

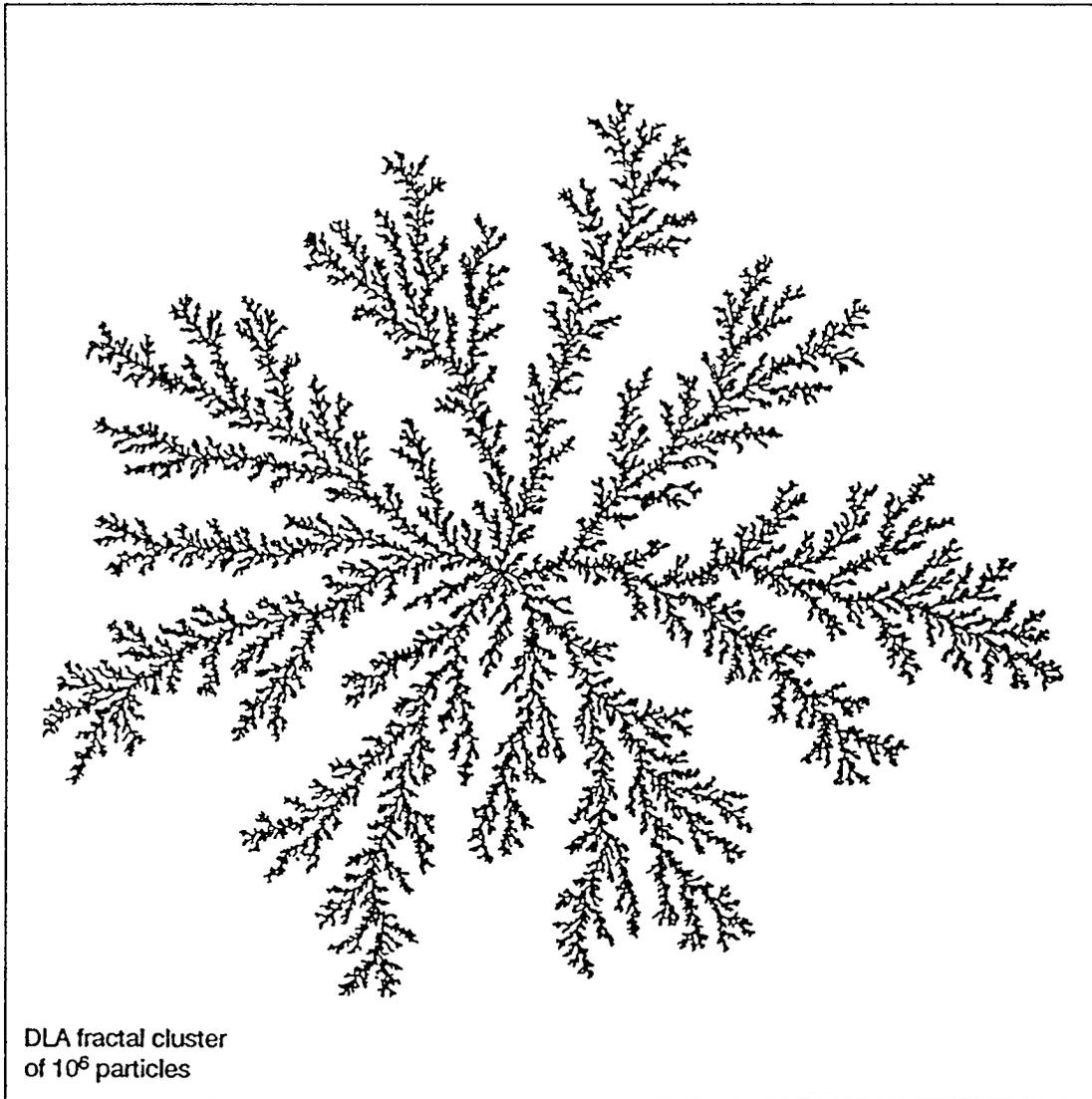
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FRACTAL PROPERTIES OF CLIMATIC RECORDS

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Abstract: *Analysis of climatic records strongly suggests that, except for some fairly obvious and easily removable deterministic component, the actual behaviour of climatic system is highly non periodic and obeys fractal statistics. Fractal nature allows for a mathematical comparison of irregularity characteristics of climatic changes obtained from a variety of observations and recorded at different time scales.*

All investigated climatic records are characterized by one general fractal dimension of 1.2–1.3 which appears to exist over the spectral range 10 to 10⁶ years. Records with such values of fractal dimension show some long-term persistence: even sufficiently distant from each other observations are not completely independent. If these conclusions become confirmed through analysis of a wider set of climatic records, long-run climatic prediction (in statistical sense) on different time scales will appear feasible.

