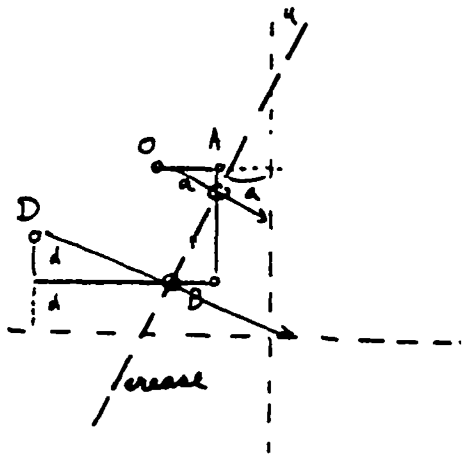


Figure 8a

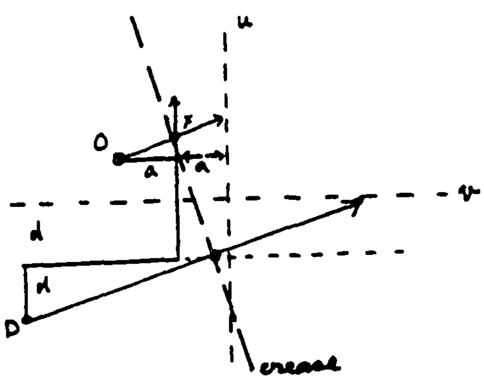


Figure 8b

Fold superposing point O onto line u, and at the same time point D onto line v. The crease determines the bounce points at line b and line c as crossing points with the crease U and V. OU is the initial direction of the ball. The slope of OU is, therefore, the zero of the equation.

Now let solve the equation

$$Z^3 + Z^2 - 2Z - 1 = 0$$

The grid and lines u and v turn out:

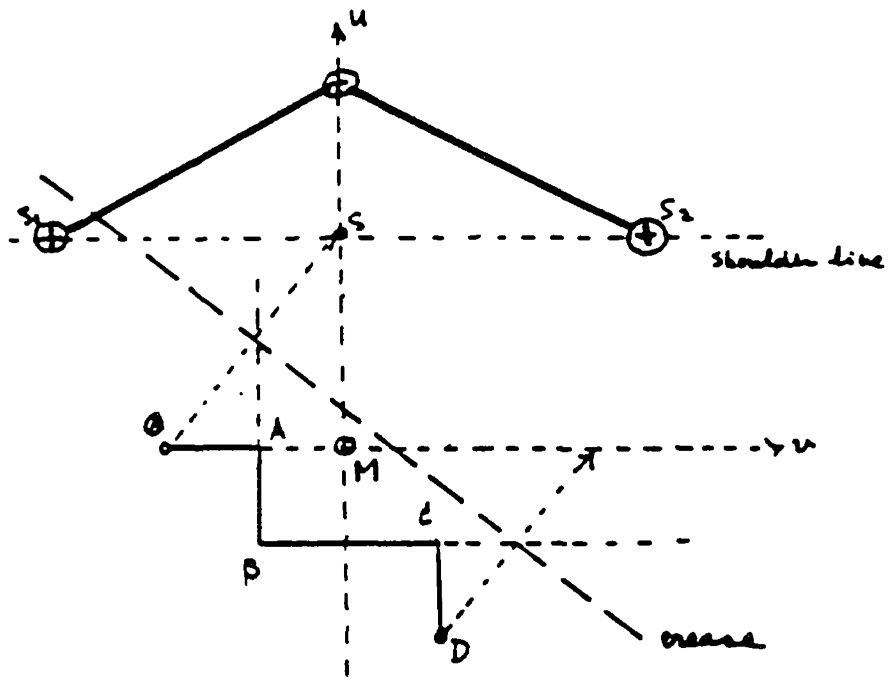


Figure 9

Now let us solve the equation:

(1) fold, superimposing O onto line u, and D onto line v at the same time. The new position of point O, or S, should be the shoulder height of a heptagon of radius 4 (since unit $OA = 1$). We now have all the information to draw a heptagon of radius 4;

(2) call the crosspoint of lines u and v the point M. This is the center of the heptagon of radius 4;

(3) make a point, called T the top point, 4 units upward from M, or twice of OM;

(4) make line s that passes point S and is parallel to line v;

(5) fold, making point O the pivot, superimposing T onto line s. We get the shoulder points, $S1$ and $S2$;

(6) fold, superimposing T onto the S's points. The new positions of S are hip points $H1$ and $H2$;

(7) repeating same procedure we get the foot point.

Here is an interesting characteristics of the polygon. Let us take into consideration the hip height. Let us again call a heptagon of radius 2, Z. That is

$$A^2 + 1/A^2 \equiv Z$$

$$(A^2 + 1/A^2) = A^4 + 2 + 1/A^4 = 1/A^3 + 2 + A^3 \quad \text{then } 1/A^3 + A^3 = Z^2 - 2$$

and

$$(A^2 + 1/A^2)^3 = A^6 + 3A^2 + 3/A^2 + 1/A^6 = 1/A + 3A^2 + 3/A^2 + A$$

then

$$A + 1/A = Z^3 = 3Z$$

Finally we get the same original equation,

$$Z^3 + Z^2 - 2Z - 1 = 0$$

For the foot level,

$$A^3 + 1/A^3$$

satisfies the equation. This means for given radius (or top point) and the center position, all the other points of a heptagon are directly given by three real roots of the equation

$$Z^3 + Z^2 - 2Z - 1 = 0$$

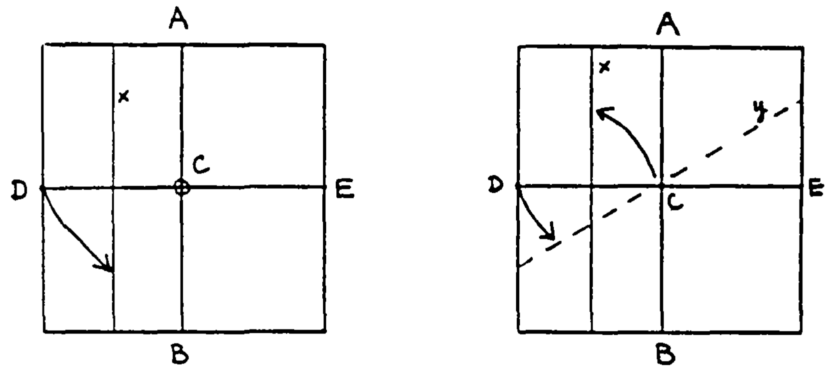
These phenomena are very natural if you consider that the number 7 is a prime number and multiplication corresponds a rotation of one section of the heptagon.

In conclusion, a polygon of $2^n3^m + 1$ can be made by folding paper. On the other hand, Euclidian methods can make only polygons of $2^n + 1$, since its capacity is only up to second order. As you see, the monodecogon (11 sided polygon) is the problem. At present origami cannot solve it. It is a problem of the fifth order. As you will see later, it is not impossible to realise by origami, since the origami geometry is not a closed system like the Euclidean methods but completely open. You can just invent a new way of folding! One possibility is shown in the Proceedings of the First International Meeting of Origami Science and Technology held at Ferrara, Italy 1989, page 53. (Those who are interested in this, please contact the author.)

2 NONAGON

2.1 How to *draw* a nonagon using origami

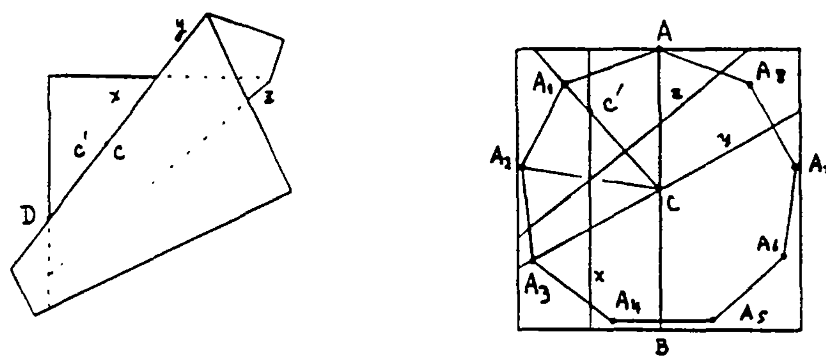
(1) Make two medians called AB (vertical), DE (horizontal) and their crossing (center C). Fold, moving point D onto C and call the crease line x. Fold, making point C a pivot and moving point D onto crease x. Call the new crease line y.



Fold 1 Fold 2

Figure 10

(2) Fold moving point C onto line x and point D onto line y at the same time and call the crease line z.



Fold 3 Get the wanted apexes

Figure 11

(3) Call the new position of point C on line x, point C', and call the crossing of line x and line z point F. Make a line through C and C' and another through C and F. On these lines and on line y make points $A1$, $A2$ and $A3$ an equal distance of $|CA|$

from the center C. Fold along line y to obtain $A4, A5$ and $A6$ at new positions of $A2, A1$ and A. By the reflection at AB, obtain $A7$ and $A8$. $A, A1, A2, A3, A4, A5, A6, A7$, and $A8$ are apexes of the wanted regular nonagon.

2.2 Explanation of the method

First of all, the number 9 is not a prime number but can be factorized into prime numbers 3×3. This means that the nonagon can be made by a regular triangle and a trisection of the angle of 120 degrees. To make an angle of 120 degrees is easy, as Euclidean methods can also make it. The problem is the trisection of 120 degrees. Let me introduce the elegant method of Hitoshi Abe of 1980 in a little detail, since it seems an important and also an interesting topic.

2.3 Trisection of an angle

It is the famous multimillennial problem, apparently easy to solve but in fact very difficult, which remained unsolved until 1837 when M. L. Wanzel in Paris showed the impossibility of trisecting an angle into equal parts by using ruler and compass or the Euclidean method. If you use other apparatus, it is certainly possible. In one hour you can invent several of them – something like carpenter's tools. In origami geometry which we have so far followed, it is possible merely by folding actions, that is without any tool.

Here is *H. Abe's method*, the earliest (published 1980) in the world and most elegant. An angle is given at point A by half lines a and b as below.

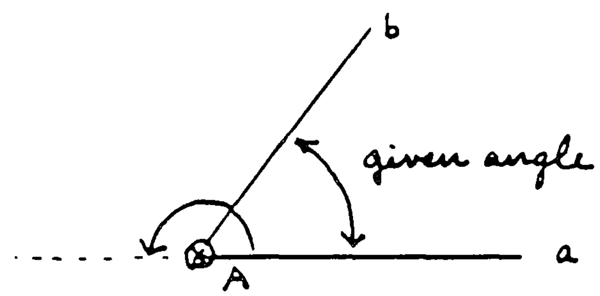


Figure 12

(1) Make line r vertical to line a at A by folding ($A \longrightarrow A, a \longrightarrow a$), then take any point B on line r and make the bisector c by folding ($A \longrightarrow B$). Call the crossing point of lines r and c, C.

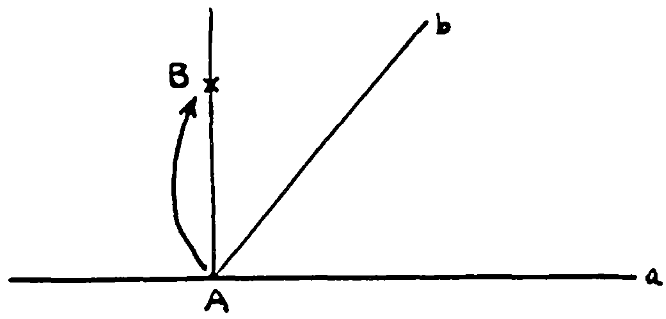


Figure 13

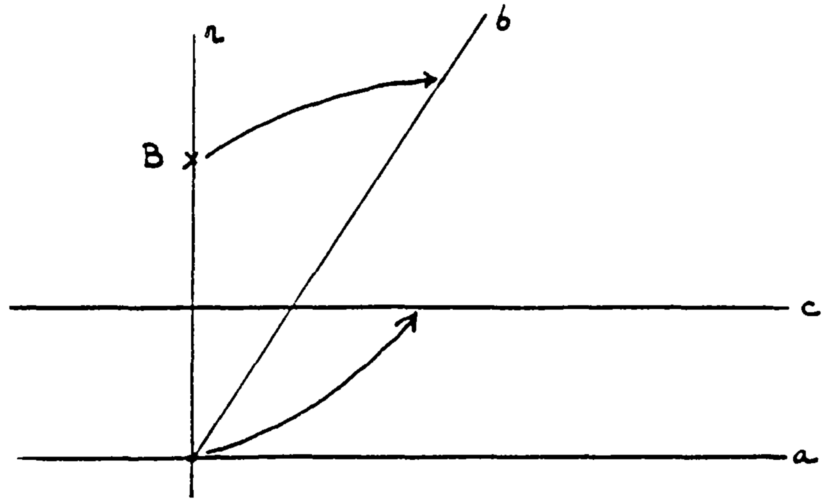


Figure 14

Now fold ($A \longrightarrow a, B \longrightarrow b$) (fold type # 6, see Fig. 15).

Let us call the folded position of points A, B and C, A', B' and C' respectively, and the crossing point of line c and the new crease d, D. Then point A' and point D (i.e., the half lines OA' and OD) trisect the given angle. The proof is simple. Let us call the folded position of line r line r', and the crossing points of line r' with line a and line r, E and F respectively. Then

$$\angle FAA' = \angle AA'C$$

since they are internal alternate angles of the parallel lines a and c. And

$$\angle AA'C = \angle BA'C$$

since triangle $A'BA$ is isosceles ($A'A = A'B$).

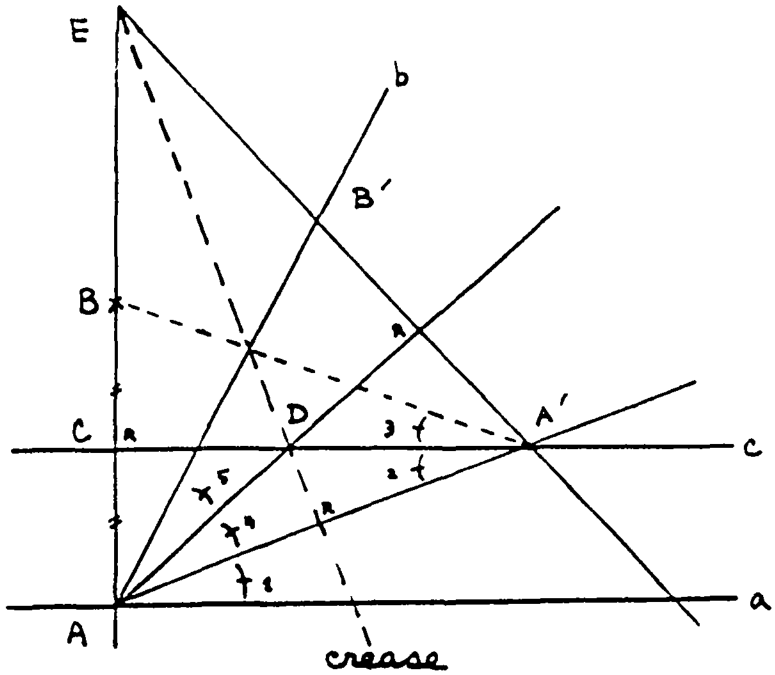


Figure 15

Finally

$$\angle A'AD = \angle AA'D$$

and

$$\angle B'AD = \angle BA'D$$

because of the superposition by the last folding. Therefore,

$$\angle FAA' = \angle A'AD = \angle DAB'$$

i.e., angle *FAB'* (angle of *a* and *b*, or the given angle) is trisected into equal parts by *AA'* and *AD*.

