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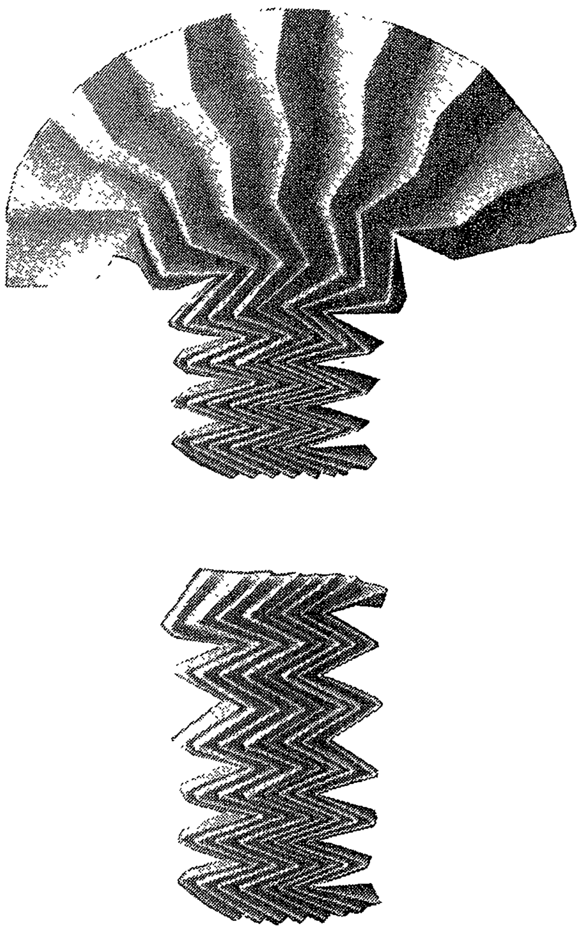

The *Miura-ori*
opened out like a fan

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BREAKING SYMMETRY: ORIGAMI, ARCHITECTURE, AND THE FORMS OF NATURE

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Abstract: *Most origami figures that attempt to replicate the forms of nature exhibit bilateral or rotational symmetry like the mammals, birds, and flowers they take as their model. In this paper I will show how I have attempted to capture some of nature's* asymmetric *forms such as leaves, mountains, and a coiled rattlesnake. Drawing analogies from nature and the formation of human settlements, I will examine different processes of growth and show how I have utilized those processes to 'break symmetry', transforming patterns that are symmetric into ones that are asymmetric. I will conclude by offering some subjective opinions on the philosophy and aesthetics of asymmetry.*

1. INTRODUCTION: THE SYMMETRIES OF THE SQUARE

It is easy to see why paperfolders are drawn to natural subjects such as birds, mammals, and simple flowers. The square, with its many symmetries, lends itself to capturing forms like these because they have bilateral or rotational symmetry. My own origami work for many years was devoted to discovering the seemingly limitless potential contained within the square's geometry. Inspired by the investigations of a handful of scientists and artists into the mathematical structure of nature (most notably by Loeb, 1971; Mandelbrot, 1982; Smith, 1981; Stevens, 1974; Thompson, 1917; and, more recently, Hargittai, 1994), I applied symmetry operations such as reflection, rotation, translation, change of scale, and the grafting of one pattern onto another to generate complex forms from simple ones. Some examples are shown in Figures 1 and 2. Such models of mine as a sailboat, motorcar, tortoise, reindeer, tiger, and elephant demonstrate bilateral mirror symmetry. A squid, scorpion, lighthouse, hot-air balloon, sun, and star employ rotational symmetry. A kangaroo, octopus, alligator, and butterfly use grafting and changes of scale. Three examples of symmetry operations taken from my first origami book (Engel, 1989) show how complicated forms like these were generated (Fig. 3).

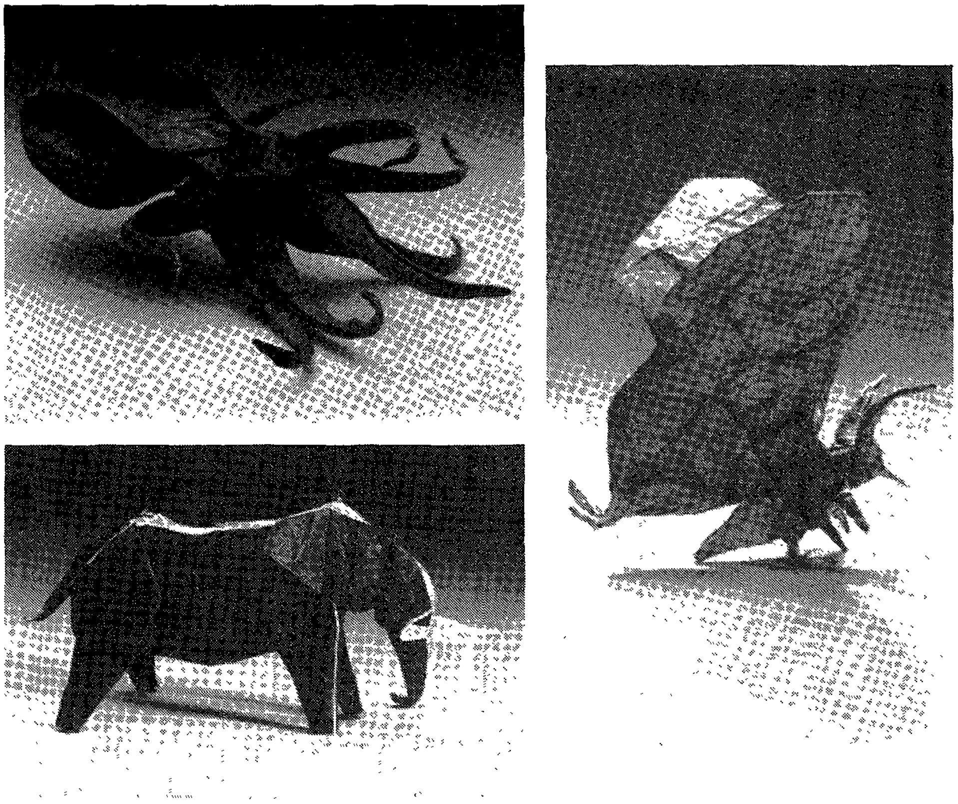


Figure 1: Some of the author's original models exhibit a high degree of symmetry. As befits an octopod, the author's octopus has eight-fold rotational symmetry as well as mirror symmetry. Its form is generated through a process of grafting. An elephant has only bilateral symmetry while a butterfly begins with four-fold rotational symmetry but in its final stage exhibits only bilateral symmetry, a process that may mirror the cellular development of the organism itself.

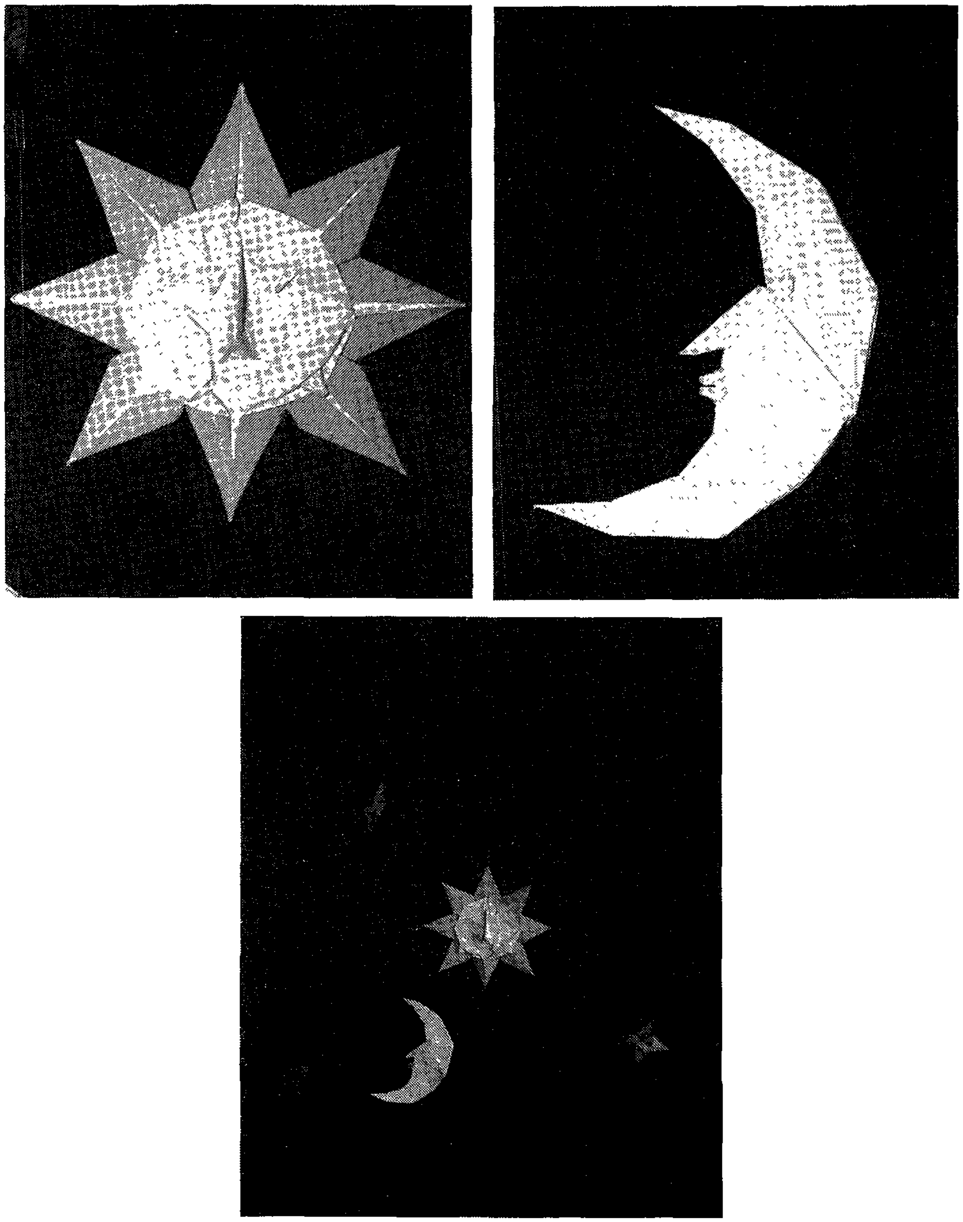


Figure 2: Mobile of a sun, moon, and stars reveals varying types of symmetry and asymmetry. The rays of the origami sun have eight-fold rotational symmetry and many axes of mirror symmetry, although the face is only bilaterally symmetric. In order to capture the twinkling of a real star, the origami star has rotational but no mirror symmetry, which tends to make an object appear static. Only the moon is asymmetric; although the real moon is a sphere, when it is lit by the sun from one side, as shown here, it takes on the asymmetry of a human face seen in profile.

