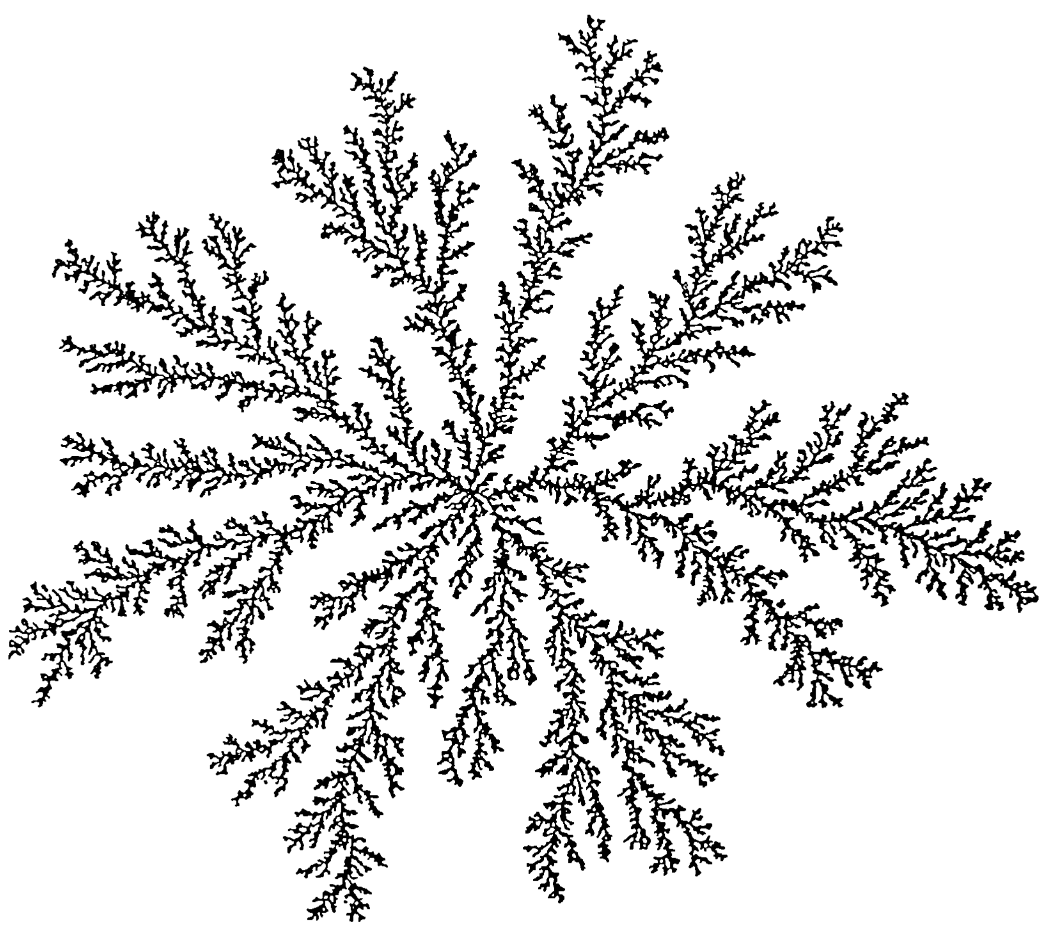
Symmetry: Culture and Science

SPECIAL ISSUE
Fractals

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 4, Number 3, 1993

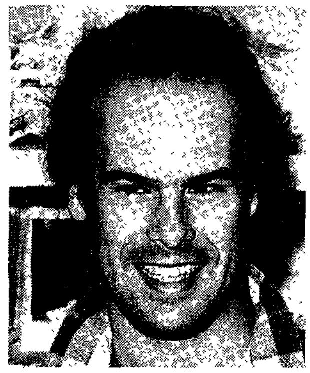


DLA fractal cluster
of 10^6 particles

Symmetry: Culture and Science
Vol. 4, No. 3, 1993, 293-302

FRACTAL ASPECTS OF INTERSTELLAR CLOUDS

Thomas Zimmermann and Jürgen Stutzki



Zimmermann, T., astrophysicist, (b. Wermelskirchen, Germany, 1962).
Address: I. Physikalisches Institut, Universität zu Köln, Zülpicher Str. 77, 50937 Köln, Germany;
E-mail: zimmermann@ph1.uni-koeln.de .
Fields of interest: Mathematics, physics, astrophysics, interstellar clouds, star formation, fractals.
Publications: *Astronomy and Astrophysics*, (co-author) 238 (1990), 337; In: *Evolution of Interstellar Matter and Dynamics of Galaxies*, (co-author), Cambridge: Cambridge University Press, 1992; *Physica A*, (co-author), 191 (1992), 79.