1. INTRODUCTION

Climate data are scattered by region and discipline and climatic studies are being conducted in many subject areas. As a result, climatic data are greatly diverse and largely dispersed. Except for historical and instrumental records, mostly from the last few hundred years, climatology data are 'proxy' data – measurements of natural phenomena which contain a signature of past climate (e.g. tree rings, lake and bog sediments, ice cores, etc.). When assembling diverse paleoclimate data into global data bases or reconstructing any continuous past climate interval one comes

across two major problems. First, climatic variables (e.g. temperature, pressure, wind velocity) are not measured directly. Their evaluation requires an interpretation of a variety of physical, chemical or biologic processes. The assembling or comparison of data obtained from diverse sources are very problematic. Second, the records cannot be fully sampled either spatially or temporally. Data vary significantly as to time span and time resolution. Taken together climatic records give consequently incomplete picture of climatic variations.

In such situation fractal analysis, which allows comparison of irregularity characteristics of records over a wide range of scales and is independent of units of measure, provides an extremely successful basis when studying processes governing climate change.

2. FRACTAL GEOMETRY FOR CLIMATIC RECORDS

Many physical processes at the Earth appear scale invariant and may be studied within the framework of fractal geometry (e.g., Turcotte, 1989). The definition of a fractal distribution (Mandelbrot, 1982) is that the number of objects N with a characteristic size greater than r scales with the relation

$$N = Cr^{-D} \quad (1)$$

where D is the fractal dimension and C is a constant of proportionality. Mathematical representation (1) could be valid over an infinite range and yields a possibility to characterize phenomena by a single parameter, the fractal dimension. However, for any physical application there will be an upper and lower limit on the applicability of the fractal distribution. If scale invariance extends over a sufficient range of length scales, the fractal distribution provides a useful description of the applicable statistical distribution. There exist several studies which argue that climate obeys fractal statistics.

The long-range paleoclimatic signatures are oxygen isotope ratios in carbonates deposited on the ocean bed. Oxygen isotope ratios in the ocean bed carbonate remains of benthic species depend on the temperature of the water in which these organisms lived, and thus directly indicate the prevailing temperatures of the sea floor (e.g., Lamb, 1977). These paleothermometers may indicate temperature variations to 1,000,000 years BP and beyond with 100 to several thousand years resolution. Analysing the oxygen isotope curve pertaining to Pacific core V28-238, Nicolis and Nicolis (1984) have established the existence of an attractor with fractal dimensionality associated with the long-term climatic evolution of the past million years. Fluegeman and Snow (1989) have investigated the oxygen isotope curve of Pacific core V28-239. The results rescaled range analysis (R/S analysis) indicated fractal geometry of the δO^{18} record for time scale 2,000,000 years (with 5000 yr resolution), with fractal dimension equal to 1.22. The authors considered this value as characteristic of global climate change over the last 2 m.y. as expressed in changing δO^{18} ratios.

Plotnick (1986) has argued that the distribution of the stratigraphic hiatuses at discontinuous sedimentation in a number of sedimentary environments exhibits fractal behaviour and may be modelled with a Cantor set, which allows the estimation

of the relative number and durations of stratigraphic hiatuses within some given section.

Mud and clay sediments formed at the bottom of lakes (varves) also contain a record of past climate. These are produced by regularly alternating sequences of colour, grain size, and mineral and organic composition, due to seasonal differences in the relative amounts of organic material and wind- and water- originated mineral particles deposited. A thicker or thinner varve is attributable to some climatic event. These records may have time span to 100,000 years BP, with 1-10 year resolution for varves. For five lakes in Sweden, with data for the time interval from 20th century to 0 A.D., Mandelbrot and Wallis (1969) obtained a mean fractal dimension of 1.17 ± 0.07 .