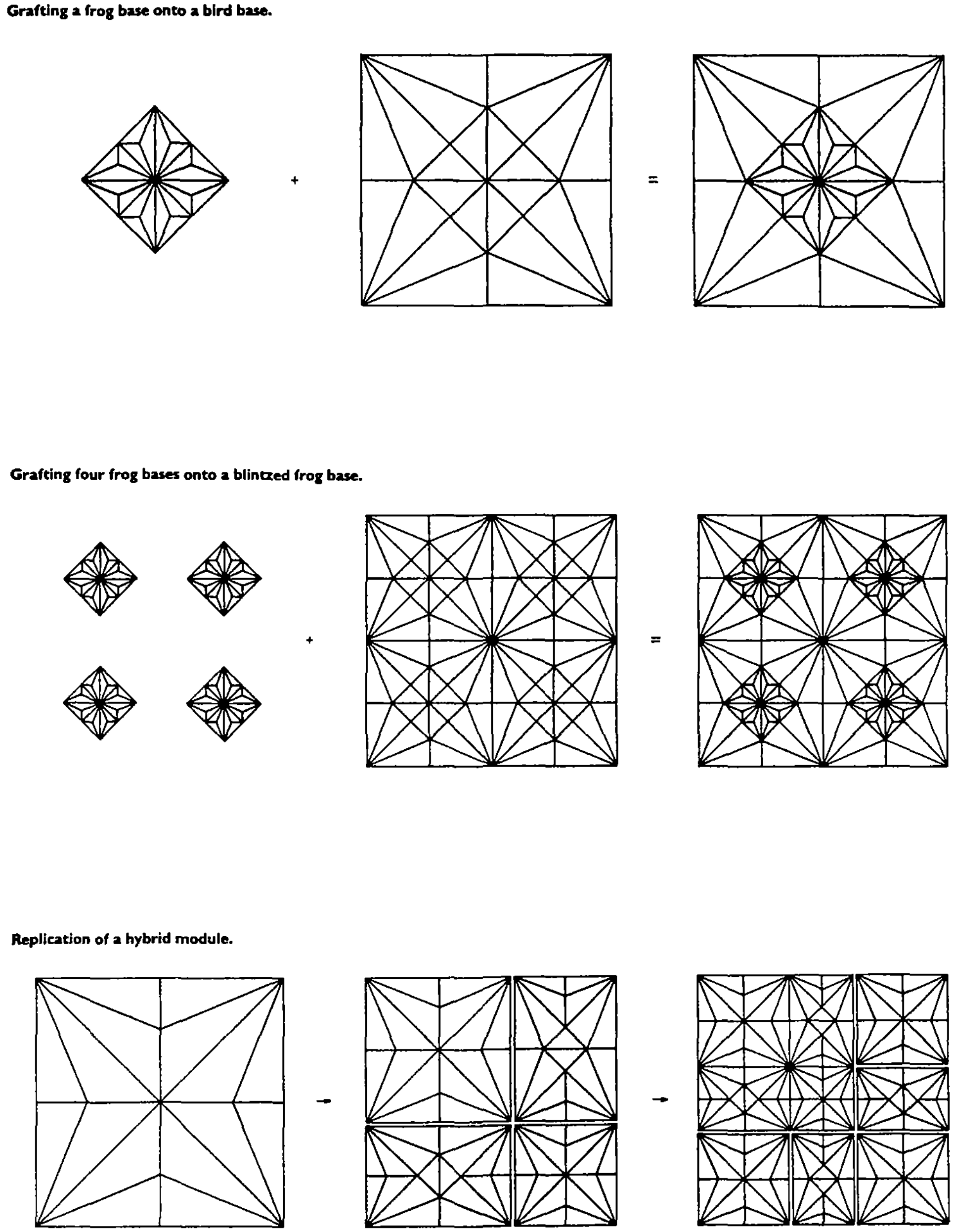


Figure 3: Three diagrams reveal the generation of complicated forms from simple ones by the application of symmetry operations. These diagrams, called folding patterns, reveal the network of creases in a finished model when is opened out to the original square.

In these investigations I have not been alone. Other authors such as Kasahara and Takahama (1985); Maekawa and Kasahara (1983); and Montroll and Lang (1990) were simultaneously exploring the symmetries of the square and venturing into areas unknown to me. The more recent accomplishments of paperfolders such as Shuzo Fujimoto, Toshikazu Kawasaki, Fumiaki Kawahata, Seiji Nishikawa, Chris Palmer, Jeremy Schafer, and Issei Yoshino, to mention just a few with whose work I am familiar, reveal the extraordinary riches that this geometrical approach has mined.

2. THE ASYMMETRIES OF NATURE

But while the unfolded square clearly offers a treasure trove of symmetric patterns, the forms of nature rarely live up to the purity and perfection of the square. Nature is, in fact, full of imperfections: twisted branches and vines, gnarled trunks, varied and ever-changing cloud formations, the mottled coloration of hair and fur, the ragged chasms and promontories of a mountain chain. The branching of trees and of river deltas, the dividing of cells and of soap bubbles, the multiplying of whorls in water and of puffs in a cloud, the cracking of mud and of eggshells, and the slow accretion of crystals and of chambers in a snail shell are all examples of forms that are coherent even though they may lack clear-cut symmetry. What all of these forms do possess, however, is great beauty, a beauty borne not of their perfection but of their imperfection – of the delicate balance they maintain between order and chaos.

As an example of how asymmetric patterns occur in nature, consider the cracking of a drying drop of oil. First we behold the fluid, formless drop. Outside, its black skin glistens; within, its molecules swim in a vast, slippery sea. Now the drop falls, it lands on a smooth, flat surface and slowly dries in the sun. As time passes, it contracts and cracks. At first, there may be only a single, large fracture that extends in a crooked arc across the surface of the shrinking oil. Then, smaller fissures occur. They meet the first fracture at angles that are close to perpendicular, but never exact. As the oil continues to constrict, dry, and flake, still smaller lines appear, bent and fragmentary, filling the empty gaps left by the earlier cracks (Fig. 4). The result is a complex and beautiful pattern – an asymmetric pattern – that is poised somewhere between order and chaos.

Take a second look at the pattern of cracks in the dried oil drop and imagine, for a moment, that you are in an airplane looking down at the ground. What do you see but the myriad patterns of human habitation – boulevards and avenues, streets and alleys, the entire circulation network of an ancient human settlement. The resemblance to an aerial photograph of a desert town in Iran (Fig. 5) is uncanny. What do these similarities tell us? Are they merely accidental, or do they reveal a larger truth about the forms created by nature and by man? As an architect as well as a paperfolder, I was truck by these similarities while I was conducting architectural

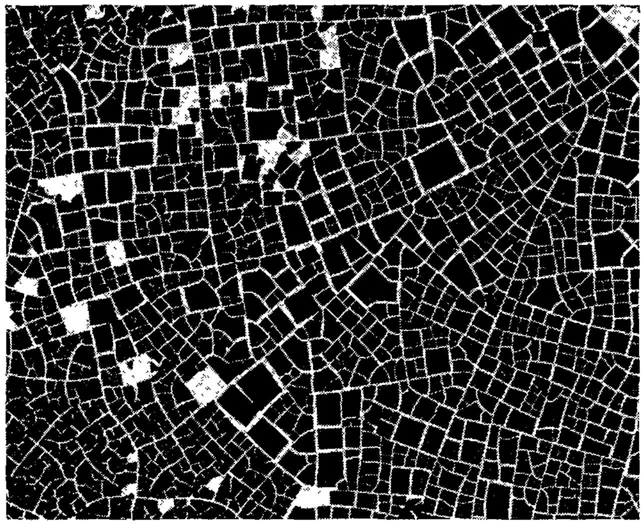


Figure 4: Typical cracking pattern in nature, the fissures in a gelatinous preparation of tin oil, has order but no symmetry. (Copyright by Manfred Kage/Peter Arnold Inc.)

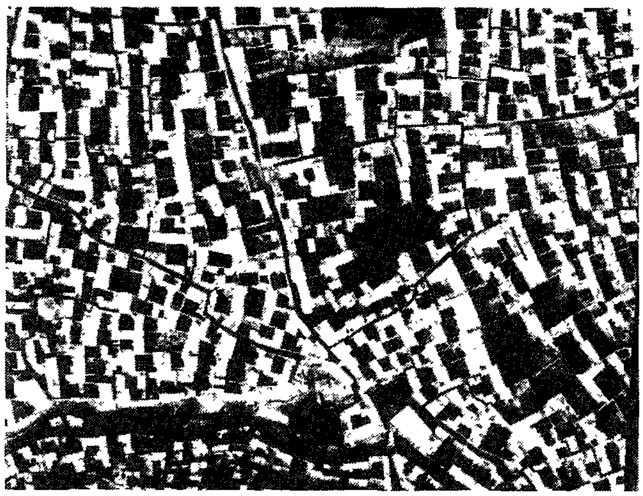


Figure 5: Aerial photograph of an Islamic settlement, the town of Hamadan, Iran, bears an uncanny resemblance to the cracking of a drop of oil.
(Copyright of the Oriental Institute of the University of Chicago.)

work for several years in India and Sri Lanka. In stark contrast to the rigid, highly geometric forms of most contemporary housing designs, the ancient human communities that I visited felt as natural as the actual products of nature. And because they were equal parts order and chaos, they were extraordinarily beautiful.

My investigations revealed that traditional human settlements come into being in much the same way as the forms of nature. While traditional settlements vary tremendously with region, culture, and climate, they are nonetheless the product of a few consistent environmental, technological, economic, and social rules. Like the cracks in the drying oil drop, a traditional community is formed incrementally, through many small modifications and interventions, over a prolonged period of time. A settlement may begin with only a single path, a path that connects two nearby towns or the center of town with a well or a watering hole. As the settlement grows, the path may be enlarged or paved to form a street, and a bazaar may form along its sides; new footpaths may be added that extend in opposite directions, connecting the street to a new temple, church, or mosque; additional homes may be built by individual families who are migrants to the town or the children of the original settlers. Over a period of time, families expand, families divide, the community expands, the community divides, and the buildings and streets multiply. Through this process of expansion and partition, the community takes the form of its maturity.

The organic quality we observe in nature and in traditional settlements is thus the result of *repetition* and *randomness*: repetition because a particular process has occurred repeatedly over the course of time, and randomness because it never occurs exactly the same way twice. The imperfections in the finished product demonstrate that the pattern was not hewn by a single creator or executed in a single master stroke. In the dried oil drop, for example, none of the areas of dried oil is square, but *almost* all have four sides. The cracks meet not at perfect right angles, but *nearly* so. Even that most repetitive of natural processes, the transcription of genetic information by DNA, is never perfect; the resulting mutations create the genetic variations that allow species to adapt and survive.

3 GENERATING ASYMMETRY IN ORIGAMI

If repetition and randomness are the blueprint for the forms of nature and the ancient settlements of man, then we should be able to employ them to capture the asymmetries of nature through origami. To do so requires paying close attention to the irregularities of nature, the erratic shapes and subtle curves that mark a form as organic and natural instead of machine-made and mechanical. But because we are working with a square of paper and not with living cells, crystals, or water droplets, our approach demands equally that we exploit the intrinsic properties of paper and square.