Abstract: *Interstellar matter consisting of gas and dust shows structures which appear to have self-similar projected shapes similar to the properties of terrestrial clouds. Moreover, the completely different physics in these interstellar clouds reveals several additional self-similar/fractal aspects which are investigated by extensive astronomical observations, aiming especially at a better understanding of the processes of star formation which take place in interstellar molecular clouds.*

1. INTRODUCTION

Only a short time after the fractal concept had been established by Mandelbrot (1982), it was applied to the shape of terrestrial clouds (Lovejoy, 1982; Hentschel and Procaccia, 1984; Rys and Waldvogel, 186). Based on the apparent similarity between terrestrial and interstellar clouds (ICs) of gas and dust (the naming 'clouds' not being accidental), subsequent work applied the concept to ICs (Beech, 1987; Basell and Désert, 1988; Dickman *et al.*, 1990; Scalo, 1990; Wakker, 1990; Vogelaar *et al.*, 1991; Falgarone *et al.*, 1991; Zimmermann and Stutzki, 1992; Hetem and Lépine, 1993). Understanding the structure of interstellar molecular clouds (IMCs) and its possible links to turbulence (e.g. Falgarone and Phillips, 1990) is crucial for understanding IMC evolution and star formation. As stars (range of masses: 0.1-30$M_\odot$) form out of IMCs (10-10$^6 M_\odot$), the structure of clouds and the cloud mass spectrum are likely to be closely linked to such fundamental quantities as the star formation rate and efficiency and the stellar initial mass distribution (Larson, 1992; Stutzki, 1992; Lada *et al.*, 1993).

Our investigations are based on an extended data set covering many size scales, as is necessary to extract fractal and hierarchical properties of ICs. This includes data from the most extended carbon monoxide (CO) maps of IMCs, covering 100 pc size structures, down to observations of the closest molecular cloud L1457 at 65 pc distance, which provides the smallest scales at 0.005 pc resolution (chap. 2). The self-similar 2-dimensional projected shapes of IMCs suggest a possible self-similarity in the 3-dimensional structure. This is stressed by the internal velocity distribution of IMCs which can be measured along the line of sight (chap. 3). In addition IMCs show scaling and power-law relations in their mass distribution and excitation conditions, which can be regarded as another aspect of the fractal structure of IMCs (chap. 4). Since the limitation to only 2-dimensional projections is a crucial point in all astronomical observations, we model 3-dimensional distributions of IMCs by *fractal/hierarchical subdivision* and compare their projections to observations by a box-counting method, which in addition yields an independent characterisation of IMC structure (chap. 5). Summarizing our results we find the fractal structure of IMCs being reflected in several more aspects than just in their projected shapes, which, regarded as a whole, might be crucial to understanding the inner structure of IMCs and the physical processes leading to coagulation and star formation (chap. 6).

2. OBSERVATIONS OF INTERSTELLAR MOLECULAR CLOUDS ON DIFFERENT SIZE SCALES

Star formation occurs in IMCs which constitute the densest regions of interstellar matter (composition: 1 % dust, 99 % gas; the latter: 50 % molecular, 50 % atomic/ionized gas) (e.g., see reviews [18,19]). Although the general processes governing star formation are understood, the details still remain unclear. Observations of molecular gas spectral line emission not only trace the overall projected spatial distribution of IMCs in our and in other galaxies, but also yield information about the velocity distribution (along the line of sight) by the Doppler-relation, and the excitation conditions (temperature, density, chemical abundances) by observing different molecular species and isotopomers in their lowest rotational transitions (e.g., see Winnewisser *et al.*, 1992). During the last decade the second most abundant interstellar molecule, carbon monoxide (CO), and its isotopomers have proven to be the most important trace molecules.