It should be interesting to note that there are always three realizations (solutions in algebra) and as you can easily see in the figures below the other two correspond to the trisection of (2**R** − given angle) and the trisection of (2**R** + given angle). The former has a classic name, supplementary angle, but the latter has no name in the Euclidean methods. Since in the terminology of navigation it is commonly called the ambiguous angle (for example, the direction to a light house from a ship and that of the ship from the light house), let us borrow the term.

Now we can say that *by origami we can trisect an angle. This method, however, gives us always three answers, one is, of course, the trisection of the given angle. The other two correspond to the trisection of its supplementary angle and its ambiguous angle.*

2.4 Why are there three?

It is a good question! However I would like stop here. I hope that you are now very curious about the mysterious secret of folding papers.

The second realization:

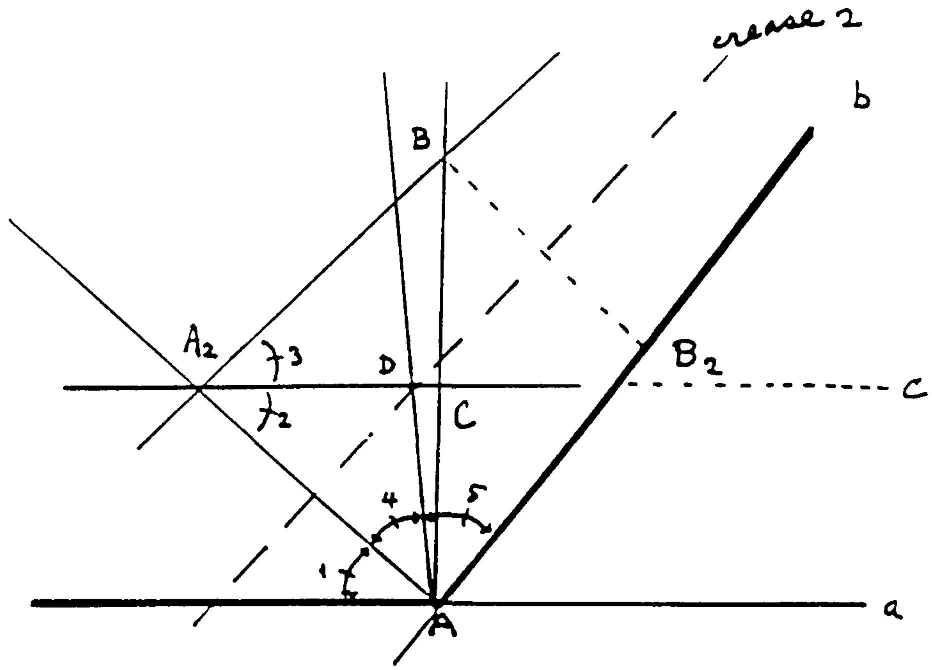


Figure 16

The third realization is shown on Figure 17.

It will be clear what you should do to avoid this ambiguity:

(1) when you define the wanted or given angle, it should be done strictly in terms of half-lines, not complete lines (well defined angle),

(2) also, when you make line c, don't consider the whole line but only the half-line, even in the case of the segment (or sometimes the whole line) which is inside the given angle.

This is well indicated by full lines and dotted lines in the above and below diagrams.

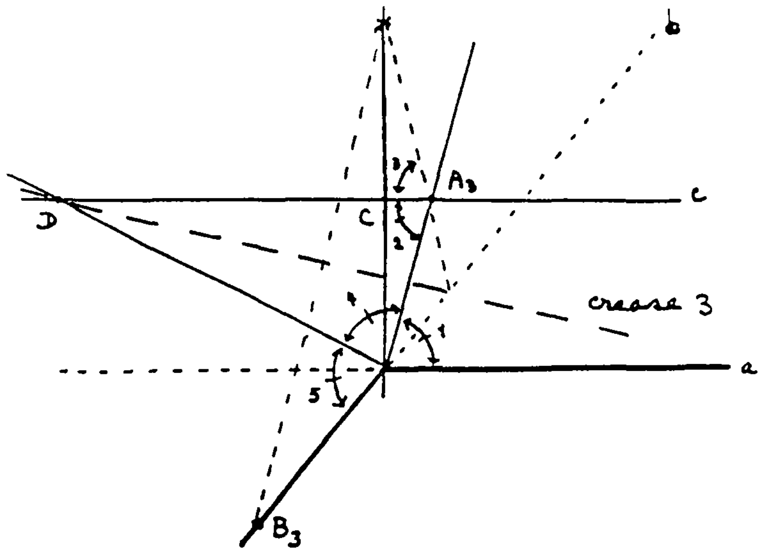


Figure 17

Probably you may point out: why does not the other, more directly related angle (4R – the given angle), come out? You want too much? I limit myself to showing how to obtain it (or to trisect this angle). You will see the clear difference. Just follow the Abe method with the case noted above.

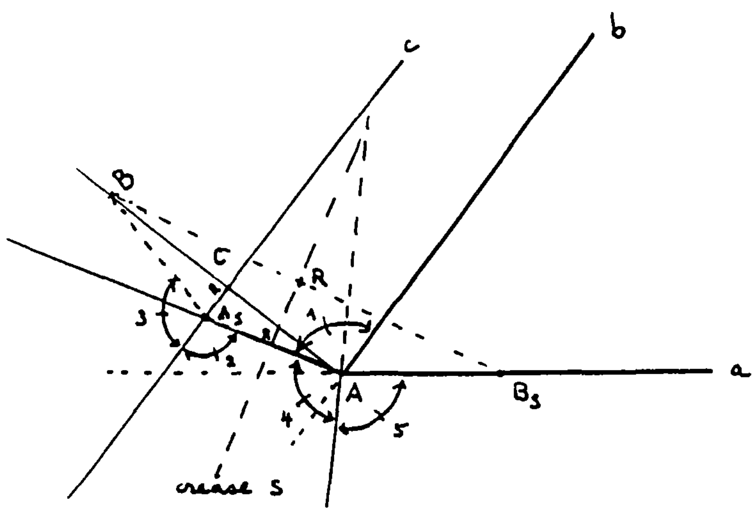
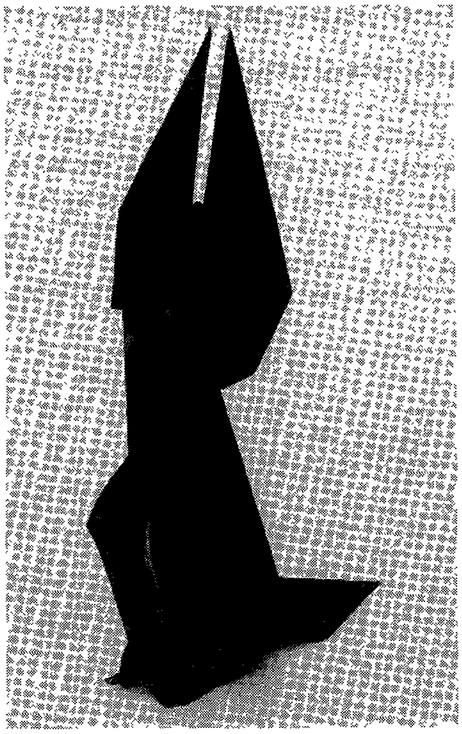


Figure 18

Now you know what the two other hidden (or unwanted) solutions are.

It is very popular now to say that software is often more powerful than hardware. We can understand these words as intelligence and mechanical tools. Origami geometry (or better origami science in general) is purely based on software having no hardware at all. Euclidean methods are strictly limited by their tools (ruler and compass) and there is no possibility of expansion. Its system is closed. On the contrary, origami is a completely open system of the intelligence and there are possibilities for developing it to a higher level by inventing new folds and admitting them as new fold types that will not necessarily be as simple as folds so far treated.



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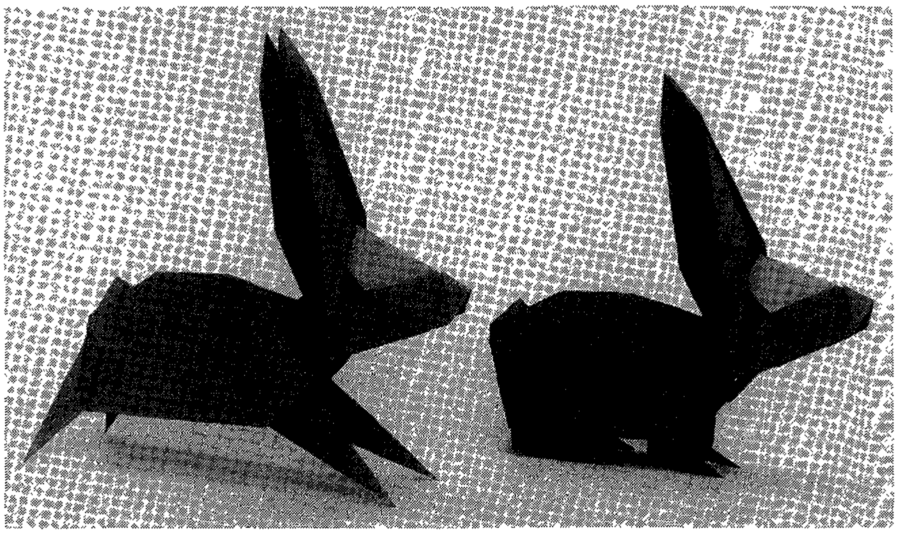

Rabbit

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NEW FOLDED CREATURES

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Many of us are able to fold paper.

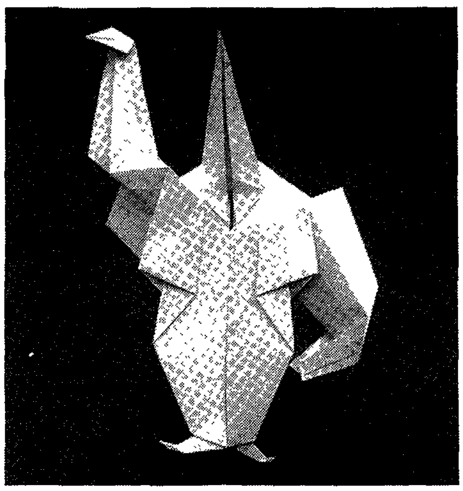
Many of us are able to produce many different foldings.

But ORIGAMI means not only being able to make a certain figure following instructions. Though the available bounds are limited (the two sides of the sheet, the known skills of folding), new solutions can be found to the same problem by applying the different elementary foldings in an individual order. These procedures qualify those who have invented them.

I have tried it and some kind of lyrical-abstract world came to life as I folded the paper.

My opinion is that each of the convenient means must be used for self-expression.

Keeping oneself to the rules may give freedom as well. The rules of ORIGAMI gave me this kind of freedom.



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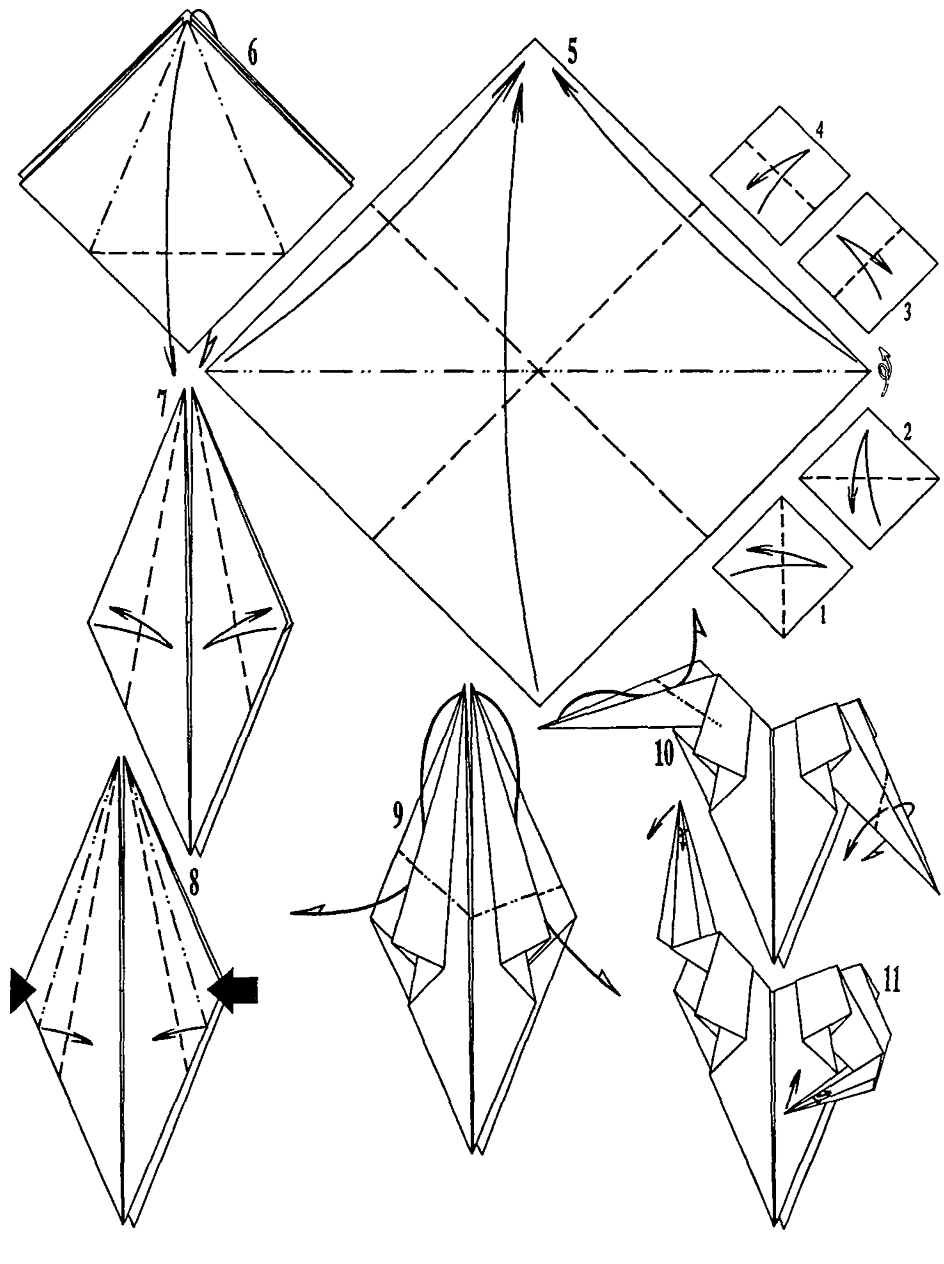