THE FLOATING SQUARE: SIX STAGES IN THE EVOLUTION OF THE RATTLESNAKE

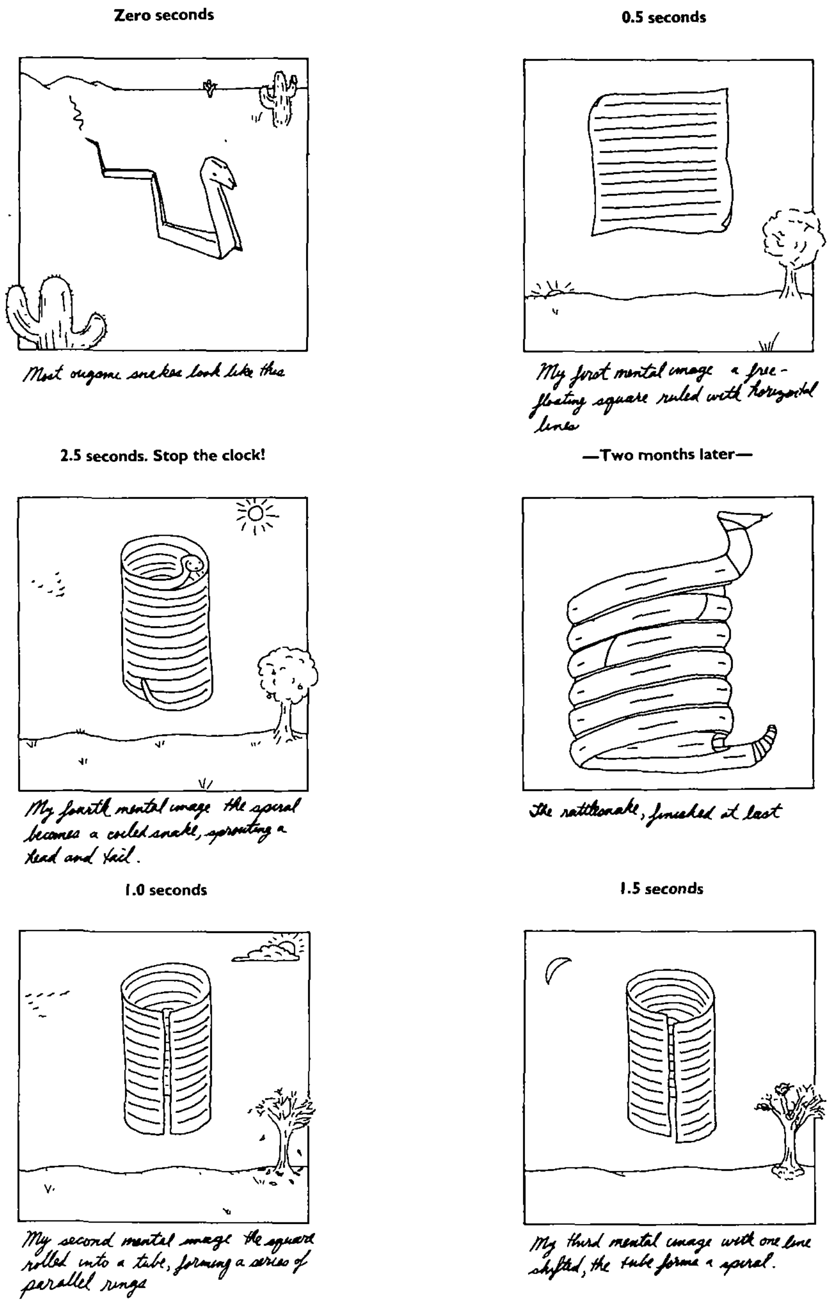


Figure 6: Conceptual diagrams illustrate the evolution of the author's origami rattlesnake.

My first example of breaking symmetry is one that I would consider an unsuccessful, early attempt in this direction. In my first origami book, I explained the process behind the formation of my model of a rattlesnake. As the conceptual diagrams I included to illustrate (Fig. 6), I wanted to create a snake that was different from all others in the origami repertoire. To make the longest possible snake from a square, other folders had lined up the body of the snake with the diagonal of the square and collapsed the two other corners accordion-style to narrow the body. That was it. Subtle variations in the position of the head and tail were the only clues to distinguish one model from another.

How was I to make my snake different? In order to begin, I conjured up images of snakes that I had seen and sought the traits that most stood out. What do snakes do that no other animal does? What aspects of their anatomy, their evolution, their movement define 'snakeness'? In short, what makes a snake a snake? Snakes slither, I thought, they undulate, they hang from trees, they strike – and they coil. I couldn't think of any other animal that coils. I made up my mind to invent a coiled snake.

At that moment, images flashed before me as my mind raced to find a solution. I saw a square of paper floating through space. On the square was a pattern of horizontal lines. I pictured the square rolled into a tube. The horizontal tubes turned into parallel rings running up and down the tube. Then, suddenly, in the conceptual breakthrough that was the decisive break with bilateral symmetry, I shifted the edges of the square by one line. Now, instead of rings, there was a spiral – one long coil running all the way around the tube, like the stripe on a barber pole. The pattern changed again; the edges sealed, and a head and a tail sprouted from each end of the spiral. I had my snake. The conceptual part was done. The rest of the execution required great ingenuity, but it never matched the simplicity and clarity of that first conceptual leap. To my mind, the resulting model (Fig. 7) is clever, but it is not beautiful. Because it remains bound by rigid geometry, it does not partake of the orderly chaos, the randomness within repetition, that marks the true forms of nature.

Before we leave the snake, it is worth examining the model's step-by-step diagrams to locate the exact moment in the folding process when the snake becomes bilaterally asymmetric. For readers familiar with the model, the decisive step is number 11 (Fig. 8). Up until this point, except for the tiny triangular flaps at the four corners of the square (which could, in fact, be turned in any direction), the model is bilaterally symmetric along the diagonal of the square. Step 11 asks the folder to swivel clockwise the long flaps that protrude from the top and bottom of the model. (These will be the head and rattle of the snake. If the flaps are swiveled counterclockwise, the snake will coil with the opposite handedness.) By layering rotational symmetry on top of bilateral symmetry, the bilateral symmetry is broken, never to be regained.

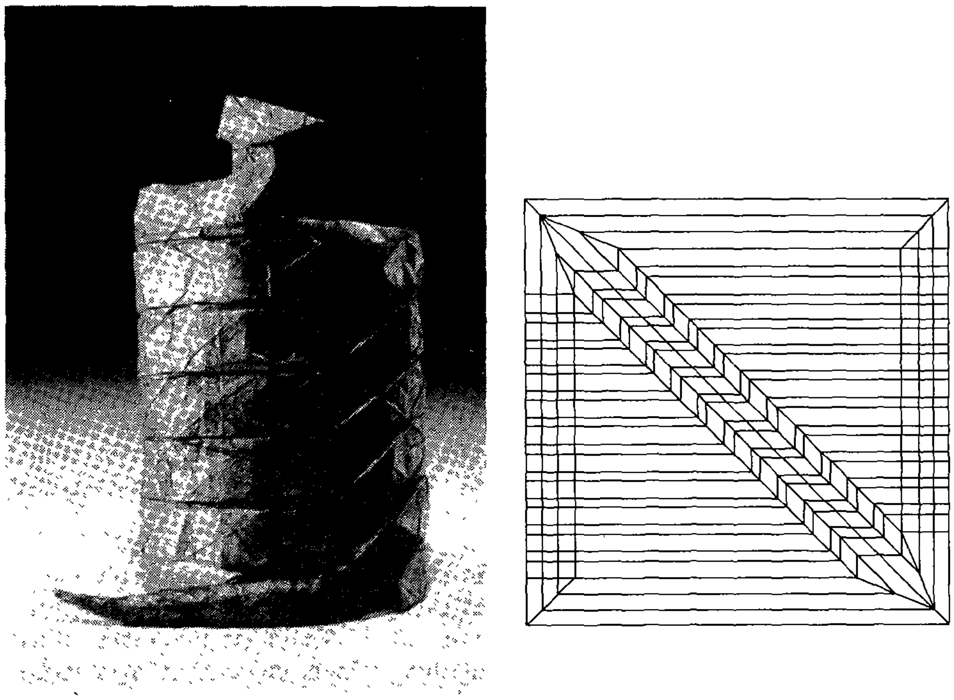


Figure 7: Completed rattlesnake is a helix that can coil clockwise or counterclockwise. Its folding pattern is similarly asymmetric.

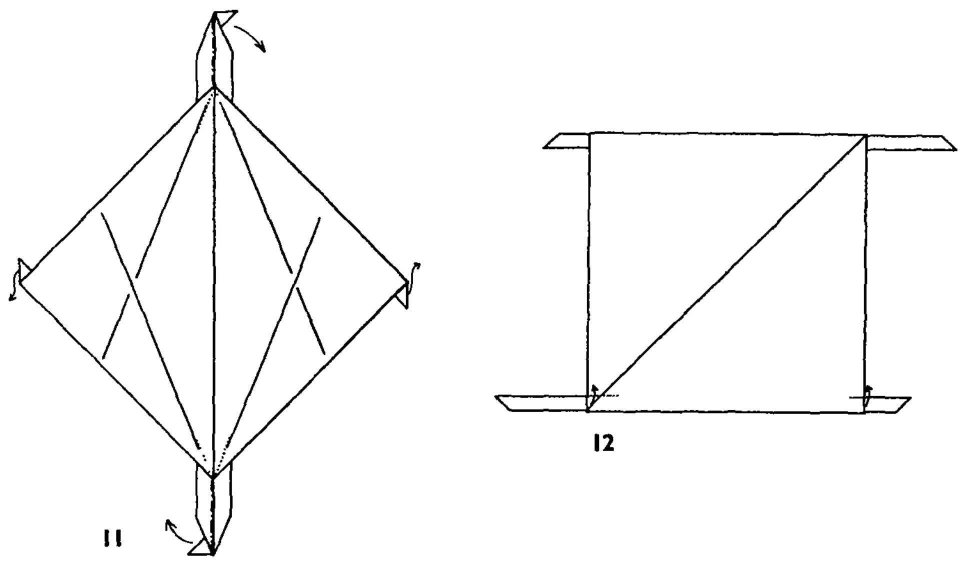


Figure 8: Between steps 11 and 12 of the rattlesnake, bilateral symmetry has been broken and only rotational symmetry remains.

My second experiment in breaking symmetry was constructing a mountain range. Again, as in my investigation of the rattlesnake, I began by letting images spin through my mind. I knew that in order to capture the ragged and craggy quality of rock, I would have to avoid obvious symmetries – nothing could look more artificial than a rock that is identical on all sides – as well as strict horizontals and verticals. But if the resulting figure was too uneven – if, say, I were simply to crumple the paper – it would lack the aspect of self-similarity that characterizes a real mountain chain. (Self-similarity, or scaling, is a property of forms that possess fractal geometry: when it is enlarged, each part resembles the whole.)

I concluded, then, that the mountains would involve a repetitive geometric pattern but still strive to appear 'natural' and 'organic' when viewed in perspective. After much experimentation, I produced a mountain that is fundamentally a pyramid with three sides and a bottom. To shape the different sides, I devised a spiraling sequence of 'closed-sink' folds that avoids unnatural-looking horizontal or vertical creases. The folds are similar in shape, but because they rotate and reduce in size with each turn, no two faces of the mountain are the same, and the resulting origami model is markedly asymmetric. It is possible to assemble dozens of mountains of varying sizes (and opposite handedness) to make a mountain chain that ranges from tiny foothills to towering peaks. When seen from directly above (Fig. 9), the sinks reveal their exacting geometry and do not look at all natural (though they do resemble the logarithmic spirals of a sunflower or snail shell). When seen in perspective, however (Fig. 10), they capture something of the orderly chaos of nature.

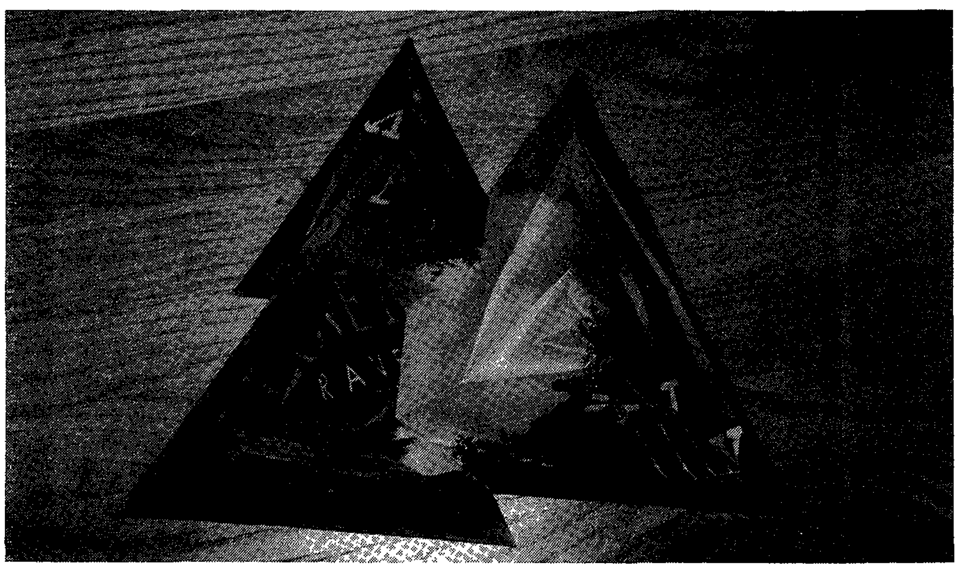


Figure 9: A view of the mountains from directly overhead reveals their exacting geometry. Note the spiraling pattern of closed sinks.