Observations of molecular clouds are carried out with millimeter- and submillimeter- radiotelescopes which limit the highest possible angular resolution ϵ by diffraction to $\epsilon \propto \lambda/D$ where D is the main reflector diameter and λ the wavelength. Although in principle one can mimic the low angular resolution of a small telescope with a large one, this is not possible ad infinitum in practice due to the limited stability of the integration and the data transfer time needed. Thus the resolution, i.e., telescope and wavelength used for observations, has to be appropriate with respect to the extent of the region to be mapped. We started our investigations with the Taurus-Perseus-Auriga region (Fig. 1.a), containing some of the nearest IMCs, which has been mapped by Ungerechts and Thaddeus (1987) as part of the whole Milky Way CO survey (Dame *et al.*, 1987). For more detailed observations we chose at the edge of the Taurus region the closest known IMC, L1457, (distance:

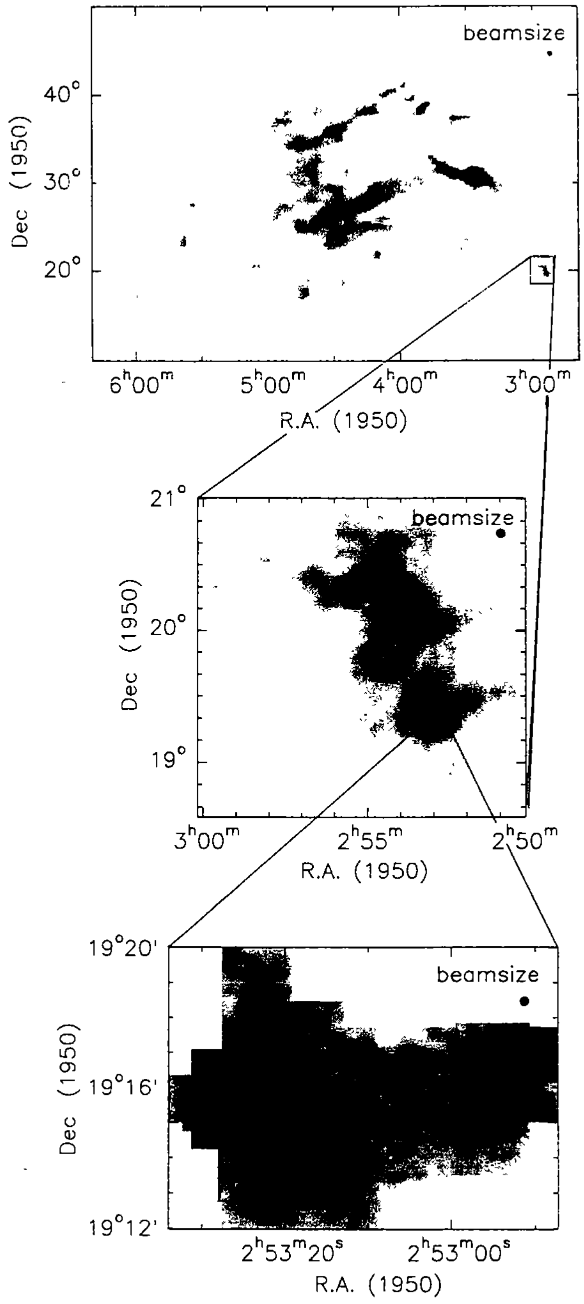


Figure 1: CO surveys of (a) Taurus (Ungerechts and Thaddeus, 1987), (b) L1457 (Zimmermann and Ungerechts, 1990), (c) L1457 core (Zimmermann, 1993), showing the self-similarity in the projected shapes of IMCs over a wide range of scales (from 4 pc down to 0.005 pc spatial resolution).

65 pc) (Figs. 1.b, 1.c) which is particularly well suited for studying IMC structures and dynamics down to very small linear scales. During the last years we mapped L1457 with successive higher resolutions in different isotopomers and transitions of CO (Zimmermann and Ungerechts, 1990; Zimmermann *et al.*, 1992; Zimmermann, 1993) to get a more complete picture of molecular gas over a wide range of size scales (spatial resolution: Taurus region: 4 – 0.6 pc, L1457: 0.08 – 0.02 pc, L1457 core: 0.005 pc).