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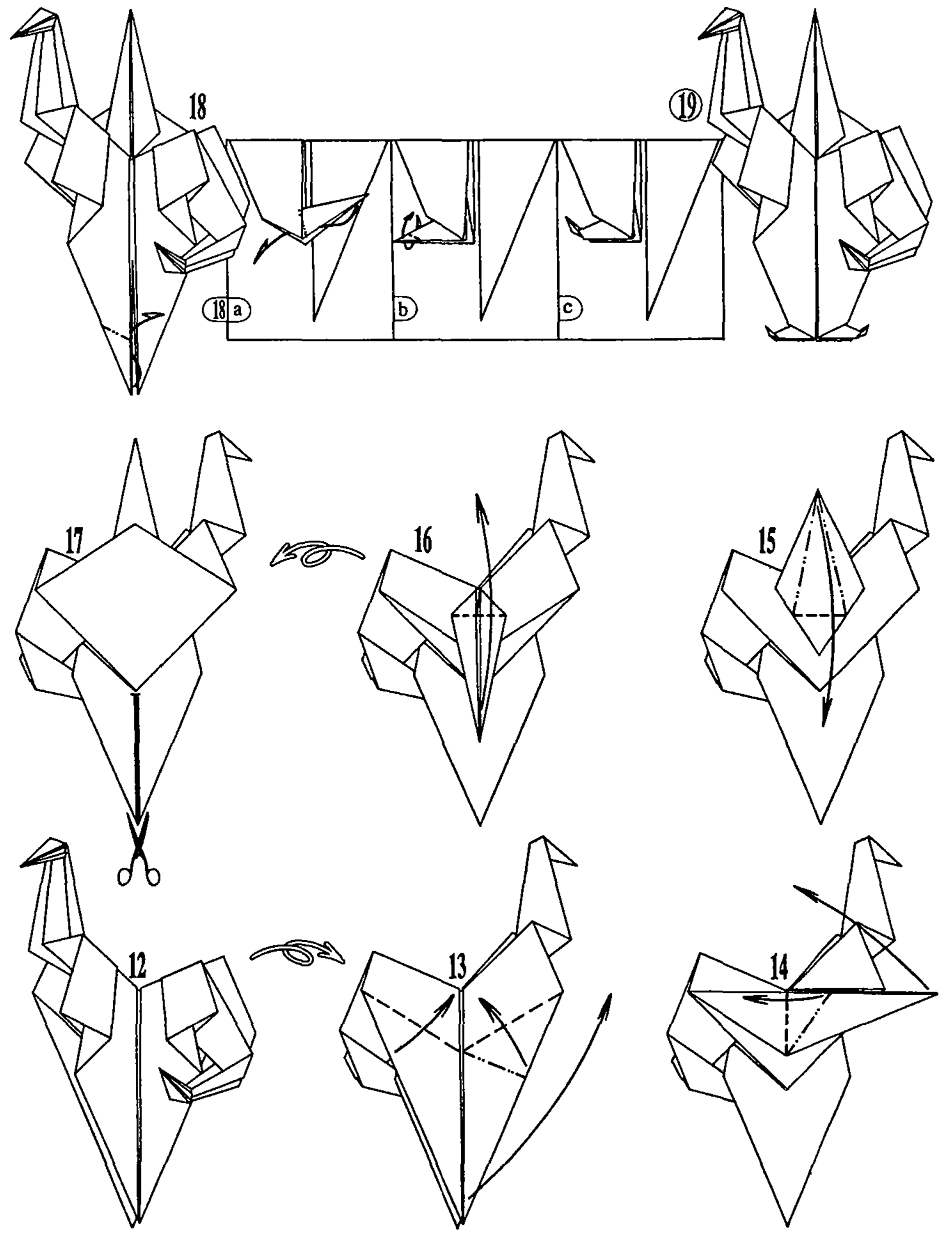

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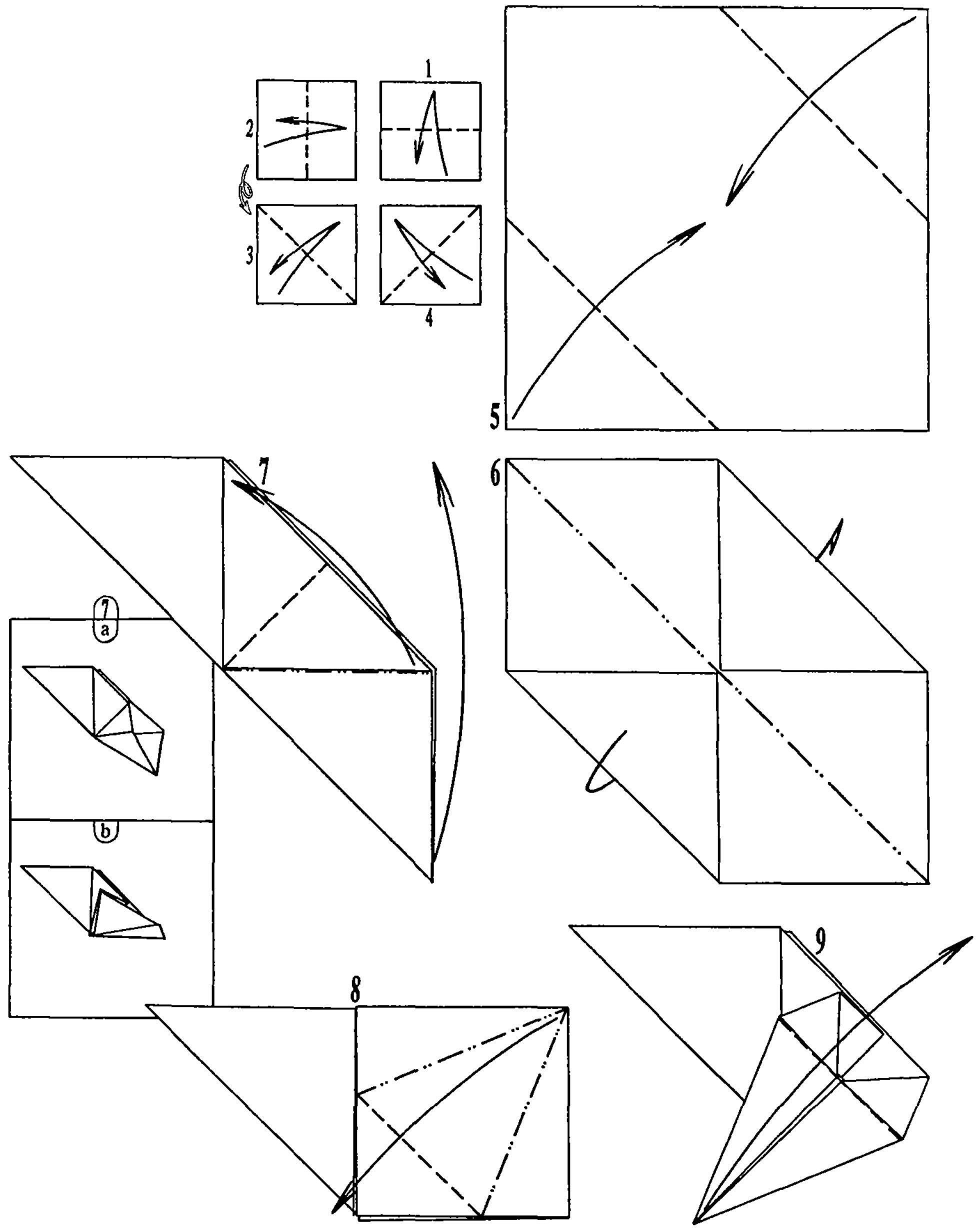

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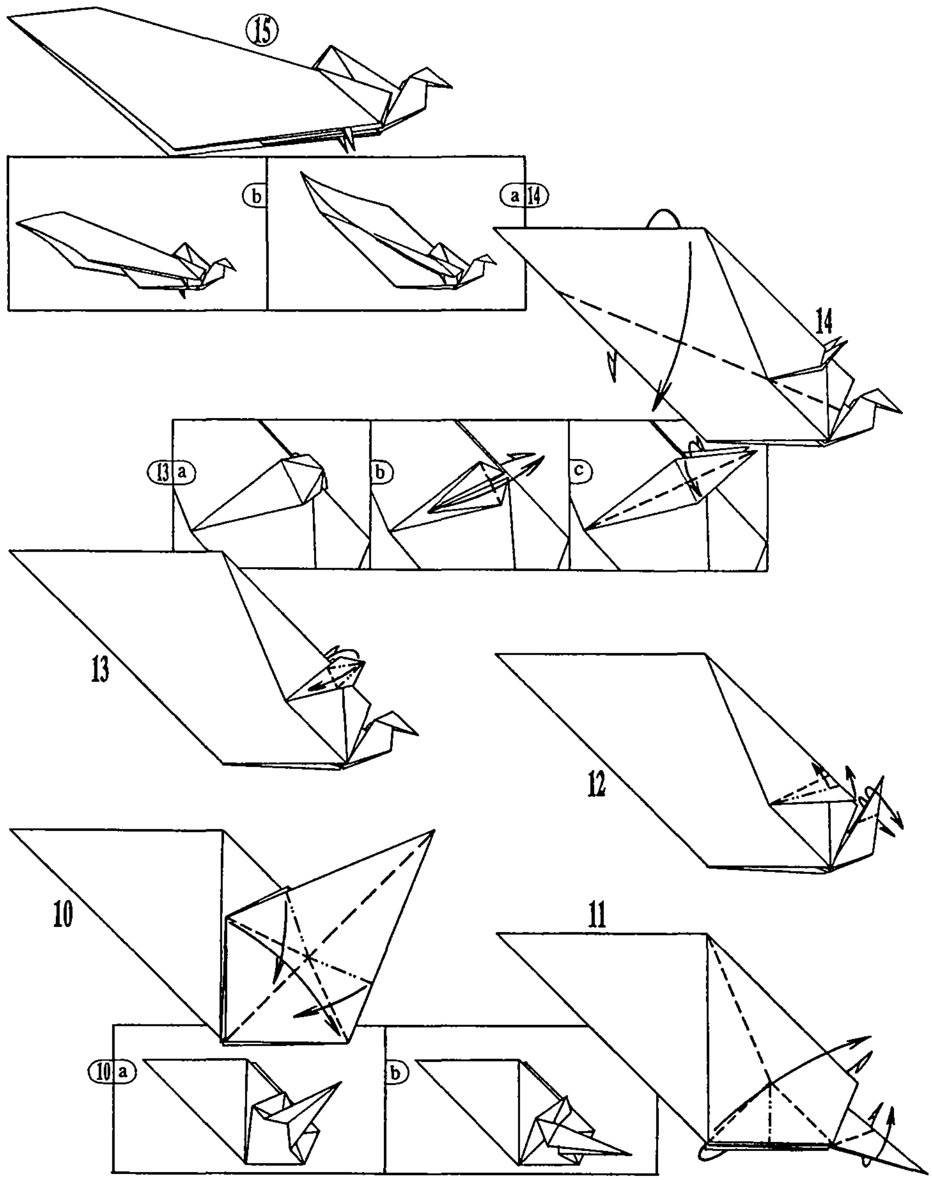
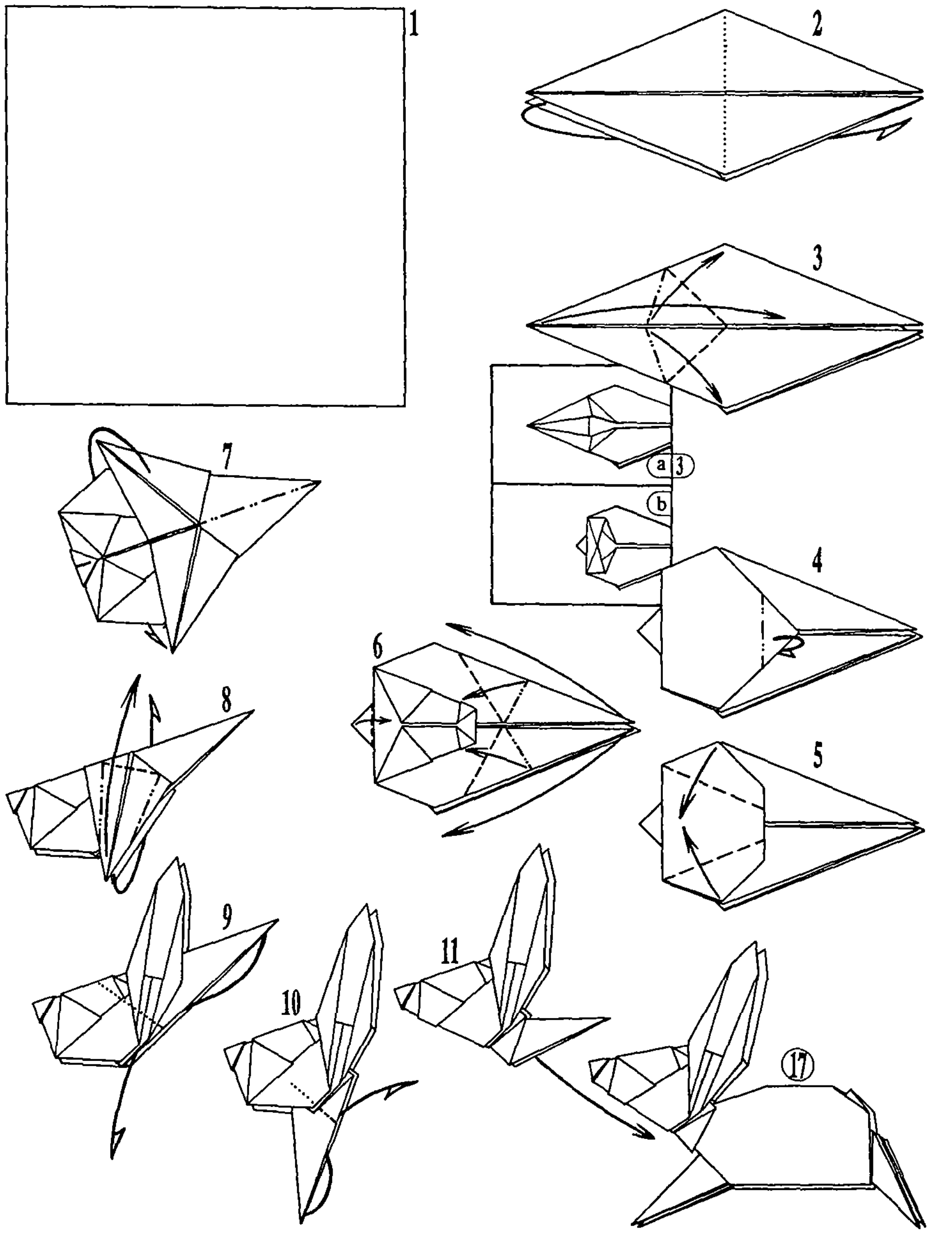