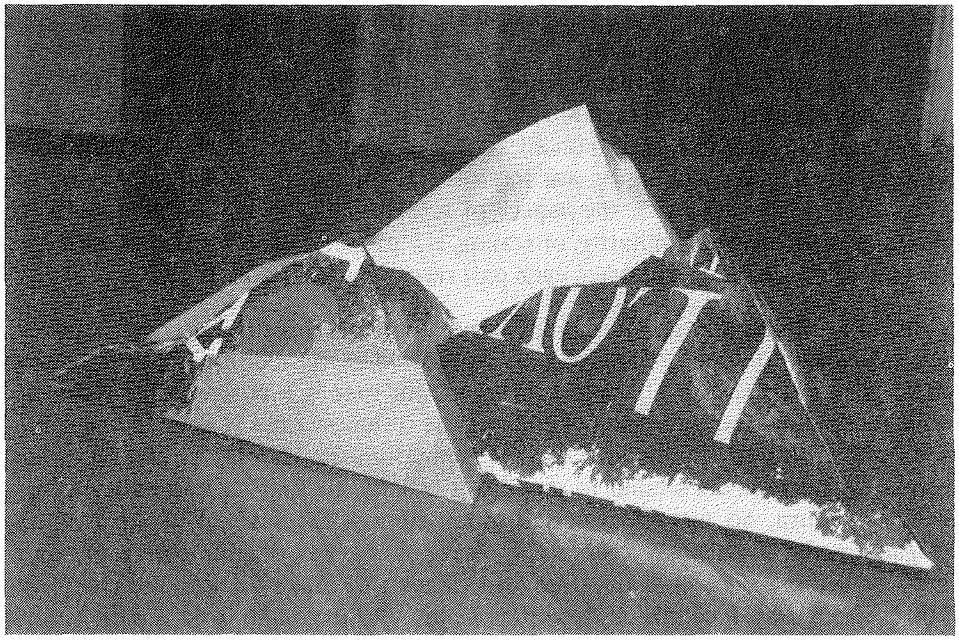


Figure 10: Seen in perspective, the origami mountains reveal the irregular crags and promontories, the 'orderly chaos', of their counterparts from nature.

The inspiration for my next investigation in breaking symmetry was the death, earlier this year, of my friend and fellow paperfolder, Mark Turner. Mark was an imaginative folder who furiously devoted his energies in the last few months of his life to creating new plant forms. Plants are a particularly challenging subject for origami. Their attenuated branches and stems, rounded leaves, shaggy fronds, and varied branching patterns constitute a poor fit for the taut geometries of the square. But in what appeared to be a single, sustained burst of invention, Mark originated a highly individual approach to folding and then, with the fastidiousness of a botanist, pursued its implications from family to family and species to species throughout the plant kingdom. His sensuous, curving plant forms breathed new life into the familiar technique of box-pleating, which to my mind once seemed destined to produce only replicas of matchboxes, cars, and modular furniture, the mechanical handiwork of man. Among Mark's accomplishments revealed in his as-yet-unpublished manuscript (Turner, 1993) are several models with bilateral asymmetry, such as the grasses and sunflower that possess an alternating branching pattern (Fig. 11).

As a tribute to Mark, I decided to undertake a model of a plant with great spiritual symbolism: a leaf from the tree *Ficus religiosa*, known throughout Sri Lanka and India as the *bodhi* or pipal tree (Fig. 12). 2500 years ago, Siddartha Gautama, the Buddha, achieved enlightenment while meditating beneath a *bodhi* tree. Since that time, the *bodhi* tree has been cultivated throughout the Buddhist and Hindu world as a symbol of *nirvana*, relief from suffering, which can only be attained by following the *dharma*, the code of thought and conduct laid out by the Buddha. The leaves of the tree have many variants, but are immediately recognizable by their elongated, tapering tip and asymmetrically branching veins.

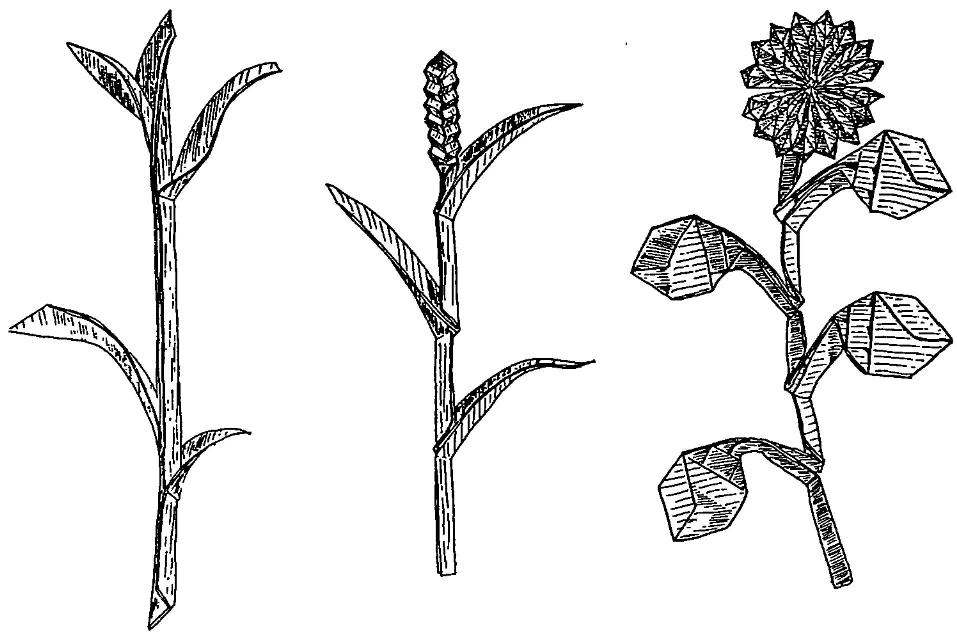


Figure 11: Mark Turner's grasses and sunflower have an alternating branching pattern.

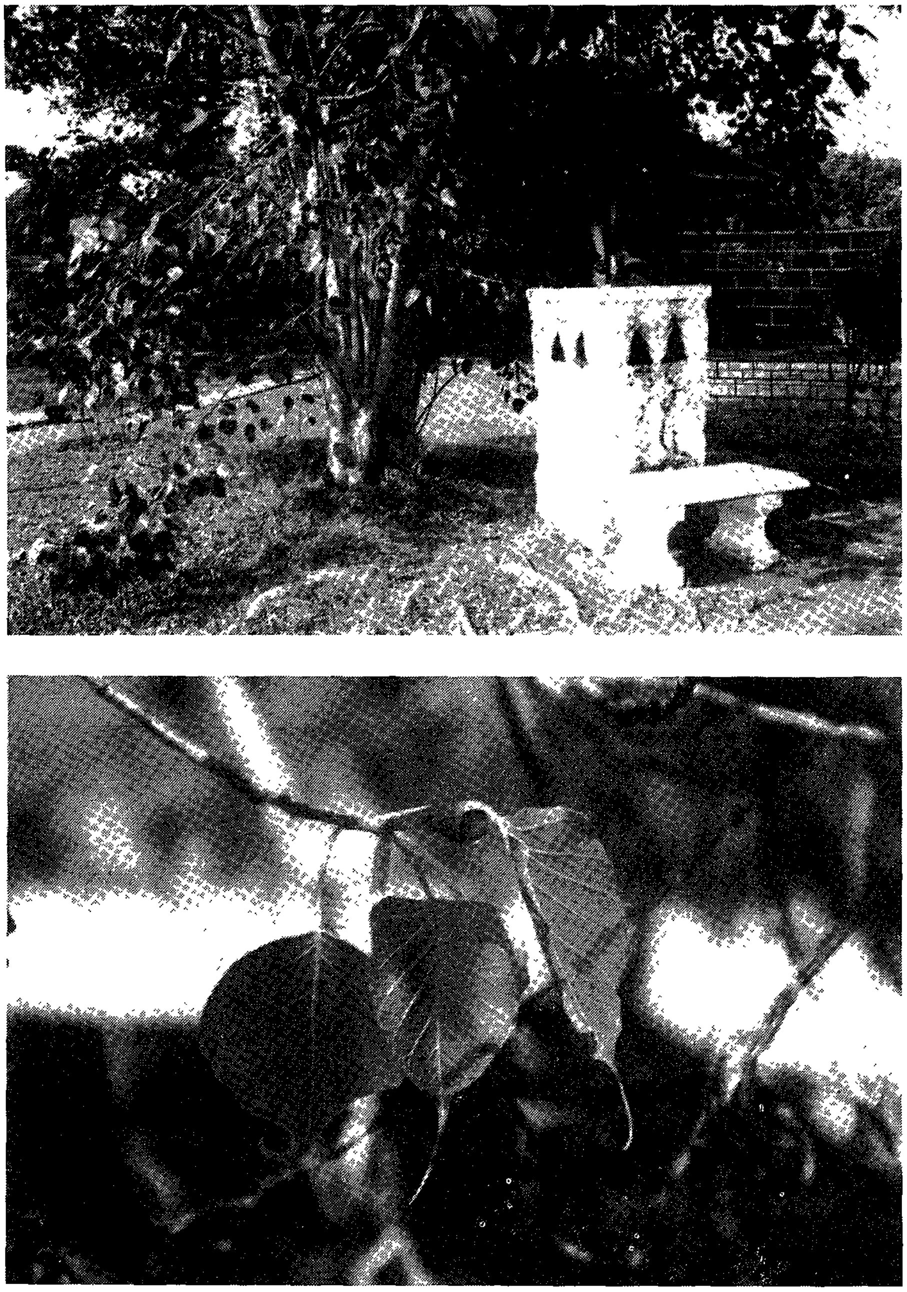
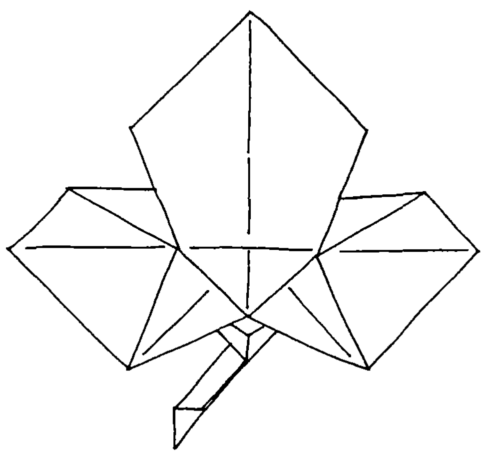


Figure 12: *Bodhi* tree with a statue of the *Buddha in samadhi*, or meditation, pose. Close-up of the leaves reveals their distinctive tapering point.

As with the rattlesnake and the mountain range, I began by sifting a series of images through my mind. Remembering the lessons I had learned from my study of natural forms and human settlements, I decided that the origami model would have to contain both systematic and random elements, and that it would begin with an orderly, symmetric structure and move gradually toward asymmetry. As I had previously designed a different form of leaf, a maple leaf (Fig. 13), I started from there. I eventually succeeded in devising a symmetric *bodhi* leaf (Fig. 14) and then set about uncovering a process by which to make it asymmetric.