3. SELF-SIMILARITY OF SPATIAL AND VELOCITY DISTRIBUTION

A first indication of ICs' self-similarity was found in the projected shapes of ICs, i.e., by log(*P*)-log(*A*) plots (*P*: perimeter, *A*: area) of iso-column density contours from different data sets (visible extinction maps (Beech, 1987; Hetem and Lépine, 1993), IRAS (Beichmann *et al.*, 1985) survey (Bazell and Désert, 1988; Dickman *et al.*, 1990; Scalo, 1990; Vogelaar *et al.*, 1991), HI surveys (Wakker, 1990; Vogelaar *et al.*, 1991), CO surveys (Falgarone *et al.*, 1991; Zimmermann and Stutzki, 1992), similar to the work of Lovejoy (1982), and Rys and Waldvogel (1986) for terrestrial

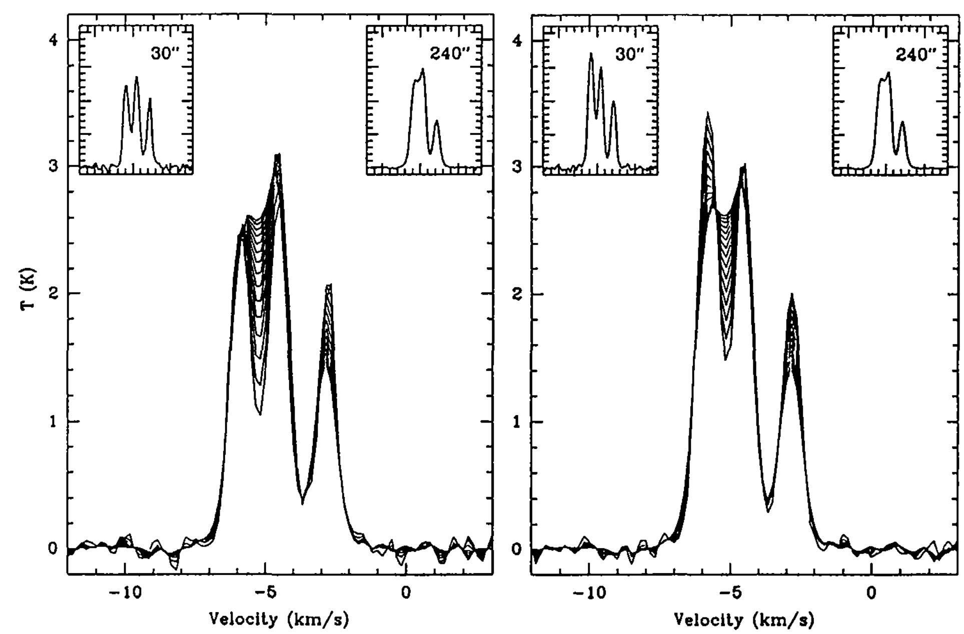


Figure 2: Variation of typical CO spectra in the direction of L1457 at different spatial resolution (30"(15")240") (intensity in units of Kelvin vs. velocity), showing a split up of the left line component with increasing resolution.

clouds. From the relation $P^{1/D_H} \alpha A^{1/2}$ these plots yield fractal dimensions D_H in a range between 1.2-1.7. The same fractal dimensions are derived independently when rescaling data sets with different resolutions ϵ to the same scale by the relation $P(\epsilon) \alpha \epsilon^{1-D_H}$ (Falgarone *et al.*, 1991; Zimmermann and Stutzki, 1992). However, the significance of these numbers is still being discussed since they may be influenced by effects like noise (Dickman *et al.*, 1990) or finite resolution (Zimmermann and Stutzki, 1992); moreover it is unknown in which way certain physical conditions lead to certain fractal dimensions of ICs' shapes.

The fractal shapes of iso-column density contours suggest fractal iso-density surfaces and thus a fractal spatial distribution, although due to the projection along the line of sight it is impossible to observe the 3-dimensional structure of ICs directly. Fortunately, the velocity distribution of cloud substructures can be observed by their Doppler-shifts in the spectra; this yields information about substructures along the line of sight assuming different structures having different velocities. Looking at CO spectra at different spatial resolutions (Fig. 2), we find that the lines, showing the dynamical distribution of the molecular gas, split up with increasing resolution into several narrow lines, similar to the spatial structures. Although no general relation between velocity and spatial distribution of molecular gas along the line of sight can be assumed, the overall presence of this phenomenon strongly points to a self-similar distribution of molecular gas in direction perpendicular to the projection plane, and thus to a self-similar 3-dimensional distribution.