Peacock (2)



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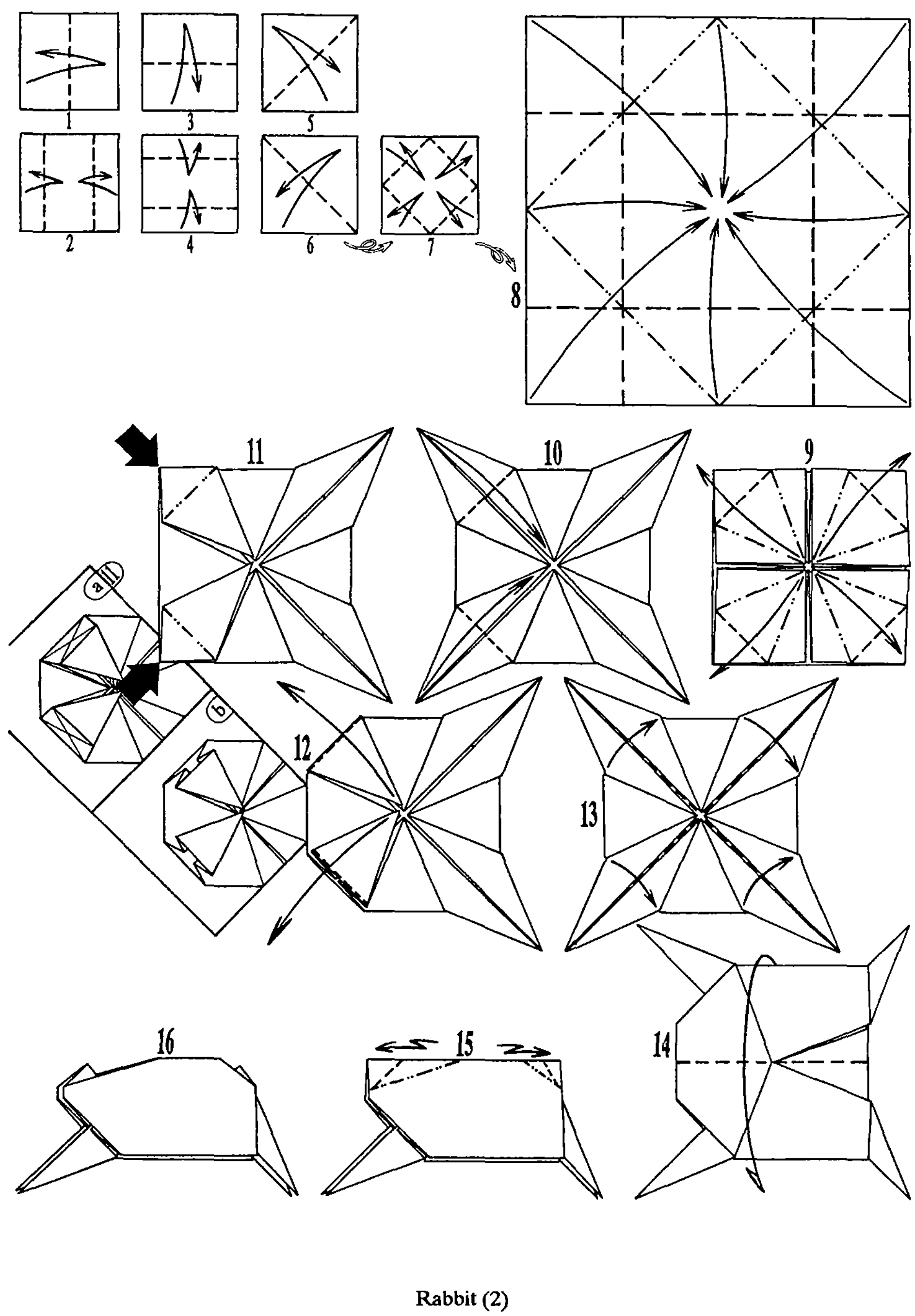
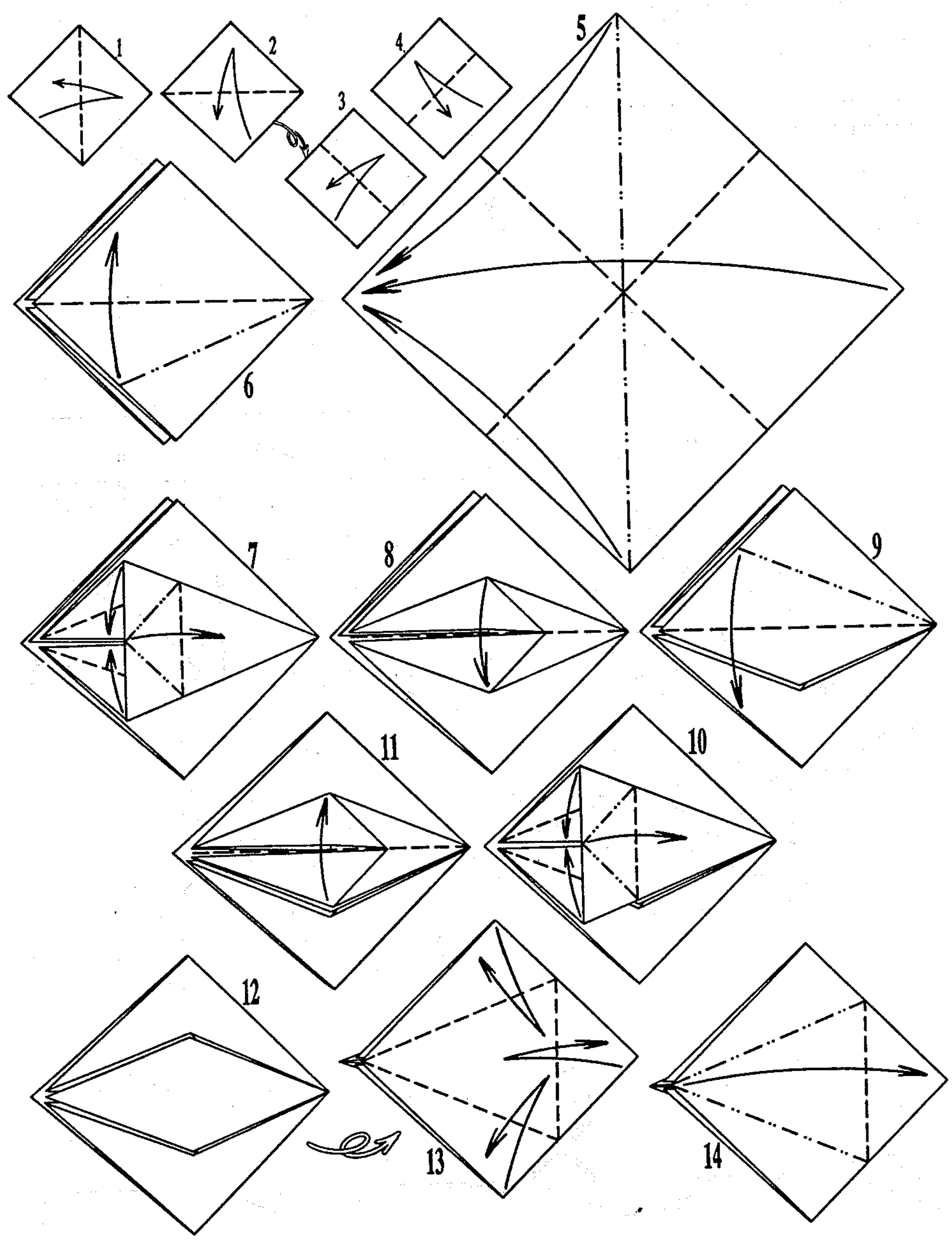
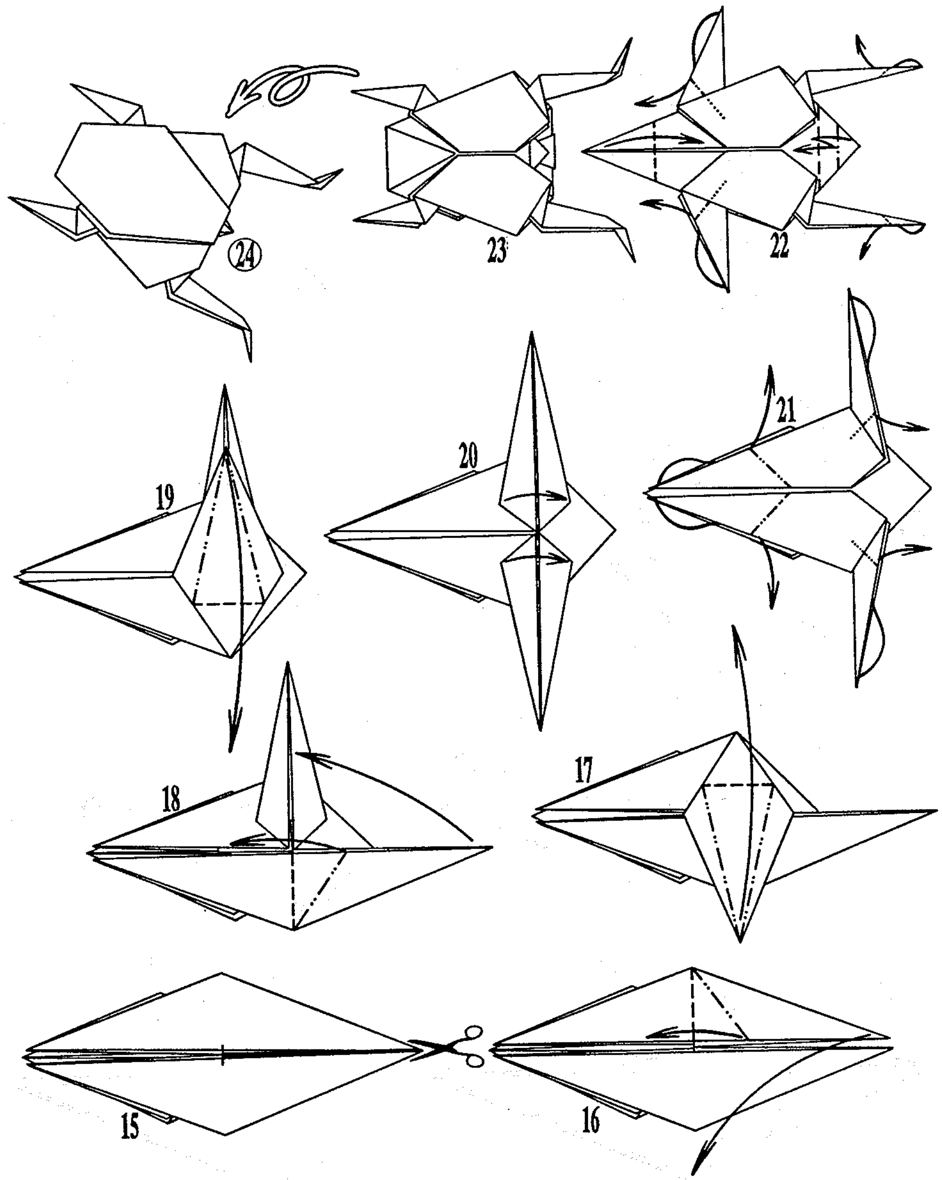


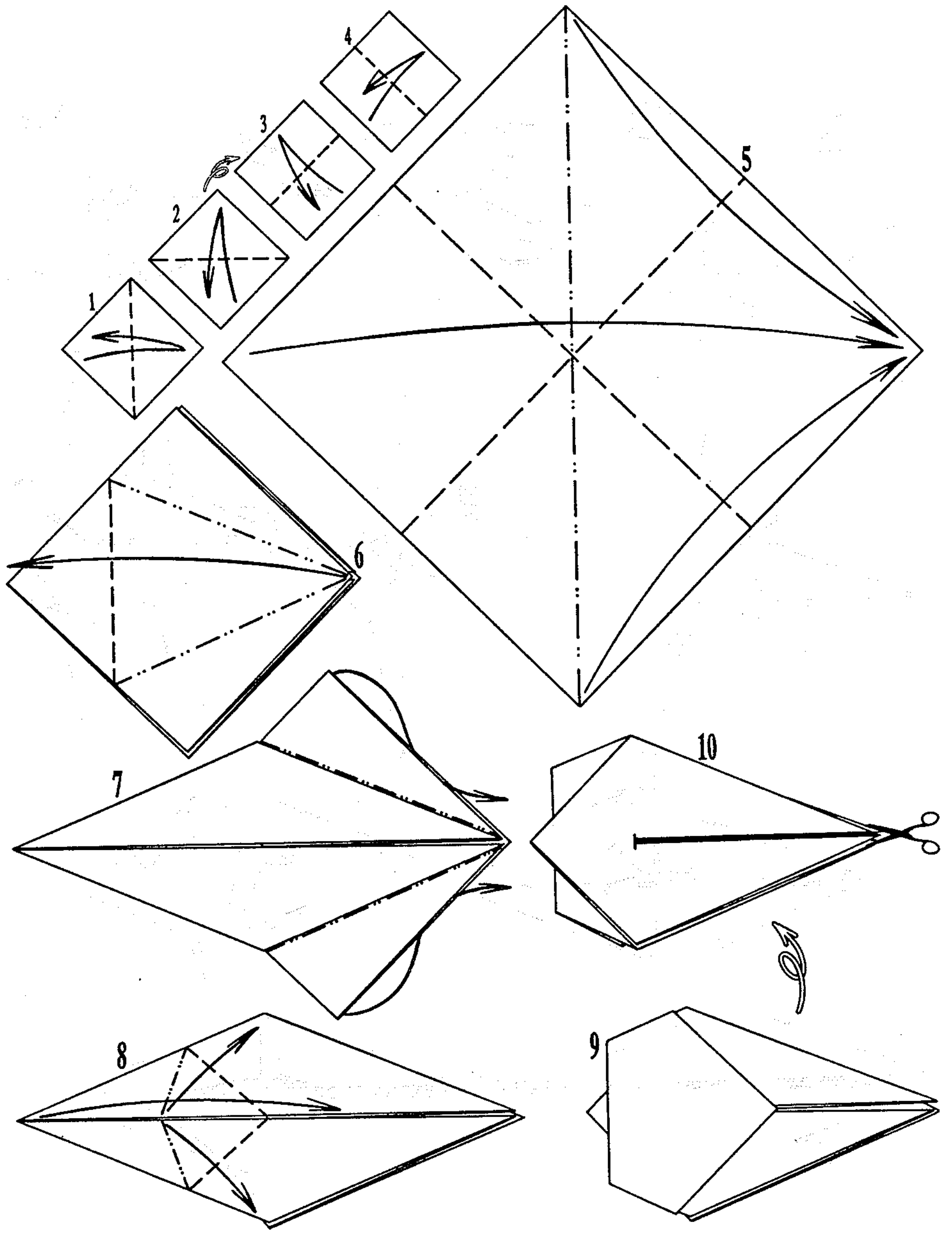
Rabbit (2)