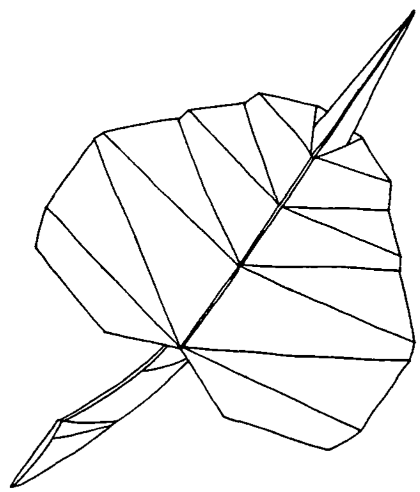


Figure 13: Maple leaf has bilateral symmetry.

Figure 14: First version of the *bodhi* leaf has bilateral symmetry.

But breaking symmetry did not prove easy! I gradually realized that to stagger the placement of the veins along the leaf's central spine required introducing a special mechanism, an operation with the paper that would shift the paper up on one side while shifting it down on the other. This mechanism turned out to correspond to a symmetry operation, introducing a small rotational symmetry within the larger bilateral symmetry of the leaf. In order to introduce rotational symmetry, I had to escape from the plane of the paper into the third dimension.

The operation is a kind of pinwheel. It begins by lying flat within the plane of the leaf, lifts out of the plane in order to rotate either clockwise or counterclockwise, and then collapses back into the plane when its work is finished. Just as the third dimension is required to turn a right-handed handprint into a left-handed one (or to lift a Flatlander out of his plane to turn him into his mirror image), the third dimension turns out to be a prerequisite for making the *bodhi* leaf asymmetric. In the completed leaf (Fig. 15), the operation is too small to be easily visible to an observer. But a sequence of photographs that I took of a larger prototype of the operation shows just how it works (Fig. 17). It was only in the preparation of this paper that I realized how much the operation has in common with the Iso-area folding theorem devised by Kawasaki and elaborated upon by Palmer and others. Clearly there is much fertile territory for exploration.

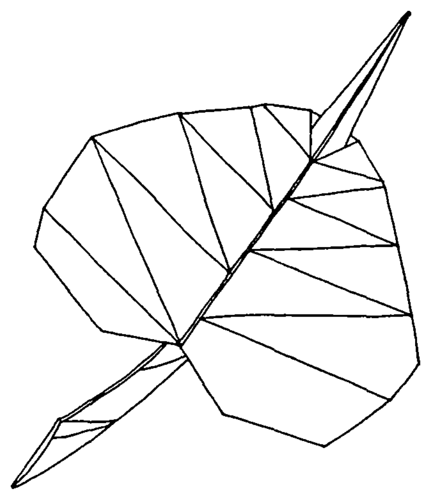


Figure 15: Final version of the *bodhi* leaf has an asymmetric branching pattern.

My last example is so new that the model is not yet completed! Seeking a completely different type of asymmetry to capture, I am now endeavoring to create a model of a leaf from the *begonia* plant. The *begonia* leaf (Fig. 16) at first appears so odd and disproportionate that it is hard to locate even the vestiges of symmetry. And yet the luminous patterns and varied textures of different *begonia* leaves clearly exhibit both repetition and randomness. A few minutes of scrutiny reveal the nature of this order (Fig. 18). At the point where the stem joins the leaf (this point is called the *hilus*), the two lobes are bilaterally symmetric. Similarly, the very tip of the leaf also exhibits bilateral symmetry. And yet these two axes of symmetry are rotated with respect to each other by as much as 90 degrees. When I examined dozens of leaves from different species of *begonia* plants, including tiny leaves just in the process of forming, I found this pattern of broken symmetry repeated over and over. How could it have come into existence?

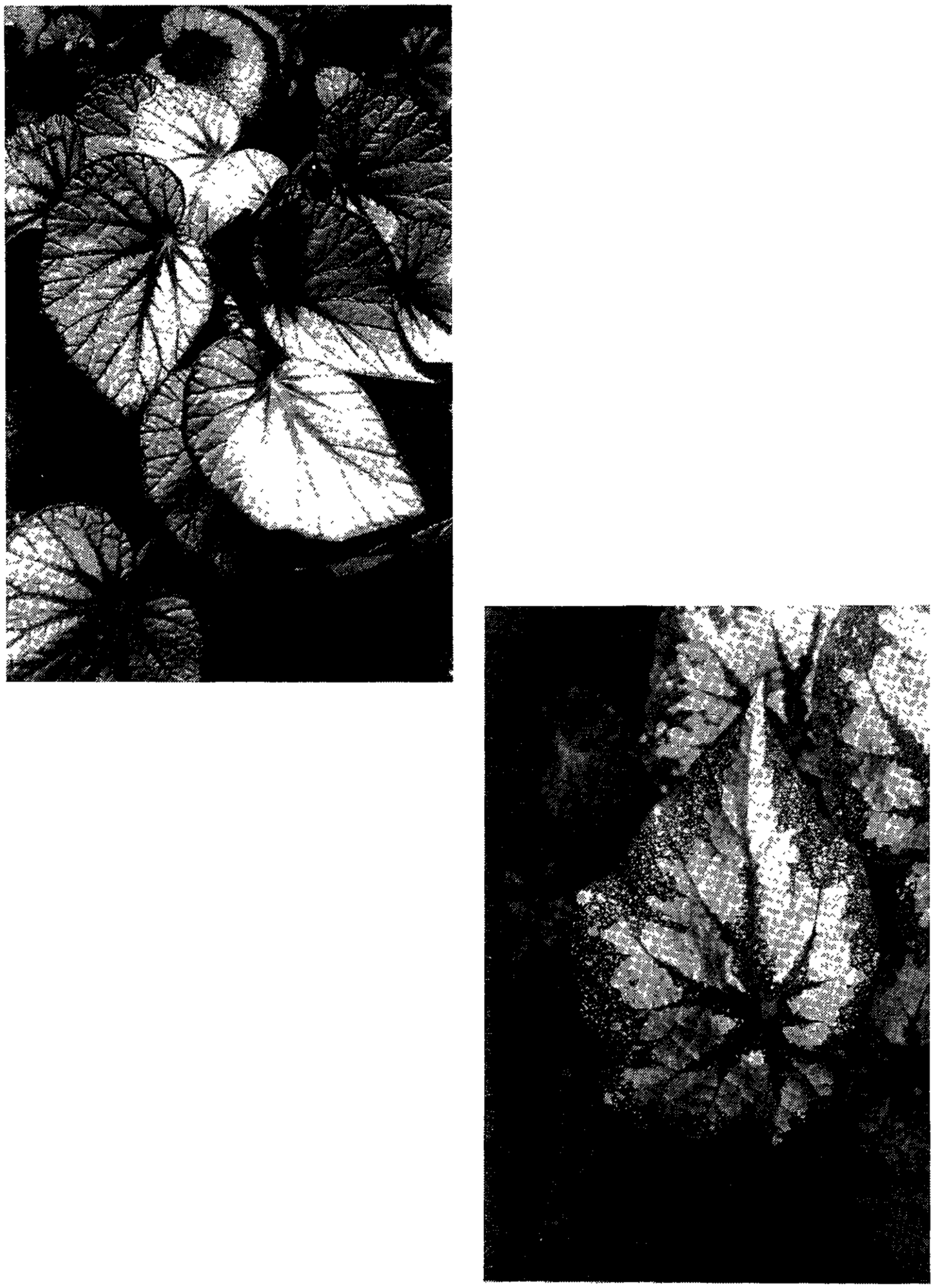
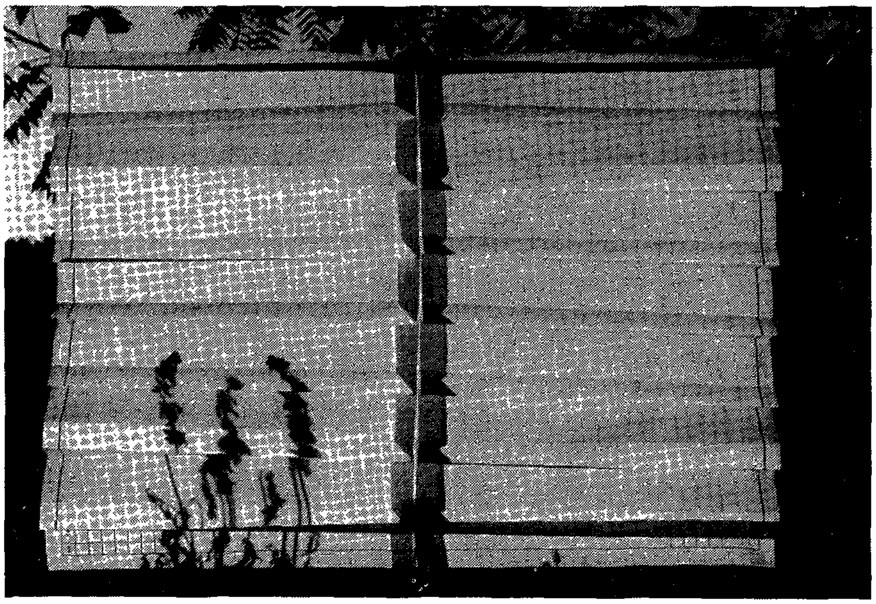


Figure 16: Two photographs of *begonia* leaves reveal their characteristic asymmetry.



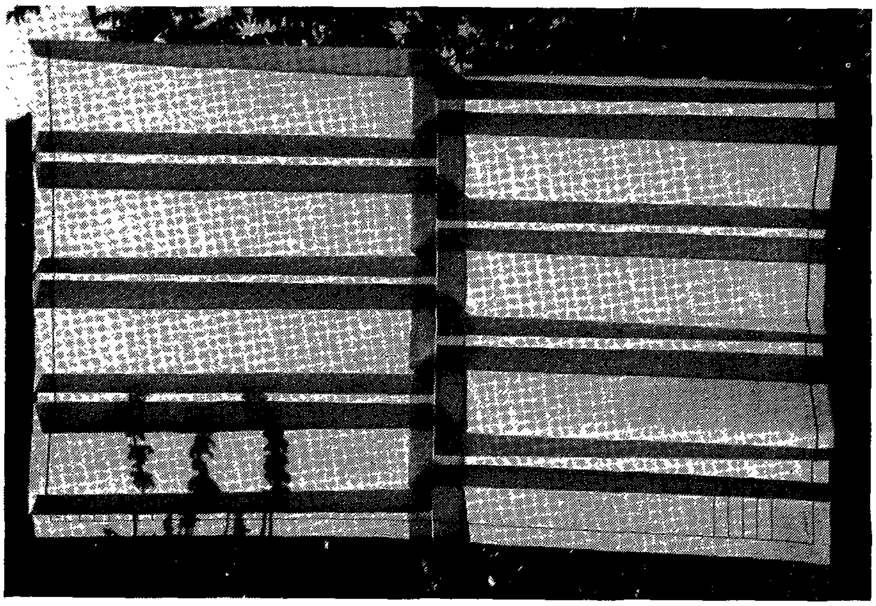
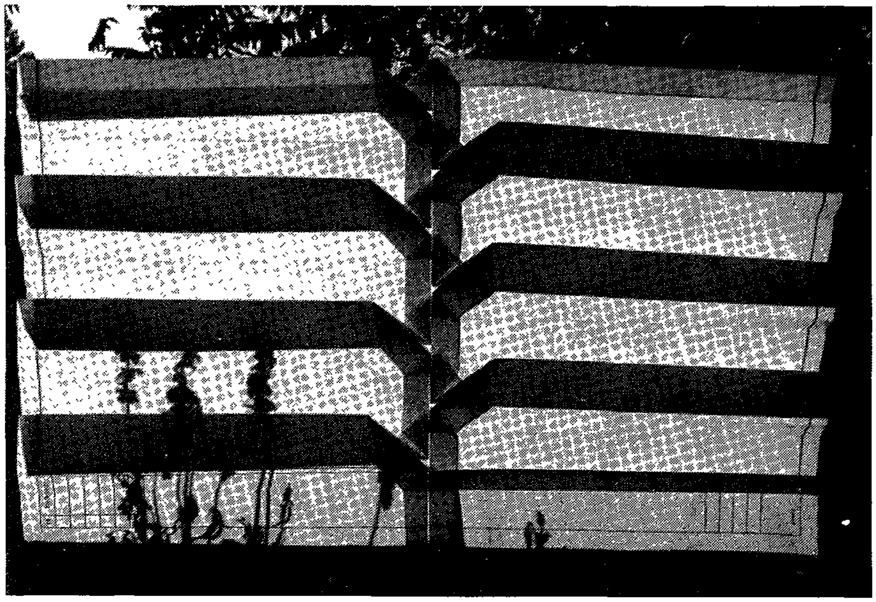


Figure 17: Four photographs show the evolution of a prototype model from symmetry to asymmetry. The key is a pinwheel mechanism that lifts the center of the paper into the third dimension, then lays it back down asymmetrically (1) and (2).