4. MASS DISTRIBUTION AND SCALING RELATIONS

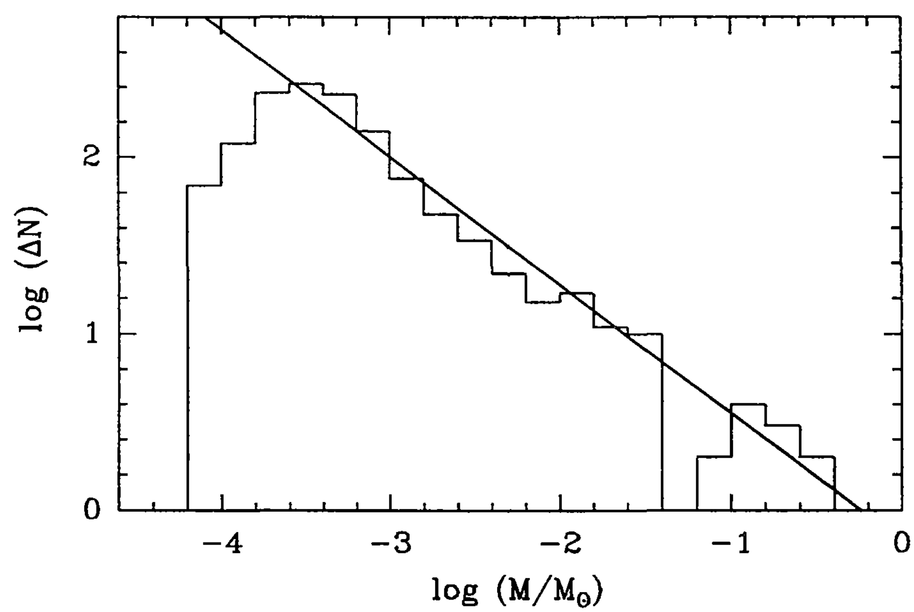


Figure 3: Mass spectrum of the decomposition of the CO map of the L1457 core (see Fig. 1.c) into Gaussian-shaped substructures, showing the power-law relation $dN/dM \alpha M^{-1.730.\pm0.06}$ (solid line).

The variation of physical excitation conditions (like density and temperature) and physical parameters (like mass and velocity dispersion) with size scale can be perceived by analysing separable substructures, or even by averaging over arbitrary subregions of observed IMCs. The latter is not surprising, keeping in mind that the definition of a 'substructure of its own' is appropriate only within the respective spatial resolution because structures dissolve and split up into smaller substructures when changing to higher resolutions.

Nevertheless authors have attempted to determine definite entities called 'fragments' and 'clumps' (Ungerechts and Thaddeus, 1987; Zimmermann and Ungerechts, 1990; Stutzki and Güsten, 1990; Herbertz *et al.*, 1991) to derive substructure mass spectra (Stutzki and Güsten, 1990; Lada *et al.*, 1991) which could be related to star formation characteristics like the stellar initial mass distribution (Larson, 1992; Stutzki, 1992; Lada *et al.*, 1993). We followed this idea and used an algorithm of Stutzki and Güsten (1990) to decompose the CO map of the L1457 core (see Fig. 1.c) into Gaussian-shaped substructures. The mass spectrum of this decomposition (Fig. 3) shows the power-law relation $dN/dM \propto M^{-1.73\pm0.06}$ (marked by solid line), where the largest structures have masses close to 0.5 solar masses ($M_\odot$). For physical parameters of the Gaussian-shaped substructures power-law scaling relations with size R according to the Larson-relations (Larson, 1981) are expected. The dispersion of the velocity distribution σ_v follows for structures larger than the resolution limit (dashed line) a power-law σ_v (kms^{-1}) $\propto R$(pc)$^\alpha$ (Fig. 4) with α between 0.38 (thin line, Larson, 1981) and 0.50 (fat line, Myers, 1983; Dame *et al.*, 1986) which were found for samples of other ICs. This is expected if the turbulent energy scales linear with the size: $\sigma_v{}^2 \propto R$. If this scaling relation still holds down to thermal velocity dispersion, a lower limit for substructure sizes will be given.