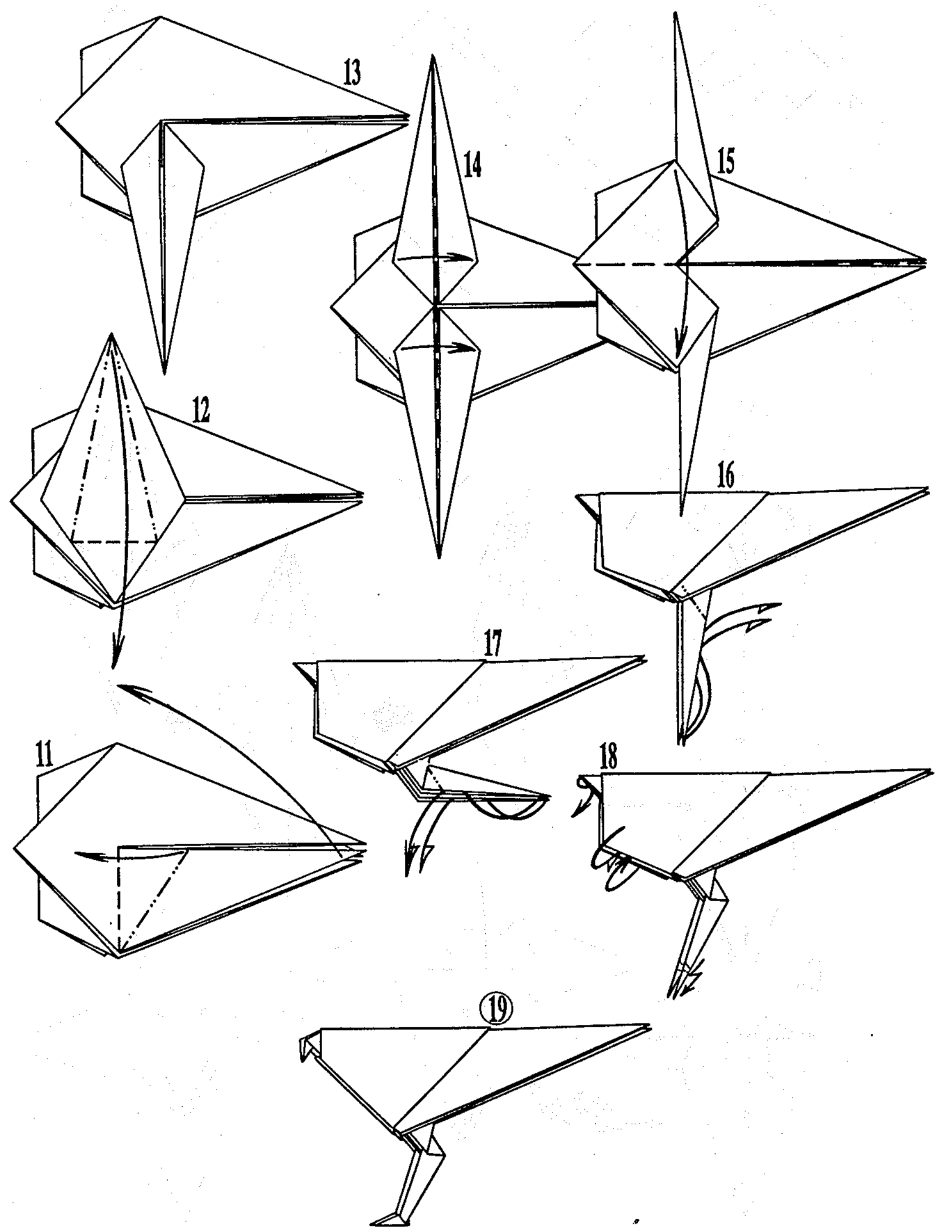
Tree-frog (1)



Tree-frog (2)



Sparrow (1)



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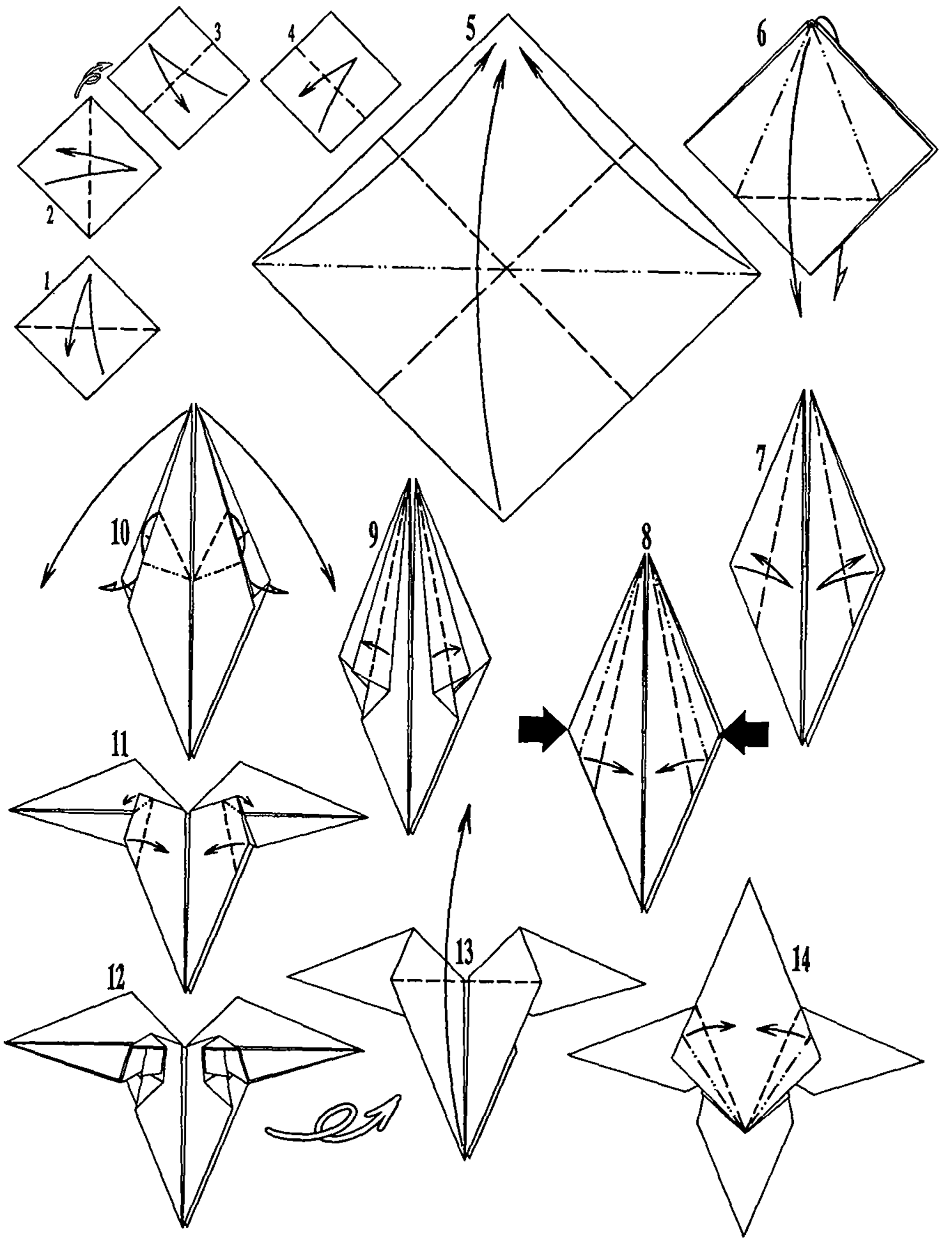
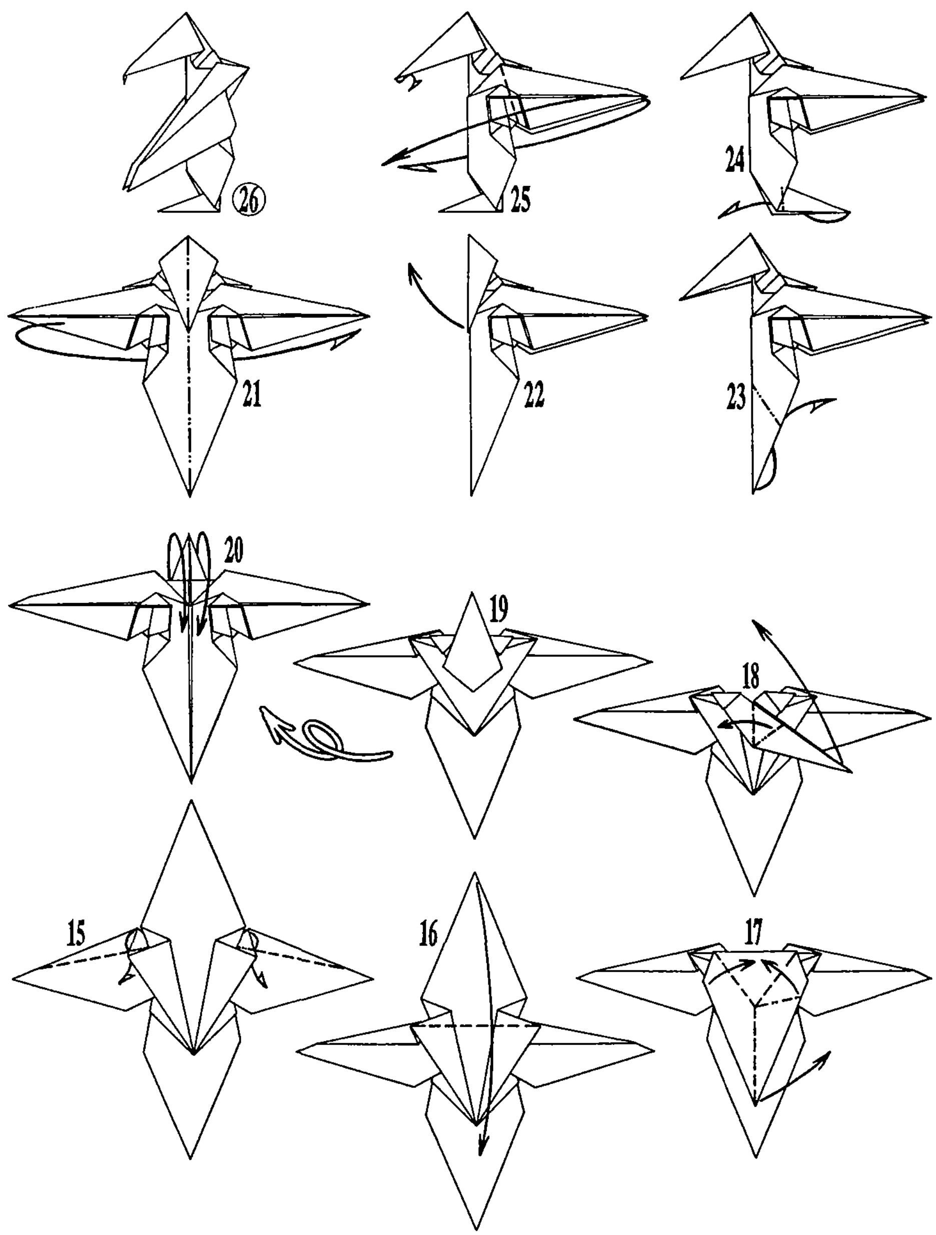


Penguin (1)