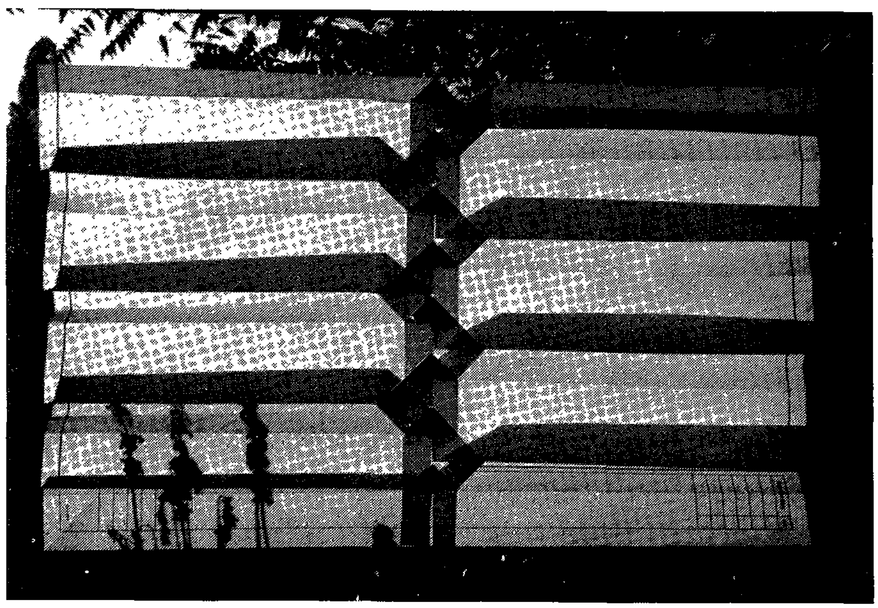


Figure 17: Four photographs show the evolution of a prototype model from symmetry to asymmetry. The key is a pinwheel mechanism that lifts the center of the paper into the third dimension, then lays it back down asymmetrically (3) and (4).

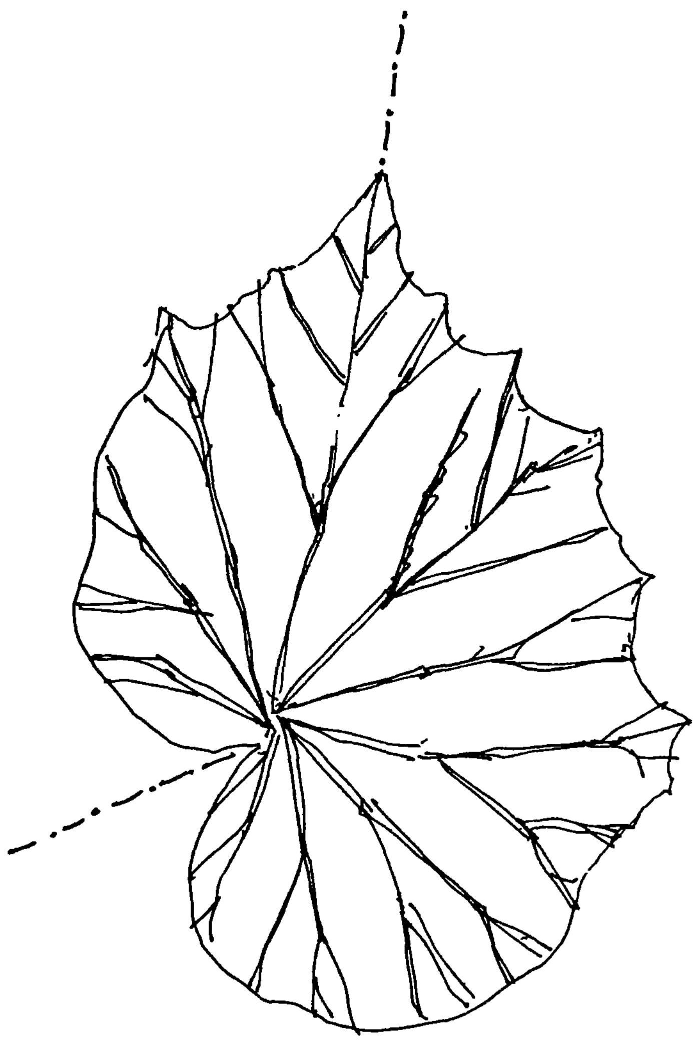
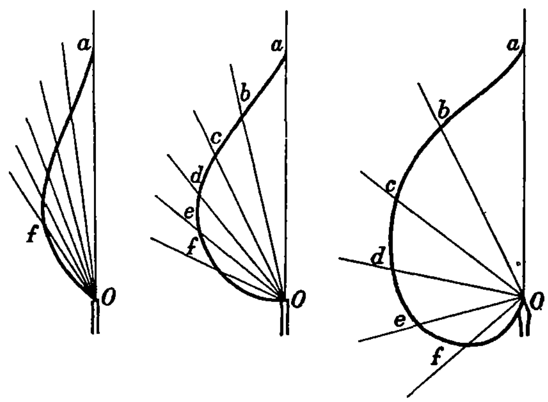


Figure 18: A page from the author's sketchbooks illustrates the *begonia* leaf's broken axis of symmetry.

I have concluded that in the original phylogenetic blueprint for the leaf, these two axes aligned. If the two sides were then to grow at vastly different rates, one half of the leaf would rotate relative to the other, and the result would be the shape that we see before us. But is this too simple an explanation for such a complicated form? Several weeks after I had begun working on an origami design of the *begonia* leaf, I found unexpected confirmation of my theory in the work of the British mathematician and biologist D'Arcy Thompson. In Thompson's analysis of bilaterally symmetric leaves, he had attributed the variation in their shapes to their differential rates of growth. Presuming a point of no growth at the *hilus*, Thompson drew vectors from the *hilus* to the edges of the leaf and analyzed those vectors in terms of their radial and tangential rates of growth. If the leaves of three different species of plants have identical tangential velocities but varying radial velocities of growth, the results will be the lanceolate, ovate, and cordiform shapes shown here (Fig. 19).



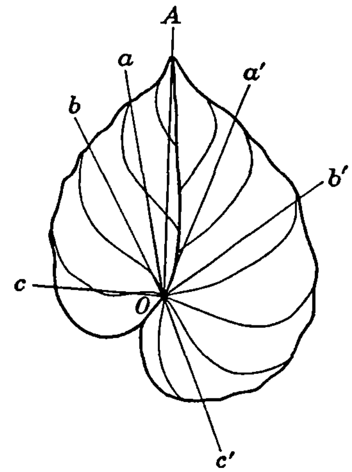


Figure 19: D'Arcy Thompson's analysis of the growth of symmetric leaves.

Figure 20: The same analysis, extended to include the asymmetric *begonia* leaf, reveals that despite the leaf's eccentric shape, orderly forces are still at work.

Extending his analysis to the *begonia* leaf (Fig. 20), Thompson measured vectors on both sides of the leaf and showed consistent but different rates of growth for the two sides. Based on my own and Thompson's mathematical analysis, I have been working to make an origami *begonia* leaf by starting with a form containing a single axis of bilateral symmetry and then varying the rates of tangential and radial growth to produce the broken symmetry revealed in my sketch. While the results are not ready to be made public, they point the way to new and unforeseen challenges in capturing the asymmetries of nature (Fig. 21).

4 CONCLUSION

From the four examples cited here it is clear that there are many approaches to breaking symmetry. It can be done in a single, decisive step, as in the rattlesnake and the *bodhi* leaf, or by a series of small alterations, as in the mountain range and the *begonia* leaf. However it is accomplished, breaking symmetry does not mean dispensing with order. Far from it! The process of breaking symmetry merely introduces a new type of order, a layering on of several types of symmetry, giving richness and diversity to phenomena that to our eyes would otherwise have been static and uninteresting.

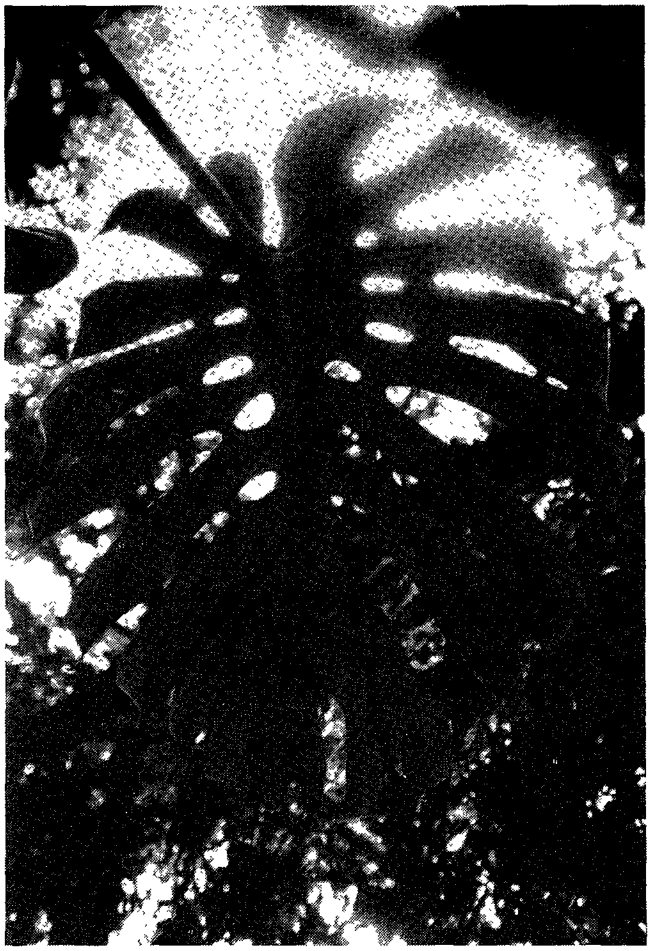


Figure 21: Nature has devised asymmetric plant forms, like this perforated *philodendron* leaf, that almost defy imagination. Can paperfolders meet the challenge of folding them from a single piece of paper?

The beauty of asymmetry, whether it is a cracking pattern in mud, the winding streets of an ancient settlement, or the alternating veins in a *bodhi* leaf, is different from the beauty of symmetry. While symmetry speaks of perfection, of idealized forms and objects that exist in a realm where they are untouched by time, asymmetry revels in the flaws and imperfections of the world, a world bound by the inexorable process of birth, growth, development, aging, and death. Like any transforming work of art, an asymmetric origami model reflects back on the life and forms that inspired it. If it reopens our eyes to the wonders of nature and sends us scurrying back to the original animals, leaves, and mountains to see them anew, the artist has certainly succeeded in his quest.