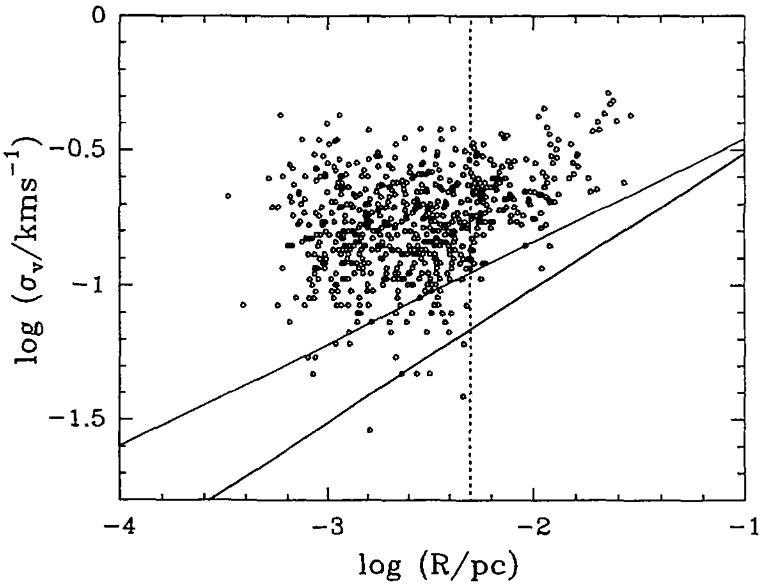


Figure 4: Velocity dispersion σ_v versus size R for Gaussian-shaped substructures (fat line: α = 0.50, (Myers, 1983; Dame *et al.*, 1986); thin line: α = 0.38, (Larson, 1981)[30]); dashed line: resolution limit.

5. MODELLING THE 3-DIMENSIONAL SPATIAL STRUCTURE OF INTERSTELLAR CLOUDS

Being led by indications of 3-dimensional self-similarity of IMCs as discussed in section 3, we have modelled 3-dimensional distributions of molecular gas and compared them with the observations. Our model-space is a cube of 64 × 64 × 64 cells, which is divided into 8 subcubes, each of these again into 8 subcubes, and so on, until the smallest subcubes consists of only 1 cell. The model-space is assumed to be filled with matter, which is distributed randomly onto N_1 of the 8 subcubes at division level L_1, i.e., $8 - N_1$ subcubes remain empty. Thus on each level L_n ($n = 1(1)6$) only the matter in filled cubes can be further distributed onto subcubes, until the lowest level L_6 is reached. Since the filled volume decreases from level to level the density increases with smaller scales like observed in real IMCs. This process produces a 3-dimensional Cantor-dust and is called 'curdling' by Mandelbrot (1982). Only if the number of filled subcubes is the same on each level this results in a (random) fractal distribution; in general it is just hierarchical and not self-similar. Already Hoyle (1953) tried to explain the distribution of galaxies with a similar 2-dimensional model, and Hetem and Lépine (1993) used a method like this to model ICs structures as well.

To compare models with observations we apply a box-counting method to the observed map and the 2-dimensional projection of the model, i.e., we count the number of boxes needed to cover map areas with intensities higher than definite values (Fig. 5.a: 10 %(10 %)50 % of peak intensity), and doing this for successive smaller box sizes. Figure 5.a shows log-log plots of the number of boxes needed versus inverse boxsize for the Taurus map and its best fit model which we obtained by varying the numbers N_n ($n = 1(1)6$) of matter filled boxes on each level. In Figure 5.b a part of the Taurus map is compared with the projection of its model. Whereas for Taurus and the whole map of L1457 (Figs. 1.a, 1.b) we find relatively high volume filling at largest and smallest scales, and relatively small filling in between, the best fit model of L1457 core (Fig. 1.c) has the same number of 7 filled subcubes on all levels except one and therefore can be characterized by a fractal dimension $D_H = \log 7/\log 2 = 2.81$.