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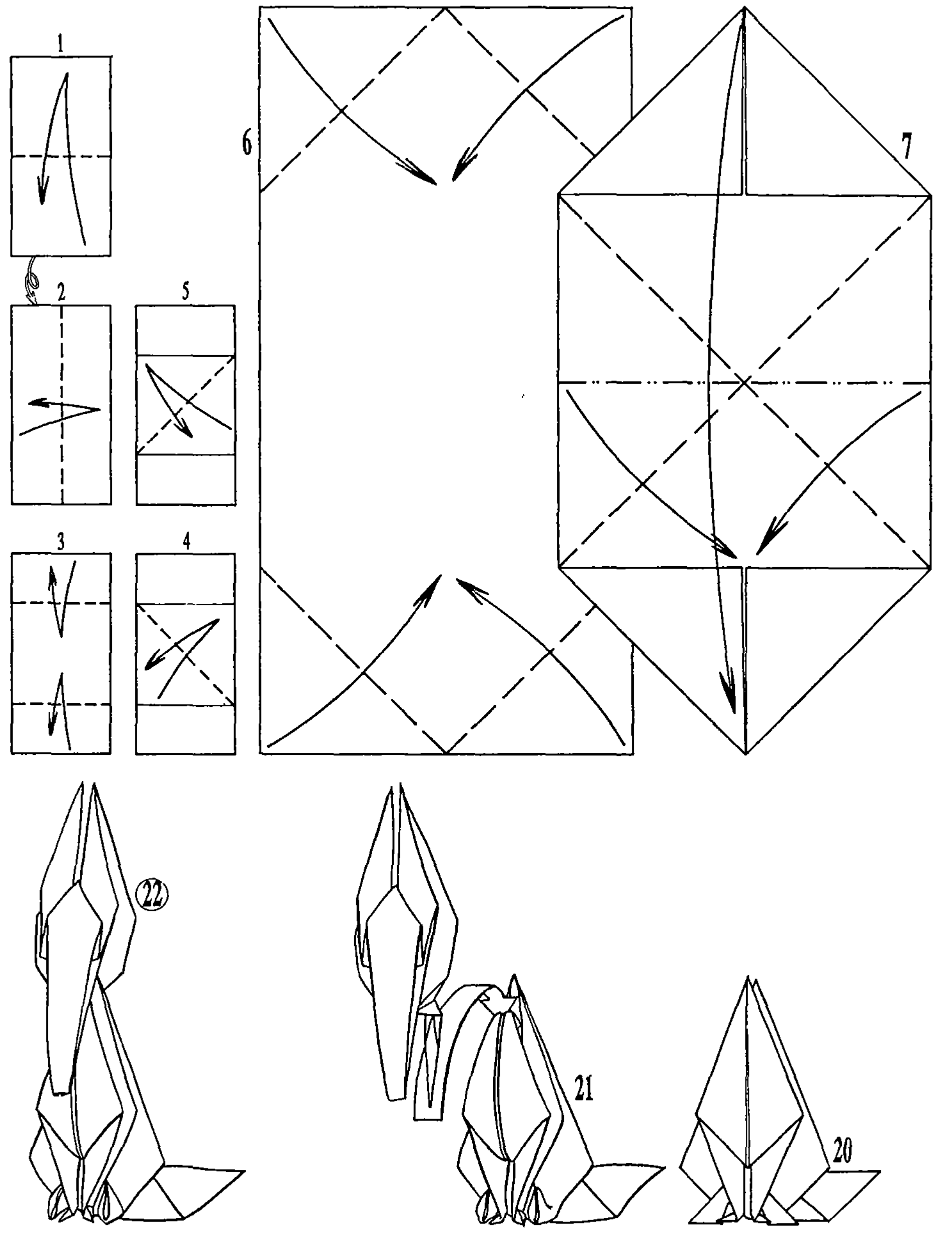
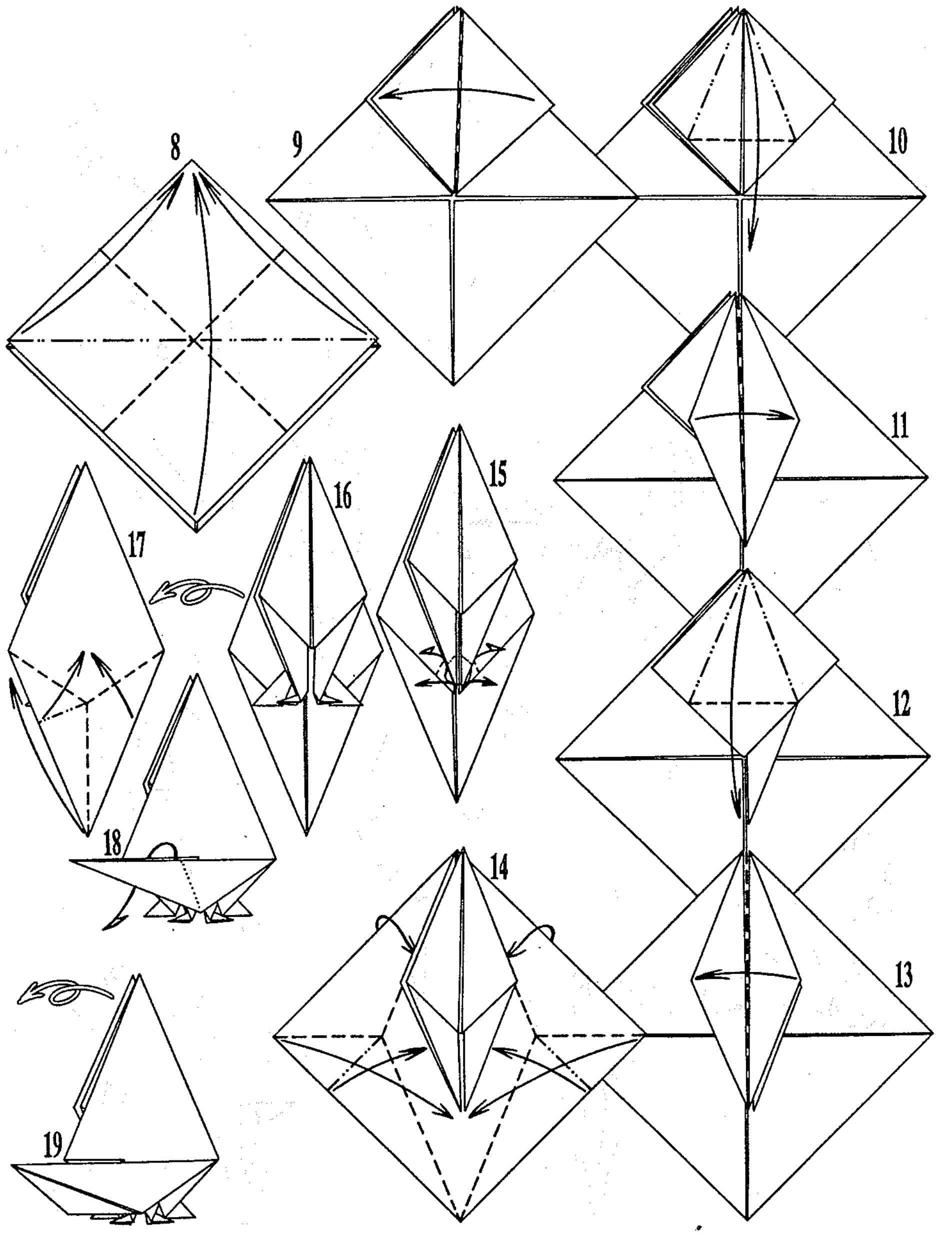
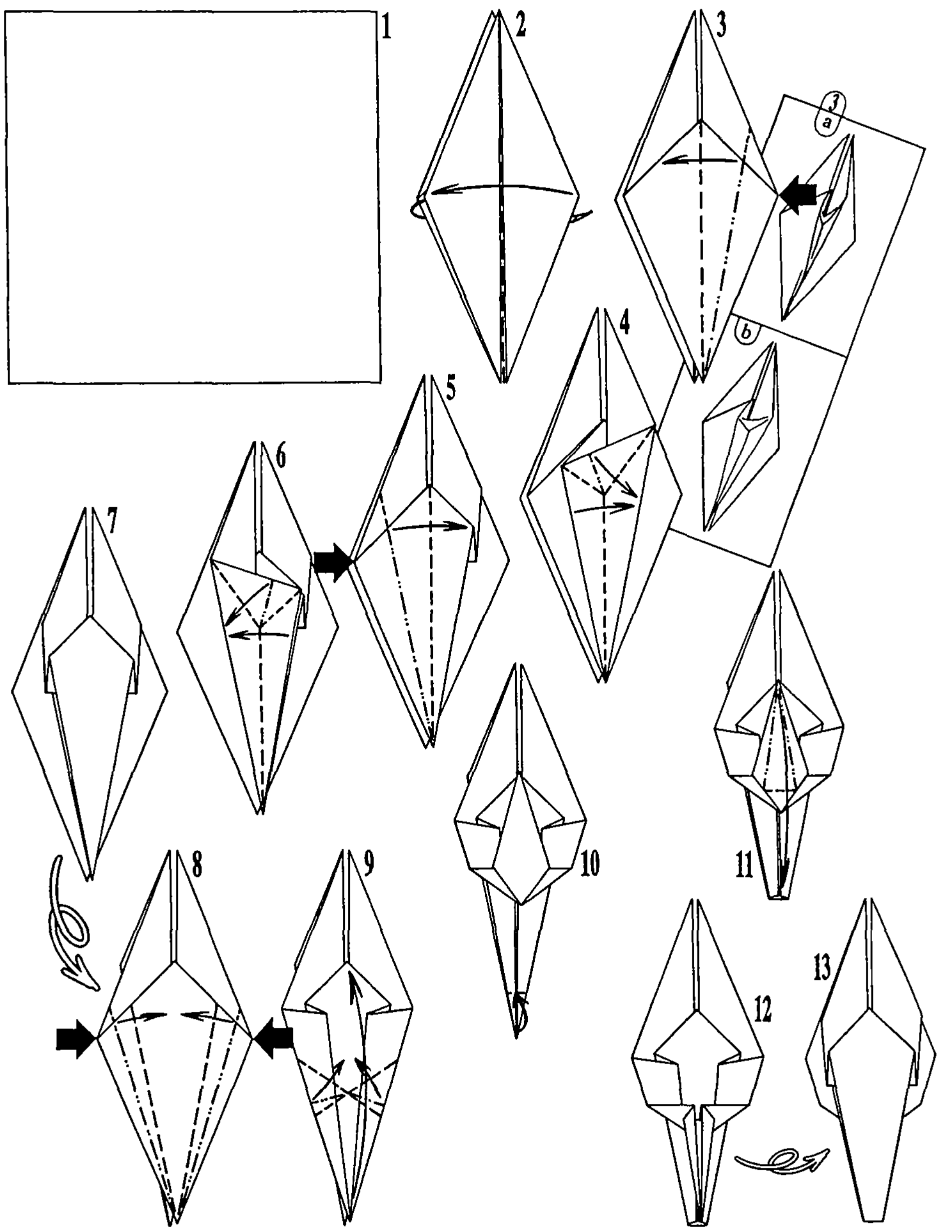


Fox (1)



Fox (2)



Fox (3)

Symmetry: Culture and Science
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SYMMETRO-GRAPHY

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BIBLIOGRAPHY

Origami, paper-folding, and related topics in mathematics and science education

This is not a systematic bibliography, just a selective list of some important items, mostly books and survey papers. After some hesitation, we decided not to include works written in Japanese, with the exception of the monograph by K. Husimi and M. Husimi (1979). (They prefer this romanization of their names; modern libraries use, however, either Hushimi or Fushimi). This work is clearly one of the most important milestones in the field and should be cited here. We hope that this book will be translated into English. The reason for not listing further Japanese works is connected with the fact that those works are not easily available outside Japan, as well as the fact that reading them usually requires some knowledge of Japanese. Note that there is an internationally excepted system of symbols to describe folding processes, thus most books on origami can be 'read' without any knowledge of the actual language of the publication. This statement is, however, less valid in the case of works about origami in science education, where more text is needed. This bibliography also includes the detailed description of the works mentioned in the introductory essay "Symmet-origami (symmetry and origami) in art, science, and technology". At the end of the bibliography the addresses of some related organizations are listed (Section 4).

Finally, we remark that the redundancy in the title of this bibliography, i.e., referring to both origami and paper-folding, intends to refer to a convention that in some countries origami means just the traditional Japanese work, while paper-folding refers to the work by Western people. This convention is less valid in the case of the English speaking countries. Indeed, Britain, Australia, and New Zealand have their own Origami Societies (using this term), while in the U.S.A. there is the noted Origami Center of America in New York, the first origami-related organization outside Japan. The same is also true in Italy, the Benelux countries, and partly in Latin America. The French and the Spanish organizations, however, do not refer to origami, but to "paper-folding".

1 History of origami and paper-folding

1.1 The spread of Japanese origami in the Western world

Harbin, R. [Williams, N.] (1956) *Paper Magic: The Art of Paper Folding*, London: Oldbourne Press, 103 pp. [Robert Harbin is the pseudonym of William Ned, a noted magician; the book is cataloged in some libraries under the original name. The Oxford English Dictionary gives credit to this work for the first usage of the expression "origami" in English, see it on p. 14.].

Honda, I. (1958) *How to Make Origami: The Japanese Art of Paper Folding*, Tokyo: Toto Shuppan, 37 pp.; American ed., New York: McDowell, Obolensky, 1959, 37 pp. [Another early work on origami in English].

Yoshizawa, A. (1983) Origami, In: *Kodansha Encyclopedia of Japan*, Vol. 6, Tokyo: Kodansha, 116-118. [A brief survey on the history of origami by the person who organized pioneering exhibitions in Amsterdam and New York in the 1950s].

1.2 An independent Western tradition: folding napkins

Ive, J. (1983) *Table Napkin Folding: An Elegant Art*, Winchester, Hampshire, England: Josephine Ive, 32 pp; Reprint, Melbourne: Keima Press, 1989, 32 pp.

2 Origami and paper-folding in science and technology

Husimi [Hushimi], K. and Husimi [Hushimi], M. (1979) *Origami no kikagaku*, [Geometry of Origami, in Japanese], Tokyo: Nihon Hyóronsha; Many reprints; Enlarged ed., 1984.

Huzita, H., ed. (1990) *Proceedings of the First International Meeting of Origami Science and Technology*, [Ferrara, Italy, December 6-7, 1989], Padova: Dipartimento di Fisica "Galileo Galilei", Università di Padova, xxvii + 391 pp. [See more details in the "Book Review" of this issue].

Miura, K. (1990) *Structural Forms in Space: The 412th Desgin Gallery Exhibition*, [Folded Exhibition Catalog, in English and Japanese - Tokyo, February 28-March 19, 1990], Tokyo: Design Gallery.

Miura, K., ed. (1994, in prep.) *Second International Meeting of Origami Science and Scientific Origami: Abstracts*, [Otsu (near Kyoto), November 29-December 2, 1994], Otsu, Japan: Seian University of Art and Design.

2.1 Origami and paper-folding in mathematics education

Gardner, M. (1959) Mathematical games: About origami, the Japanese art of folding objects out of paper, *Scientific American*, 201, No. 1, 138-143 and 174.

Johnson, D. A. (1957) *Paper Folding for the Mathematics Class*, Washington, D.C.: National Council of Teachers of Mathematics, 32 pp.

Olson, A. T. (1975) *Mathematics through Paper Folding*, Reston, Virginia: National Council of Teachers of Mathematics, iv + 60 pp.

Sundara Row [Rao], T. (1893) *Geometric Exercises in Paper Folding*, Madras: Addison, 114 pp.; Rev. ed., *T. Sundara Row's Geometric Exercises in Paper Folding*,

Ed. and rev. by W. W. Beman and D. E. Smith, Chicago: Open Court, 1901, xiv + 148 pp.; 2nd ed., *ibid.*, 1905; 3rd ed., *ibid.*, 1917; 4th ed., La Salle, Ill.: Open Court, 1941; Reprint, New York: Dover, 1966.

2.2 Paper models of polyhedra in chemical education

The 'champion' of this approach is S. Yamana who published through more than two decades short notes on his "easily constructed models" using sealed empty envelopes in the *Journal of Chemical Education*. Yamana retired very recently from Kinki University, Higashi Osaka, Japan. We strongly recommend to collect his papers in a book. In the last years the *Journal of Chemical Education* published some similar papers by both Western and Chinese authors. Beyond Yamana's publications, we only list here two papers about models of the fullerene.

Beaton, J. M. (1992) A paper-pattern system for the construction of fullerene molecular models, *Journal of Chemical Education*, 69, 610-612.

Vittal, J. J. (1989) A simple paper model for buckminsterfullerene, *Journal of Chemical Education*, 66, 282.

Yamana, S. (1968, 1980, 1982-1985, 1987-1992) [Very many short notes], *Journal of Chemical Education*. [Some of them in the early 1980s are coauthored by S. Kawaguchi. The papers can be easily located using the annual index of the journal at the end of each volume.].