REFERENCES

Engel, P. (1989) *Folding the Universe: Origami from Angelfish to Zen*, New York: Random House; Reprinted by Dover Publications in 1994 under the title *Origami from Angelfish to Zen*.

Hargittai, I. and Hargittai, M., (1994) *Symmetry: A Unifying Concept*, Bolinas, CA, USA: Shelter Publications.

Kasahara, K. and Takahama, T. (1985) *Top Origami*, Tokyo: Sanrio Publishing Co.

Loeb, A. L. (1971) *Color and Symmetry*, New York: John Wiley and Sons.

Maekawa, J. and Kasahara, K. (1983) *Viva! Origami*, Tokyo: Sanrio Publishing Co.

Mandelbrot, B. B. (1982) *The Fractal Geometry of Nature*, San Francisco: W.H. Freeman.

Montroll, J. and Lang, R. J. (1990) *Origami Sea Life*, Vermont, USA: Antroll Publishing Co.; Lang has taken his ideas further in an as-yet- unpublished manuscript, *Origami Design*, in which he uses a computer to devise new patterns to suit any given configuration of points.

Smith, C. S. (1981) *A Search for Structure*, Cambridge, MA: MIT Press.

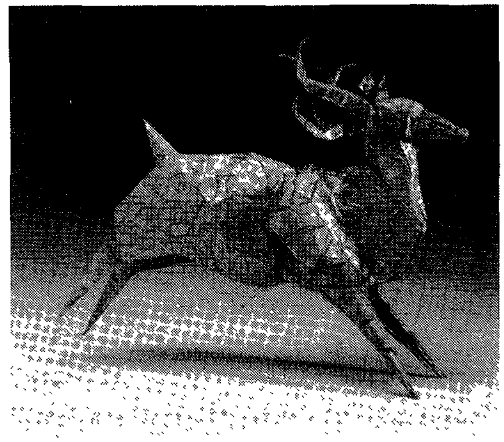
Stevens, P. S. (1974) *Patterns in Nature*, Boston: Atlantic-Little, Brown.

Thompson, D'Arcy (1961) *On Growth and Form*, Cambridge, UK: Cambridge University Press; (an abridged reprint of the original 1917 edition).

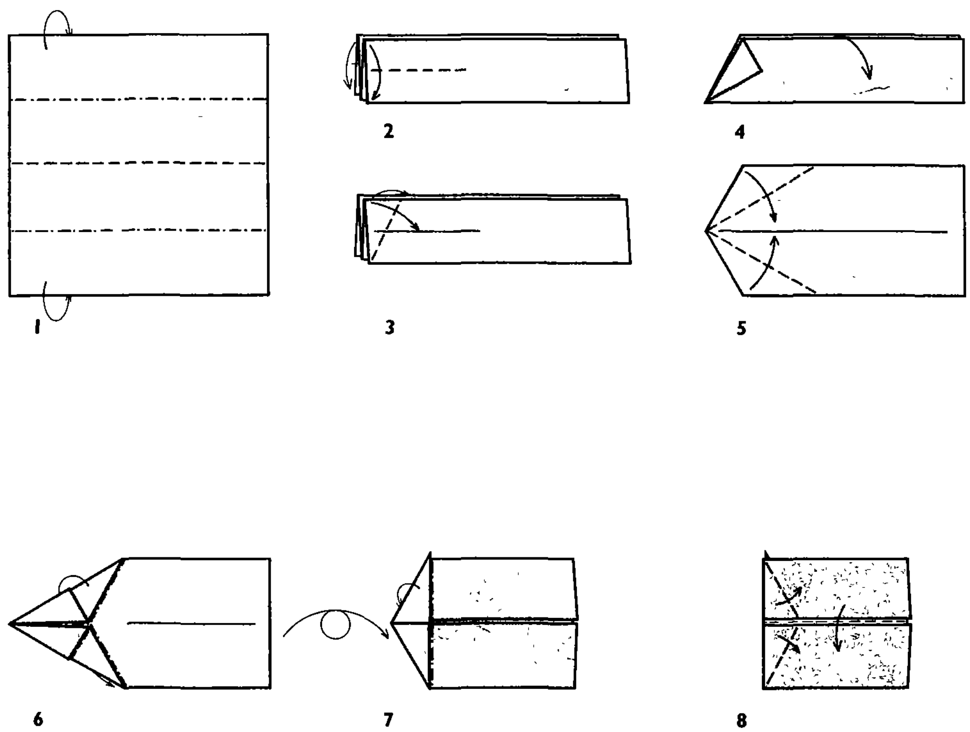
Turner, M. (1993) *Garden Folds: Origami Plant Forms for Foil Aficionados*, unpublished manuscript.

REINDEER

Use a sheet of paper colored on one side. A 10-inch square will produce a model 3 inches long. For your first attempt, use a square measuring at least 18 inches to a side.



1 Divide the square horizontally into quarters. Pleat like an accordion.
2 Crease lightly edge to edge. Repeat behind.
3 Valley-fold the corner so it meets the crease. Repeat behind.
4 Swing the front face down.
5 Valley-fold the left-hand edges to the centerline
6 Following the edges of the existing flaps, rabbit's-ear behind. Turn the model over.
7 Following the edge of the existing flap, mountain-fold behind.
8 Following the edges of the hidden flaps, inside reverse–fold and close the model.



9 Following the edge of the hidden flap, valley-fold the lower-left-hand corner to the upper right. Repeat behind.
10 Valley-fold the right-hand edge to the left. The crease should lie on top of the former lower-left-hand corner. Crease firmly and unfold to step 9.
11 Following the existing creases, crimp the entire model symmetrically.
12 Unfold the entire square.
13 Following the existing creases, refold the square. In the middle of the paper are two vertices where many lines meet. These vertices will plunge downward as the left and right sides of the paper swing upward and toward each other.

14 Inside reverse–fold the two flaps projecting from the top. Valley-fold the two side flaps down and to the right.
15 Squash one flap.
16 Lift the loose paper upward, and close the flap.

17 Valley-fold two flaps up and to the left, returning them to their position in step 14.
18 Following the edge of the hidden flap, crease and unfold. Then swivel two flaps up and to the left. When you are done, the model will not lie flat.
19 The model is now three-dimensional. The exposed white portion shows paper that is seen almost directly on edge. Squash the shaded flap.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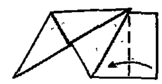
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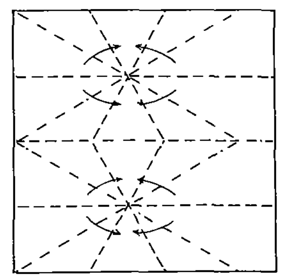
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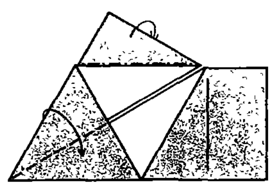
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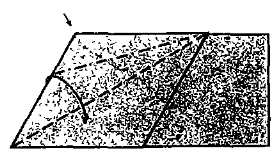
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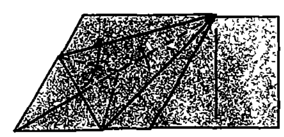
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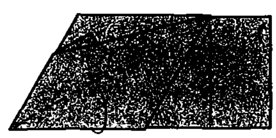
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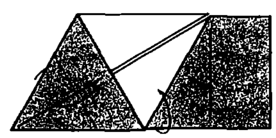
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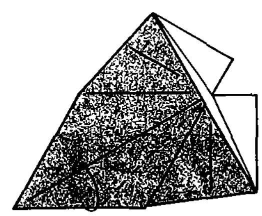
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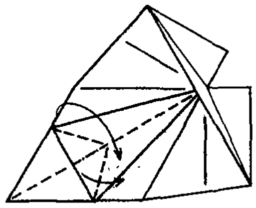

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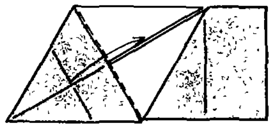

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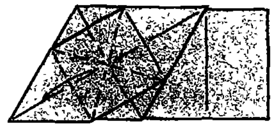

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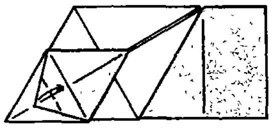

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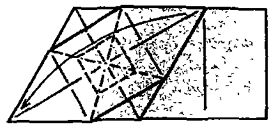

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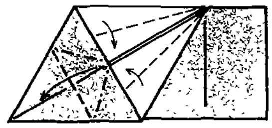

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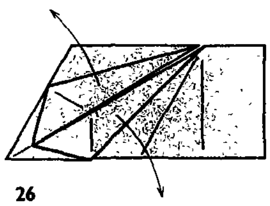

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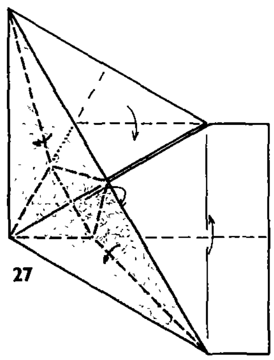

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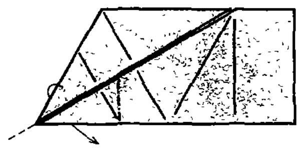

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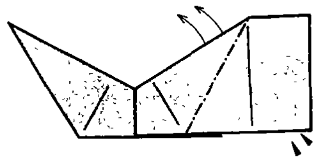
20 Lift the loose paper upward, as in step 16, and swing two flaps down and to the right.
21 Valley-fold the lower-left-hand corner up and to the right as far as it will go.
22 Push with your finger from behind to form a little pyramid of the shaded square. The square will pop forward and flatten.

23 Valley-fold the tip halfway, and unfold to step 22.
24 Following the existing creases, open double-sink and close the flap in the same motion.
25 This is a form of petal fold. Repeat behind.

26 Open up and spread the loose paper.
27 Following the existing creases, tuck the small triangle into the pocket behind and close the model in the same motion. Use tweezers. Repeat behind.
28 Swivel the flap at the left of the slot counterclockwise. The model will crimp symmetrically.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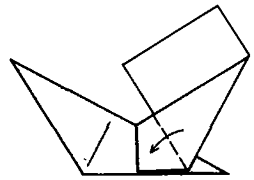
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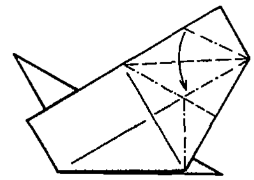
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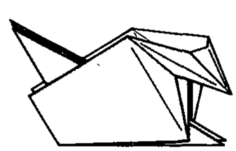
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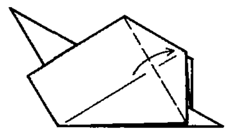
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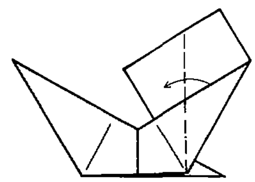
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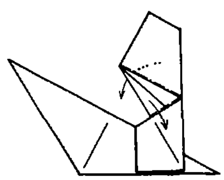
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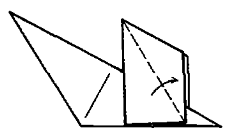
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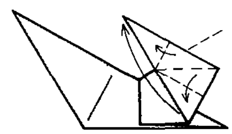
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29 Open the model slightly and inside reverse–fold the entire assembly at the right. (This procedure could be called a "closed" inside reverse fold, because it pops directly from one position to another.) The central flap turns inside out in the process.