The projections of our models are not thought to have the exact morphology of the observations, but to show the same fractal/hierarchical properties in their area filling which is measured over all size scales, inherent in an observed map, by means of the box-counting. From the models we can extract typical density profiles along the line of sight. Assuming a certain velocity field along the line of sight we can calculate line profiles and compare them with the observed ones; e.g., for a constant velocity gradient the line profiles are identical to the density profiles.

6. CONCLUSIONS

From investigating the observations and modelling its 3-dimensional structures we draw the following conclusions:

1. Our observations of IMCs cover a wide range of size scales between 100 pc and 0.005 pc, which is necessary to investigate their hierarchical and fractal properties.

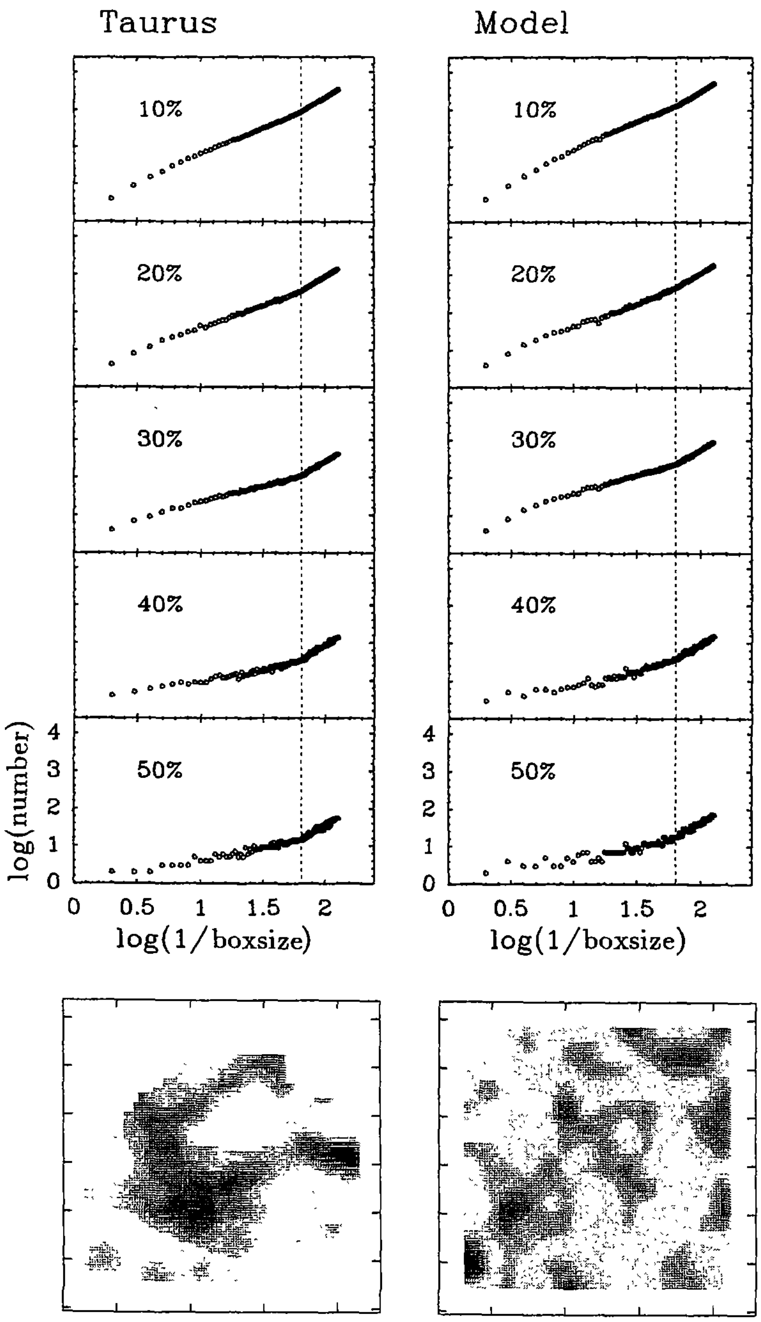


Figure 5. a: Plots from box-counting of Taurus CO map and the projection of the best fit model; **b.** Taurus CO map compared to the projection of the best fit model.

2. ICs appear to have self-similar projected shapes with fractal dimensions $D_H = 1.2\text{-}1.7$ derived from log(P)-log(A) plots.