3 Indirect connection with origami and paper-folding

3.1 Nets of polyhedra, plaiting and braiding of polyhedra

Cundy, H. M. and Rollett, A. P. (1951) *Mathematical Models*, Oxford: Clarendon Press, 240 pp.; 2nd ed., *ibid.*, 1961, 286 pp; 3rd ed., Norfolk, England: Tarquin, 286 pp.; Polish trans. of the 2nd, 1967. [Chap. 3, Polyhedra, includes the nets of various polyhedra; Sub-chapter 3.14 on plaited polyhedra was added to the 2nd ed. on the basis of the work by Pargeter (1959), see below].

Dürer, A. (1525/1977) *The Painter's Manual: A Manual of Measurement of Lines, Areas, and Solids by Means of Compass and Ruler*, [Facsimile of German original of

1525 and modern English trans.], Trans. and commentary by W. L. Strauss, New York: Abaris Books, 472 pp. [Net of some regular and semiregular polyhedra].

Gardner, M. (1971) Mathematical games: The plaiting of Plato's polyhedrons and the asymmetrical yin-yang-lee, *Scientific American*, 224, No. 3, 204-212 and 246. [About Pedersen's variation of plaiting: while Gorham (1888) and Pargeter (1959) used asymmetric strips, she braids Palatonic solids with congruent straight strips].

Gorham, J. (1888) *A System for the Construction of Crystal Models on the Type of an Ordinary Plait, Exemplified by the Forms Belonging to the Six Axial Systems in Crystallography*, London: E. and F. N. Spon, 28 pp. and 56 plates. [The pioneering work of plaiting polyhedra using strips].

Pargeter, A. R. (1959) Plaited polyhedra, *Mathematical Gazette*, 43, 88-101. [The author discovered the book by Gorham (1888) and developed the idea of plaiting into a systematic approach. Note, however, that in the citation of Gorham's book he 'added' an extra word to the title referring to "Plaited Crystal Models", which spread in this incorrect form widely, cf., Cundy and Rollett (1951/1961, p. 152), Gardner (1971, p. 204), Husimi and Husimi (1979, p. 28).].

Pedersen, J. (1988) Why study polyhedra?, In: Senechal, M. and Fleck, G., eds., *Shaping Space: A Polyhedral Approach*, Boston: Birkhäuser, 133-147. [This is a survey article on her method of braiding polyhedra with congruent straight strips, cf., Gardner (1971)].

Wenninger, M. J. (1966) *Polyhedron Models for the Classroom*, Reston, Virginia: National Council of Teachers of Mathematics; Reprint, *ibid.*, 1975, viii + 43 pp. [Traditional paper models].

Wenninger, M. J. (1971) *Polyhedron Models*, London: Cambridge University Press, xii + 208 pp.; Paperback ed., *ibid.*, 1974; Many reprints; Russian trans., 1974; Japanese trans., 1979. [Traditional paper models].

3.2 Flexible paper models

We do not refer here to the topic of flexible polyhedra in general, which was seriously boosted in the late 1970s by R. Connelly's discovery of a flexible surface with triangles. We refer here just to some paper models, including chains of polyhedra, which are flexible.

Chepizhnyi, K. I. (1991) Novye uchebnye modeli gomologicheskikh kristallov; Atlas modelei prima-tel (gomologicheskaya mineralogiya), [New educational models of homological crystals; Atlas of models of prime-bodies (Homological mineralogy), in Russian], In: Usupaev, Sh. É, and Chepizhnyi, K. I. (1991) *Kvarts v lessakh kirgizskogo Tyan'-Shanya*, Frunze [Bishkek], Kirgiz SSR [Kyrgyzstan]: Ilim, Chap. 3, 188-233, Atlas, 243-282.

Schattschneider, D. and Walker, W. (1977) *M. C. Escher Kaleidocycles*, New York: Ballantine Books, iv + 43 pp. and 16 plates; 2nd ed., ibid., 1987; Reprint, Stradbroke, England: Tarquin Publications; Rev. ed., Petaluma, Calif.: Pomegranate Artbooks; Czech, Danish, Dutch, Finnish, French, German, Hungarian, Italian, Japanese, Korean, Norwegian, Portuguese, Slovak, Spanish, Swedish trans.

Schatz, P. (1975) *Rhythmusforschung und Technik*, [Rhythm-Research and Technology, in German], Stuttgart: Freies Geistesleben, 138 pp. [A summary of the main ideas and inventions by Schatz, including the girdle of the cube of 1929 and its applications in technology].

Schatz Gesellschaft (1992) *Paul Schatz*, [Leaflet], Dornach, Switzerland: Paul Schatz Gesellschaft. [Very good illustration of how to obtain the "Würfelgürtel", i.e., the girdle of the cube, a chain of tetrahedral units, that can be endlessly turned].

Schwabe, C. (1992) Flexing polyhedra, *Symmetry: Culture and Science*, 3, 168-169 [Abstract] and 213-221. [Nets of the polyhedra].

3.3 Paper models and foldable structures as scientific topics in art and art education

Escher, M. C. (1991) *M. C. Escher: The Collection of Haags Gemeentemuseum*, [Exhibition Catalog, in English and Japanese – Nagasaki, March 11-September 25, 1991], Nagasaki: Holland Village Museum, 134 pp. [See the exhibition item No. 72, *Paper Construction*, p. 100, cf., the related note on p. 128].

Huff, W. S. (1975, 1977) *Symmetry: An Appreciation of its Presence in Man's Consciousness*, Parts 2, *The Six Isomorphic Coverage Operations*, Part 3, *The Seven Homoeomorphic Coverage Operations*, Pittsburgh: [Privately Published], 16 + [4] and 9 + [6] pp. [Paper models representing various symmetry transformations used at the Hoschule für Gestaltung in Ulm (Ulm School of Design) in the 1950s and 1960s, see p. 2.15 and p. 3.9].

Ganter, B. (1986) *Polyeder der Dreiklänge nach A. F. Möbius (1861)*, [Polyhedron of the Triads after A. F. Möbius (1861), in German – Net of the polyhedron], Darmstadt.

Nagy, D. (1990) The kaleidoscope and symmetry (or, a symmetroscope): Part 2, From science to art (20th century), *Symmetry: Culture and Science*, 1 (1990), No. 2, 119-138. [See the notes on foldable kaleidoscopes, pp. 121-122].

Waters, R. (1992) The unfolding world of Chuck Hoberman, *Discovery*, 13, No. 3, 70-78. [Various practical applications of foldable structures].

Yamawaki, I. (1930) *Studie für Papier-Knickung*, [Study of Paper-Folding, in German — Graphic illustrations], In: *Bauhaus – Revolution und Experiment der Kunstausbildung: 75 jähriges Jubiläum von Bauhaus*, [Exhibition Catalog, in Japanese and German, ed. by M. Fukagawa – Kawasaki, February 12-March 27, 1994], Kawasaki, Japan: Kawasaki City Museum, 1994, p. 54. [Note: the catalog refers to 1931, but the date on the graphic work is 1930].

4 Addresses connected with folding

4.1 Origami organizations

Australia

Australian Origami Society, 31/2 Goderich St., Perth, WA 6000, Australia.

Belgium

Belgisch Nederlandse Origami Societeit, Adolf Reydamslaan 12, B-2400 Mol, Belgium.

International Origami Center Belgium, Degrooflaam, B-2400 Mol, Belgium.

France

Mouvement Français des Plieurs de Papier, 56, rue Coriolis, F-75012 Paris, France.

Germany

Origami Deutschland, Postfach 1630, D-W-8050 Freising, F. R. Germany.

Great Britain

British Origami Society, The Chestnuts, Countesthorpe, Leicester LE8 7JL, U.K.

Italy

Centro Diffusione Origami, P. O. Box 225, I-40100 Bologna, Italy.

Centro Italiano Origami, P. O. Box 357, I-10100 Torino, Italy.

Japan

International Origami Center, P. O. Box 3, Ogikubo, Tokyo 167, Japan.

Nippon Origami Association, Domir Gobancho 1-96, Gobancho 12-7, Chiyoda-ku, Tokyo 102, Japan.

Origami Tanteidan, a Japanese association of origami designers, [they publish a *Newsletter*, in Japanese].

Mexico

Associacion Mexicana de Origami, Apartado Postal 85-063, MEX-10200 Mexico D. F., Mexico.

The Netherlands

Origami Society Netherlands (ORISON), P.O. Box 35, NL-9989 ZG Warffum, The Netherlands.

Origami Organisation Netherlands (ORION), Lindestraat 22, NL-3581 LS Utrecht, The Netherlands.

New Zealand

New Zealand Origami Society, 79 Dunbar Road, Christchurch 3, New Zealand.

Peru

Centro Latina de Origami, Caracas 2655, Dpto 13, Lima 11, Peru.

Spain

Associacion Española de Papiroflexia, Pedro Teixeira 9, Madrid 20, Spain.

U.S.A.

Origami Center of America, 15 West 77th Street, New York City, NY 10024, U.S.A.

E-mail discussion group: the origami mailing list on the Internet. To join, send the message "subscribe origami-l [yourname]" to listserv@nstn.ns.ca .

4.2 Other organizations

AHA Gallery and Shop

AHA, Spiegelgasse 14, CH-8001 Zürich, Switzerland. [This gallery is co-founded by Caspar Schwabe; his models are available in the shop].

Schatz Society

Paul Schatz Gesellschaft, Grenzweg 2, CH-4143 Dornach, Switzerland; Osterbronnstr. 72, D-W-7000 Stuttgart 80, F. R. Germany. [They have various materials on Schatz's life and work, including his flexible objects].

Dénes Nagy

SYMMETRIC REVIEWS 5.1

The "Symmetric Reviews" (SR), as a regular subsection, publish brief notes about books and papers. These are not conventional reviews; their main goal is to emphasize the connections with symmetry and, in same cases, the required background.

Correspondence should preferably be sent to both the section editor (for reviewing) and the Symmetrion in Budapest (for the data bank).

SR 5.1–1 (Origami: science and technology)
Huzita, Humiaki, ed., *Proceedings of the First International Meeting of Origami Science and Technology*, [Ferrara, Italy, December 6-7, 1989], Padova: Dipartimento di Fisica "Galileo Galilei", Università di Padova, [1990], xxvii + 391 pp.

SR 5.1–2 (Textbook: design and bionics; Origami: science and technology)
Kresling, Biruta, *Le Pliage dynamique 1: Design expérimental et bionique*, [Dynamic Folding 1: Experimental Design and Bionics, in French], [Textbook], Aulnoy lez Valenciennes, France: Sup Info Com, 1990, 77 pp.

A note under the contents emphasizes that this is an "evolutionary edition". Indeed, not only is the topic connected with biological shapes, but the related ideas in the textbook may also have an 'evolution'. Of course this process requires an author such as Kresling, who is, in a very positive sense of the expression, both a natural philosopher and an artist. It looks like she is always open to perceiving new pieces of information and adapting them very quickly to her system. Thus her textbook, after a brief introduction to the concepts of morphology and bionics, as well as to the art of origami, deals in detail with the *Miura-ori*, the folding technique by Koryo Miura, which fascinated the author at the Budapest Symposium of ISIS-Symmetry in August 1989, just a couple months before completing (at the actual 'evolutionary level', of course) this textbook. After this part, the author focuses on the wings of insects with a special regard to the folded structures. Kresling and her students also deal with a sort of plaiting technique to make cubes. We recommend to the author to also consider the plaiting technique initiated by Gorham in 1888 and further developed by Pargeter in 1959, as well as the braiding technique by Pedersen (for further details, see the bibliography "Origami, paper-folding, and related topics in mathematics and science education" in this issue). The final part of the book deals with the structure of paper and its deformations. The book is richly illustrated by drawings and photographs. Kresling knows very well that the traditional material cultures provided various items of optimal design and it is worthwhile to study and sketch them. Her textbook, using Umberto Eco's term, is an "open work", and this topic needs exactly such flexibility. We hope to see later stages of the evolving of this textbook. Address: 170, rue Saint-Charles, F-75015 Paris, France.

Dénes Nagy

BOOK REVIEW

Huzita, Humiaki, ed., *Proceedings of the First International Meeting of Origami Science and Technology*, [Ferrara, Italy, December 6-7, 1989], Padova: Dipartimento di Fisica "Galileo Galilei", Università di Padova, [1990], xxvii + 391 pp.

In this volume the most papers are in English, three ones are in French, while three papers by Beloch are reprinted in Italian (including an 1935 paper with the general solution of the cubic equation using folding geometry). The organization of this Meeting is clearly a milestone in the field of origami science and technology, and we are happy to report that it will be followed by another symposium organized by Koryo Miura: *Origami Science and Scientific Origami*, November 29-December 2, 1994–Otsu (near Kyoto), Japan.

The volume includes the following papers: Luigi Pepe, "Remembrance of prof. Margherita Beloch" [after a four-line introduction, three papers by Beloch are reprinted in Italian]; John Smith, "Origami as an art of constraints"; Roberto Morassi, "The elusive pentagon"; Koryo Miura, "Map fold a la Miura style, its physical character and application to the space science"; Wil Oosterbosch, "Towards the realization of the Miura fold by machine"; Humiaki Huzita, "A possible example of system expansion in origami geometry"; Benedetto Scimemi, "Draw of a regular heptagon by the folding"; Maria Paparo, "Origami-therapy applied to a drug addict"; Thoki Yenn, "Origami and insanity"; Emma Frigerio, "New relations in origami geometry: Proposed by J. Justin"; Toshikazu Kawasaki, "On high dimensional flat origamis"; Humiaki Huzita, "Axiomatic development of origami geometry"; Humiaki Huzita, "A problem on the Kawasaki theorem"; Yasushi Kajikawa, "Complementary unit origami: Maximum volume with minimum material, more with less"; M. L. Wantzel, "Recherches sur les moyens de reconnaître si un problème de géométrie peut se résoudre avec la règle et le compas"; Humiaki Huzita, "The trisection of a given angle solved by the geometry of origami"; Humiaki Huzita and Benedetto Scimemi, "Algebra of paper-folding (origami)"; Toshikazu Kawasaki and Masaaki Yoshida,"Crystallographic flat origamis"; Toshikazu Kawasaki, "On relation between mountain-creases and valley-creases of a flat origami"; Koryo Miura, "A note on intrinsic geometry of origami"; Jacques Justin, "Resolution par le pliage de l'equation du troisieme degre et applications geometriques"; Jacques Justin, "Aspects mathematiques du pliage de papier"; Michel Mendès France, "The Rudin-Schapiro sequence, Ising chain, and paperfolding"; Michel Mendès France, "Wire bending"; Michel Mendès France, "Folding paper and thermodynamics"; M. Mendès France and A. J. van der Poorten, "Arithmetic and analytic properties of paper folding sequences"; J. H. Loxton and A. J. van der Poorten, "Arithmetic proprties of the solutions of a class of functional equations"; Michel Dekking, Michel Mendès France, and Alf van der Poorten "Folds!"; Emma Frigerio, "Origami geometry: old and new". Note various discrepancies between the contents (which is called index) and the headings of papers, e.g., Koryo Miura's name is misspelled in the contents (as Koryu Miura) twice; the correct spelling of the name of Wil Oosterbosch is not clear, it is given in this form in the contents and the second part of his paper, while the heading of his paper and the first sentence refer to Ooberbosch; the authorship of the paper "Axiomatic development of origami geometry" is not clear because the table of contents refers to both Frigerio and Huzita, while the heading of the paper only gives Huzita's name; Wantzel is given as Wanzel in the contents, as well as various words of the title are misspelled there; etc. We listed here mostly those versions of names and titles that are given by the authors in their camera-ready papers. Some of the papers will be reviewed individually. Address: INFN c/o Department of Physics, University of Padova, Via Marzolo 8, I-35131 Padova, Italy.