30 Valley-fold the white flap down and to the left. Repeat behind.

31, 32 Make individual creases as shown. Then, in a single motion, push in at the front of the model and collapse it into a three-dimensional rabbit's ear Massage the creases into place. The result is symmetrical.

33 Valley-fold the shaded flap up and to the right. Repeat behind.

34 Inside reverse–fold through the shaded portion. Repeat behind.

35 Swing the white triangle down and to the right. The shaded portion of the flap will automatically swing down and to the left. Repeat behind.

36 Valley-fold the shaded flap up and to the right. Repeat behind.

37 Bring the three corners of the white triangle together, and collapse the loose paper like a fan. Repeat behind.

38 The flaps pointing up and to the left are the rear legs. Mountain-fold them, leaving the central tail flap in place. Valley-fold the white flap down and to the right. Inside reverse–fold the shaded tip. Repeat behind.

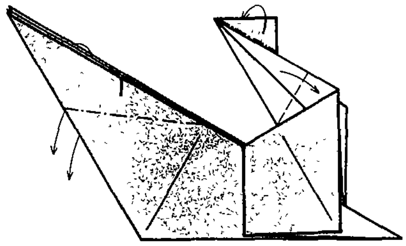
39 Crimp the legs. Swing the tail all the way underneath and forward. Narrow the white flap with valley folds, and swing it up and to the right. The shaded flaps pointing down and to the right are the front legs. Creasing lightly, lift up the flap obscuring the front leg. Repeat behind.

40 The position of the drawing has been rotated slightly. Inside reverse–fold the tail and swing it toward the rear. (This crease is hidden from view). Then, in a single motion, crimp the front legs and rotate the head and neck assembly clockwise. Narrow the hip with a mountain fold. Repeat behind.

41 Narrow the belly with mountain folds, and tuck the loose paper into the adjacent pockets formed by the tail. Narrow the front leg, valley-folding the double thickness. Swing one white flap and one shaded flap over to the left, and tuck the excess paper into the body. Repeat all folds behind.

42 Inside reverse–fold the hind leg. Without making any new creases, slide the top layer off the front leg, and tuck it into the pocket beneath. Cut-away view: Squash. Repeat all folds behind.

43 Narrow the hind leg symmetrically with valley folds, and tuck the loose paper inside with mountain folds. (*The mountain folds will pinch the back of the hips slightly.*) Slide another layer off the front leg. Turn the valley fold into a mountain fold, and tuck the layer into the pocket beneath. Cut-away view: Closed-sink the big flap. Squash the little flap at the top. Repeat all folds behind.



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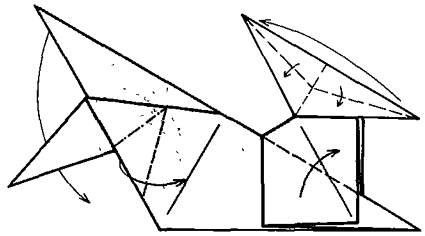

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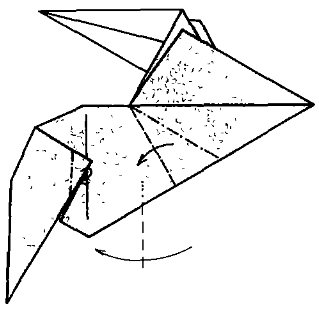

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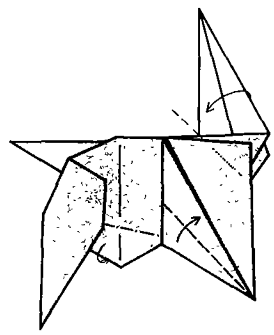

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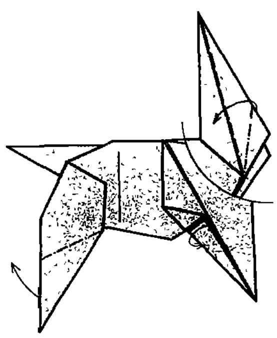

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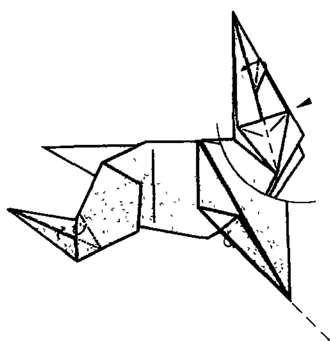

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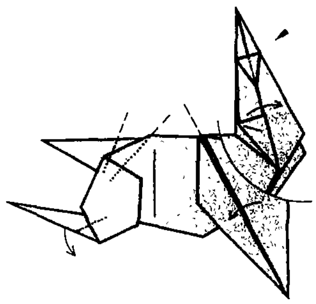

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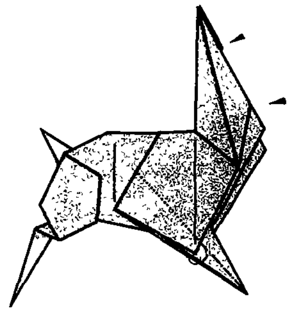

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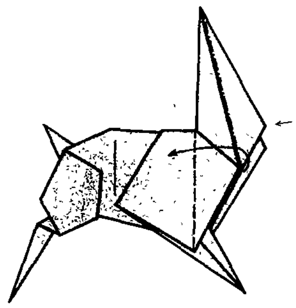

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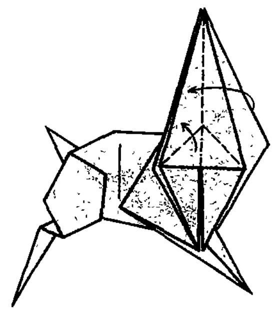

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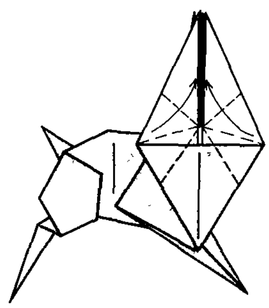

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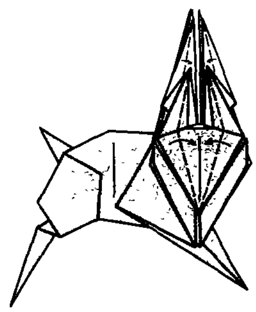

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44 Inside reverse–fold the hind leg. Close the front leg with a valley fold. Crimp the tail symmetrically. Cutaway view: Closed-sink the little flap. Swing the big flap to the right. Repeat all folds behind.

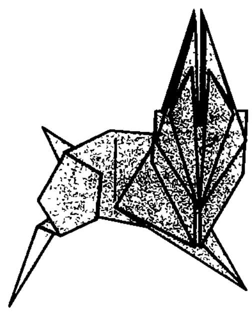
45 Closed-sink two head flaps. Mountain-fold the top layer of the front leg. (Part of this crease is hidden from view.) Repeat both folds behind.

46 Squash the next head flap, and swing it to the rear. Repeat behind.

47 Lift the tiny triangle inside the squash fold, and collapse it upward. The white flap at the center of the model contains the head and the ears. Swing it into view.

48 Mountain- and valley-fold the head assembly, and collapse it upward. Flatten it. Then pull the front flap down slightly to expose the inside.

49 Open the head assembly slightly, and narrow all the flaps with valley folds. Flatten again.



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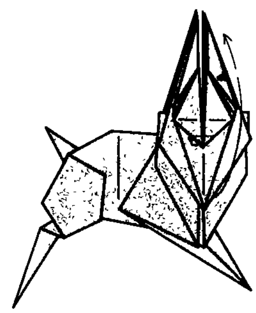

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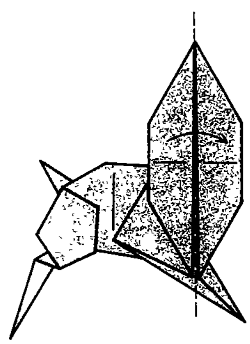

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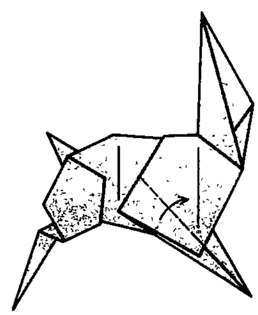

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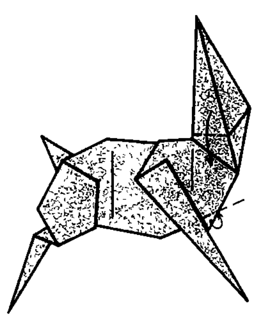

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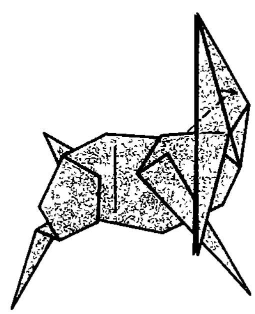

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50 Open the head assembly slightly, and pull out the loose paper. Following the existing creases, flatten it into a petal fold.

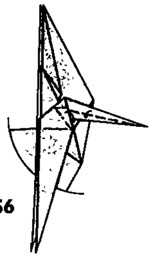
51 Mountain-fold the tip of the single-ply triangle. This will be the eyes. Following the existing crease, swing the entire head assembly upward.

52 Valley-fold the head assembly to the right.

53 The model is now entirely symmetrical. Narrow the front leg with a valley fold. Repeat behind.

54 Narrow the belly with a mountain fold. Swing down the near antler. Repeat both folds behind.

55 Inside reverse-fold the adjacent antler. Repeat behind.



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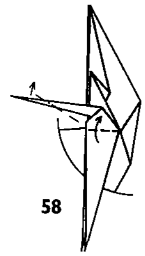

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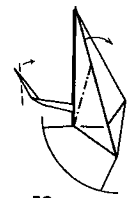

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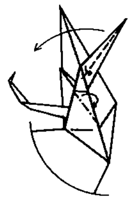

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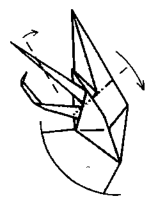

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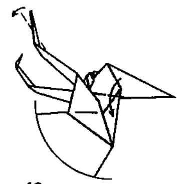

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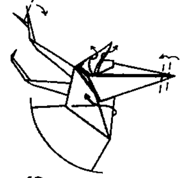

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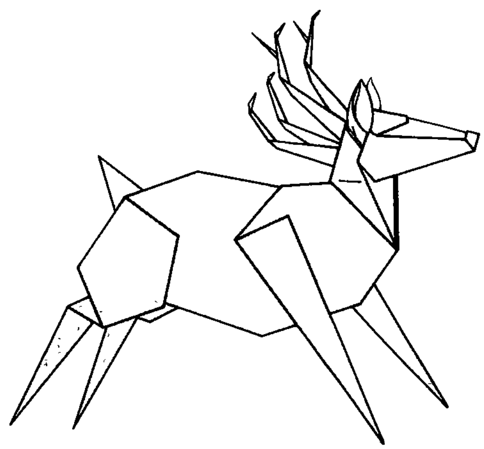
56 Here through step 63 are details of the head. Narrow the projecting antler with valley folds. Repeat behind.
57 Swing the projecting antler to the rear. Repeat behind.
58 Inside reverse–fold the rear antler. Swing up the front antler. Repeat behind.
59 Outside reverse–fold the rear antler to form the tine. Inside reverse–fold the front antler. Repeat behind.

60 Narrow the front antler with valley folds, and swing it to the rear. Repeat behind.
61 Inside reverse–fold the upper rear antler. Repeat behind. Inside reverse–fold the head through the base of the ears.
62 Separate the tines of the upper rear antler. Repeat behind. Turn the head flap inside out with valley folds on either side. The ears will pop up.
63 Outside reverse–fold the upper front tines. Spread the ears. Pull out the loose paper from either side of the neck to enlarge the jaw. Roll the tip of the head to form the nose.



The completed REINDEER.

(1976–78)