3. We find that molecular spectral line profiles split up with increasing spatial resolution into several narrow lines, pointing to a self-similar distribution of molecular gas along the line of sight as well.

4. Conclusion 2. and the overall presence of 3. point to a self-similar or at least hierarchical 3-dimensional fractal distribution of molecular gas.

5. IMCs substructures show a mass spectrum $dN/dM \propto M^{-1.73\pm0.06}$ and a power-law scaling relation of the velocity dispersion σ_v with size R of $\sigma_v \propto R^{\alpha}$ where $\alpha = 0.38\text{-}0.50$ which can be regarded as another aspect of the fractal structure.

6. Comparing observations and models by box-counting we succeed in modelling the observations by a method of hierarchical subdivision; from the models, density profiles along the line of sight and realistic spectral line profiles can be derived.

REFERENCES

Bazell, D., Désert, F. X. (1988) *Astrophysics Journal*, 333, 353.

Beech, M. (1987) *Astrophysics and Space Sciences*, 133, 193.

Beichmann, C. A. *et al.*, eds. (1985) *IRAS (Infrared Astronomical Satellite) Catalogues and Atlases Explanatory Supplement*, Washington, D.C.: U.S. Government Printing Office, .

Dame, T. M. *et al.* (1986) *Astrophysics Journal*, 305, 892.

Dame, T. M. *et al.* (1987) *Astrophysics Journal*, 322, 706.

Dickman, R. L. *et al.* (1990) *Astrophysics Journal*, 365, 586.

Evans, N. J. (1990) In: *Frontiers of Stellar Evolution.*

Falgarone, E., Phillips, T. G. (1990) *Astrophysics Journal*, 359, 344.

Falgarone, E. *et al.* (1991) *Astrophysics Journal*, 378, 186.

Hentschel, H. G. E., Procaccia, I. (1984) *Physical Review A*, 29, 1461.

Herbertz, R. *et al.* (1991) *Astronomy and Astrophysics*, 249, 483.

Hetem, A. Jr. Lépine, J. R. D. (1993) *Astronomy and Astrophysics*, 270, 451.

Hoyle, F. (1953) *Astrophysics Journal*, 118, 513.

Lada, E. A. *et al.* (1991) *Astrophysics Journal*, 368, 432.

Lada, E. A. *et al.* (1993) In: *Protostars & Planets III*, Tucson: University of Tucson Press.

Larson, R. B. (1981) *MNRAS*, 194, 809.

Larson, R. B. (1992) *MNRAS*, 256, 641.

Lovejoy, S. (1982) *Science*, 216, 185.

Mandelbrot, B. B. (1982) *The Fractal Geometry of Nature*, New York: Freeman, 460 pp.

Myers, P. C. (1983) *Astrophysics Journal*, 270, 105.

Rys, F. S., Waldvogel, A. (1986) *Physical Review Letters*, 56, 784.

Scalo, J. M. (1990) In: *Physical Processes in Fragmentation and Star Formation*, Dordrecht: Kluwer.

Shu, F. H. *et al.* (1987) *Annual Review of Astronomy and Astrophysics*, 25, 23.

Stutzki, J., Güsten, R. (1990) *Astrophysics Journal*, 356, 513.

Stutzki, J. (1992) *Review of Modern Astronomy*, 6.

Ungerechts, H., Thaddeus, P. (1987) *Astrophysics Journal Supplement*, 63, 645.

Vogelaar, M. G. R. *et al.* (1991) In: *Fragmentation of Molecular Clouds and Star Formation*, IAU Symp. No. 147, Dordrecht: Kluwer.

Wakker, B. P. (1990) [Ph.D. thesis], Groningen: University Groningen.

Winnewisser, G. *et al.*: (1992) In: *Spectroscopy of the Earth's Atmosphere and Interstellar Medium*, San Diego: Academic Press.

Zimmermann, T., Ungerechts, H. (1990) *Astronomy and Astrophysics*, 238, 337.

Zimmermann, T. *et al.* (1992) In: *Evolution of Interstellar Matter and Dynamics of Galaxies*, Cambridge: Cambridge University Press.

Zimmermann, T., Stutzki, J. (1992) *Physica A*, 191, 79.

Zimmermann, T. (1993) [Ph.D. thesis], Köln: University Köln.