Dénes Nagy

Contributions to *SYMMETRY: CULTURE AND SCIENCE* are welcomed from the broadest international circles and from representatives of all scholarly and artistic fields where symmetry considerations play an important role. The papers should have an interdisciplinary character, dealing with symmetry in a concrete (not only metaphorical!) sense, as discussed in "Aims and Scope" on p. 112. The quarterly has a special interest in how distant fields of art, science, and technology may influence each other in the framework of symmetry (symmetrology). The papers should be addressed to a broad non-specialist public in a form which would encourage the dialogue between disciplines.

Contributors should note the following:

- All papers and notes are published in English and they should be submitted in that language. The quarterly reviews and annotates, however, non-English publications as well.

- In the case of complicated scientific concepts or theories, the intuitive approach is recommended, thereby minimizing the technical details. New associations and speculative remarks can be included, but their tentative nature should be emphasized. The use of well-known quotations and illustrations should be limited, while rarely mentioned sources, new connections, and hidden dimensions are welcomed.

- The papers should be submitted either by electronic mail to both editors, or on computer diskettes (5 ¼" or 3.5") to György Darvas as text files (IBM PC compatible or Apple Macintosh); that is, conventional characters should be used (ASCII) without italics or other formatting commands. Of course typewritten texts will not be rejected, but the preparation of these items takes longer. For any method of submission (e-mail, diskette, or typescript), four hard-copies of the text are also required, where all the necessary editing is marked in red (inserting non-ASCII characters, underlining words to be italicized, etc.). Three hard-copies, including the master copy and the original illustrations, should be forwarded to György Darvas, while the fourth copy should be sent to Dénes Nagy. No manuscripts, diskettes, or figures will be returned, unless by special arrangement.

- The papers are accepted for publication on the understanding that the copyright is assigned to ISIS-Symmetry. The Society, however, aiming to encourage the cooperation, will allow all reasonable requests to photocopy articles or to reuse published materials. Each author will receive a complimentary copy of the issue where his/her article appeared.

- Papers should begin with the title, the proposed running head (abbreviated form of the title of less than 35 characters), the proposed section of the quarterly where the article should appear (see the list in the note 'Aims and Scope'), the name of the author(s), the mailing address (office or home), the electronic mail address (if any), and an abstract of between 10 and 15 lines. A recent black-and-white photo, the biographic data, and the list of symmetry-related publications of (each) author should be enclosed; see the sample at the end.

- Only black-and-white, camera-ready illustrations (photos or drawings) can be used. The required (approximate) location of the figures and tables should be indicated in the main text by typing their numbers and captions (Figure 1: [text], Figure 2: [text], Table 1: [text], etc.), as new paragraphs. The figures, which will be slightly reduced in printing, should be enclosed on separate sheets. The tables may be given inside the text or enclosed separately.

- It is the author's responsibility to obtain written permission to reproduce copyright materials.

- Either the British or the American spelling may be used, but the same convention should be followed throughout the paper. *The Chicago Manual of Style* is recommended in case of any stylistic problem.

- Subtitles (numbered as 1, 2, 3, etc.) and subsidiary subtitles (1.1, 1.1.1, 1.1.2, 1.2, etc.) can be used, without over-organizing the text. Footnotes should be avoided; parenthetic inserts within the text are preferred.

- The use of references is recommended. The citations in the text should give the name, year, and, if necessary, page, chapter, or other number(s) in one of the following forms: ... Weyl (1952, pp. 10-12) has shown...; or ... as shown by some authors (Coxeter et al., 1986, p. 9; Shubnikov and Koptsik 1974, Chap. 2; Smith, 1981a, Chaps. 3-4; Smith, 1981b, Sec. 2.12; Smith, forthcoming). The full bibliographic description of the references should be collected at the end of the paper in alphabetical order by authors' names; see the sample. This section should be entitled *References*.

Sample of heading (Apologies for the strange names and addresses)

SYMMETRY IN AFRICAN ORNAMENTAL ART
BLACK-AND-WHITE PATTERNS IN CENTRAL AFRICA
Running head: Symmetry in African Art
Section: Symmetry: Culture & Science

Susanne Z. Dissymmetrist and Warren M. Symmetrist
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Sunflower City, CA 11235, U.S.A.

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69 Harmony Street, San Symmetrino, CA 69869, U.S.A.
E-mail: symmetrist@symmetry.edu

Abstract

The ornamental art of Africa is famous ...

Sample of references

In the following, note punctuation, capitalization, the use of square brackets (and the remarks in parentheses). There is always a period at the very end of a bibliographic entry (but never at other places, except in abbreviations). Brackets are used to enclose supplementary data. Those parts which should be italicized — titles of books, names of journals, etc. — should be underlined in red on the hard-copies. In the case of non-English publications both the original and the translated titles should be given (cf., Dissymmetrist, 1990).

Asymmetrist, A. Z. (or corporate author) (1981) *Book Title: Subtitle*, Series Title, No. 27, 2nd ed., City (only the first one): Publisher, vii + 619 pp.; (further data can be added, e.g.) 3rd ed., 2 Vols., ibid., 1985, viii + 444 + 484 pp. and 2 computer diskettes; Reprint, ibid., 1988; German trans., *German Title*, 2 Vols., City: Publisher, 1990, 986 pp.; Hungarian trans.

Asymmetrist, A. Z., Dissymmetrist, S. Z., and Symmetrist, W. M. (1980-81) Article or e-mail article title: Subtitle, Parts 1-2, *Journal Name Without Abbreviation*, [E-Journal or Discussion Group address: journal@node (if applicable)], B22 (volume number), No. 6 (issue number if each one restarts pagination), 110-119 (page numbers); B23, No. 1, 117-132 and 148 (for e-journals any appropriate data).

Dissymmetrist, S. Z. (1989a) Chapter, article, symposium paper, or abstract title, [Abstract (if applicable)], In: Editorologist, A.B. and Editorologist, C.D., eds., *Book, Special Issue, Proceedings, or Abstract Volume Title*, [Special Issue (or) Symposium organized by the Dissymmetry Society, University of Symmetry, San Symmetrino, Calif., December 11-22, 1971 (those data which are not available from the title, if applicable)], Vol. 2, City: Publisher, 19-20 (for special issues the data of the journal).

Dissymmetrist, S. Z. (1989b) *Dissertation Title*, [Ph.D. Dissertation], City: Institution, 248 pp. (Exhibition Catalogs, Manuscripts, Master's Theses, Mimeographs, Patents, Preprints, Working Papers, etc. in a similar way; Audiocassettes, Audiotapes, Compact Disks, Computer Diskettes, Computer Software, Films, Microfiches, Microfilms, Slides, Sound Disks, Videocasettes, etc. with necessary modifications, adding the appropriate technical data).

Dissymmetrist, S. Z., ed. (1990) *Dissimmetriya v nauke* (title in original, or transliterated, form), [Dissymmetry in science, in Russian with German summary], Trans. from English by B. W. Antisymmetrist, etc.

Phyllotaxist, F. B. (1899/1972) *Title of the 1972 Edition*, [Reprint, or Translation, of the 1899 ed.], etc.

[Symmetrist, W. M.] (1989) Review of *Title of the Reviewed Work*, by S. Z. Dissymmetrist, etc. (if the review has an additional title, then it should appear first; if the authorship of a work is not revealed in the publication, but known from other sources, the name should be enclosed in brackets).

In the case of lists of publications, or bibliographies submitted to *Symmetro-graphy*, the same convention should be used. The items may be annotated, beginning in a new paragraph. The annotation, a maximum of twenty lines, should emphasize those symmetry-related aspects and conclusions of the work which are not obvious from the title. For books, the list of (important) reviews, can also be added.

Sample of biographic entry

Name: Warren M. Symmetrist, Mathematician, (b. Boston, Mass., U.S.A., 1938).

Address: Department of Dissymmetry, University of Symmetry, 69 Harmony Street, San Symmetrino, Calif. 69869, U.S.A. *E-mail:* symmetrist@symmetry.edu .

Fields of interest: Geometry, mathematical crystallography (also ornamental arts, anthropology — non-professional interests in parentheses).

Awards: Symmetry Award, 1987; Dissymmetry Medal, 1989.

Publications and/or *Exhibitions*: List all the symmetry-related publications/exhibitions in chronological order, following the conventions of the references and annotations. Please mark the most important publications, not more than five items, by asterisks. This shorter list will be published together with the article, while the full list will be included in the computerized data bank of ISIS-Symmetry.

There are many disciplinary periodicals and symposia in various fields of art, science, and technology, but broad interdisciplinary forums for the connections between distant fields are very rare. Consequently, the interdisciplinary papers are dispersed in very different journals and proceedings. This fact makes the cooperation of the authors difficult, and even affects the ability to locate their papers.

In our 'split culture', there is an obvious need for interdisciplinary journals that have the basic goal of building bridges ('symmetries') between various fields of the arts and sciences. Because of the variety of topics available, the concrete, but general, concept of symmetry was selected as the focus of the journal, since it has roots in both science and art.

Symmetry: Culture and Science is the quarterly of the *International Society for the Interdisciplinary Study of Symmetry* (abbreviation: *ISIS-Symmetry*, shorter name: *Symmetry Society*). ISIS-Symmetry was founded during the symposium *Symmetry of Structure* (*First Interdisciplinary Symmetry Symposium and Exhibition*), Budapest, August 13-19, 1989. The focus of ISIS-Symmetry is not only on the concept of symmetry, but also its associates (asymmetry, dissymmetry, antisymmetry, etc.) and related concepts (proportion, rhythm, invariance, etc.) in an interdisciplinary and intercultural context. We may refer to this broad approach to the concept as *symmetrology*. The suffix *-logy* can be associated not only with knowledge of concrete fields (cf., biology, geology, philology, psychology, sociology, etc.) and discourse or treatise (cf., methodology, chronology, etc.), but also with the Greek terminology of proportion (cf., *logos*, *analogia*, and their Latin translations *ratio*, *proportio*).

The basic goals of the *Society* are
(1) to bring together artists and scientists, educators and students devoted to, or interested in, the research and understanding of the concept and application of symmetry (asymmetry, dissymmetry);
(2) to provide regular information to the general public about events in symmetrology;
(3) to ensure a regular forum (including the organization of symposia, congresses, and the publication of a periodical) for all those interested in symmetrology.

The Society organizes the triennial *Interdisciplinary Symmetry Congress and Exhibition* (starting with the symposium of 1989) and other workshops, meetings, and exhibitions. The forums of the Society are *informal* ones, which do not substitute for the disciplinary conferences, only supplement them with a broader perspective.

The Quarterly - a non-commercial scholarly journal, as well as the forum of ISIS-Symmetry - publishes original papers on symmetry and related questions which present new results or new connections between known results. The papers are addressed to a broad non-specialist public, without becoming too general, and have an interdisciplinary character in one of the following senses:
(1) they describe concrete interdisciplinary 'bridges' between different fields of art, science, and technology using the concept of symmetry;
(2) they survey the importance of symmetry in a concrete field with an emphasis on possible 'bridges' to other fields.

The Quarterly also has a special interest in historic and educational questions, as well as in symmetry-related recreations, games, and computer programs.

The regular sections of the Quarterly:
- **Symmetry: Culture & Science** (papers classified as humanities, but also connected with scientific questions)
- **Symmetry: Science & Culture** (papers classified as science, but also connected with the humanities)
- **Symmetry in Education** (articles on the theory and practice of education, reports on interdisciplinary projects)
- **SFS: Symmetric Forum of the Society** (calendar of events, announcements of ISIS-Symmetry, news from members, announcements of projects and publications)
- **Symmetro-graphy** (biblio/disco/software/ludo/historio-graphies, reviews of books and papers, notes on anniversaries)

Additional non-regular sections:
- **Symmetrospective: A Historic View** (survey articles, recollections, reprints or English translations of basic papers)
- **Symmetry: A Special Focus on ...** (round table discussions or survey articles with comments on topics of special interest)
- **Symmetric Gallery** (works of art)
- **Mosaic of Symmetry** (short papers within a discipline, but appealing to broader interest)
- **Research Problems on Symmetry** (brief descriptions of open problems)
- **Recreational Symmetry** (problems, puzzles, games, computer programs, descriptions of scientific toys; for example, tilings, polyhedra, and origami)
- **Reflections: Letters to the Editors** (comments on papers, letters of general interest)

Both the lack of seasonal references and the centrosymmetric spine design emphasize the international character of the Society; to accept one or another convention would be a 'symmetry violation'. In the first part of the abbreviation *ISIS-Symmetry* all the letters are capitalized, while the centrosymmetric image iSIS! on the spine is flanked by 'Symmetry' from both directions. This convention emphasizes that ISIS-Symmetry and its quarterly have no direct connection with other organizations or journals which also use the word *Isis* or *ISIS*. There are more than twenty identical acronyms and more than ten such periodicals, many of which have already ceased to exist, representing various fields, including the history of science, mythology, natural philosophy, and oriental studies. ISIS-Symmetry has, however, some interest in the symmetry-related questions of many of these fields.

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ISIS-Symmetry
Budapest, P.O. Box 4
H-1361 Hungary

CALL FOR PAPERS AND WORKSHOP TOPICS

SYMMETRY: NATURAL AND ARTIFICIAL

Third Interdisciplinary
Symmetry Congress
of the

INTERNATIONAL SOCIETY FOR
THE INTERDISCIPLINARY
STUDY OF SYMMETRY
(ISIS-SYMMETRY)

August 14 - 20, 1995
Old Town Alexandria
(near Washington, D.C.)
U.S.A.

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CALL FOR EXHIBITION ITEMS

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