In present work *R/S* analysis is performed for a series of annual layers in the mud deposits of a salty lake Saki on the west coast of Crimea, for the period from 23th century B.C. to 19th century A.D. (data from Lamb, 1977; Appendix V., Tables 29 a-c). Data appear to indicate in the main rainfall variations in the Crimea. Data do not show a linear trend, but spectral analysis detected the presence of a long periodic component of ~ 750 yrs, and also a 80-years cycle with subharmonics. The first can possibly be attributable to some long-period perturbation of the Earth's orbit, whilst the second seems to be associated with the corresponding periodicity in solar activity. Figure 1 presents the values of mean thickness (mm) of the yearly mud layers and the *R/S* diagram. The slope of the linear regression line is $H = 0.81 \pm 0.04$, which yields a fractal dimension $D = 2 - H = 1.19$ (Mandelbrot, 1982).

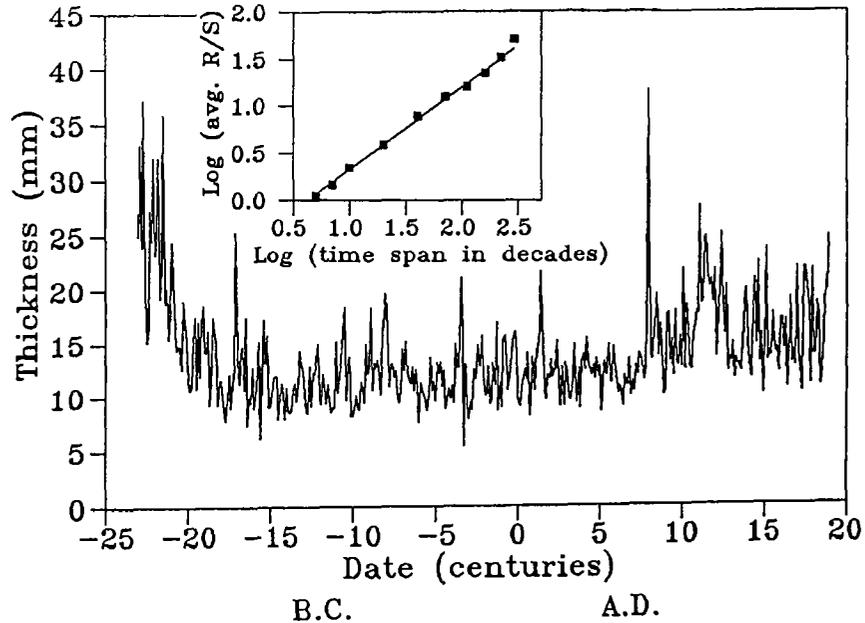


Figure 1: Mean thickness (mm) of the yearly mud layers by decades in lake Saki, Crimea, diagram. Plotted line represents a linear regression line for time spans of 50, 70, 100, 200, 400, 700, 1100, 1600, 2200, and 2900 years.

Mandelbrot and Wallis (1969) performed *R/S* analysis of the dendroclimatic data (tree ring index values) and derived an average fractal dimension of 1.32 ± 0.02 . We have calculated this number from 16 *D*-values (Mandelbrot and Wallis, 1969; Table 1) varying in the range 1.22–1.42. Most of the records were taken from forest-border sites in the southwestern U.S.A. and cover the period from the 13th to the 20th centuries. Since different tree species have their own characteristic responses to climatic changes, we have taken into account only records for *Douglas fir* and *Ponderosa pine* which behave much alike and can be used together. Besides, their growth ring widths have no autocorrelation from one year to the next. In other words their tree ring width characterizes only the weather of the given year (Lamb, 1977).

Table 1: Fractal dimensions for different climatic records

Type of record	Time interval of record (yrs)	Resolution (yrs)	Fractal dimension
Surface air temperature change			
northern hemisphere	105	1	0.89
southern hemisphere	105	1	1.25
global data	105	1	1.21
Annual surface air temperature	90	1	1.23 ± 0.01
Annual precipitation	≈ 150	1	1.26 ± 0.03
Tree ring indices	650-300	1	1.32 ± 0.02
Varve data			
Sweden	~ 2000	—	1.17 ± 0.07
Crimea	4200	10	1.19
Oxygen isotope ratio	2×10^6	5×10^3	1.22

Investigating annual precipitation from between 1817 and 1963 for nine major cities in the United States, the same authors concluded that regional precipitation was fractal with mean fractal dimension of 1.26 ± 0.03 (the range of variations: 1.12–1.36).

Another kind of climatic data on a similar time scale has been considered by Bodri (1993). This author investigated the annual mean surface air temperatures at seven meteorological stations in Hungary for the period of 1901-1991 yrs and has shown that the considered temperatures were fractals with a mean fractal dimension of 1.23 ± 0.01 (with an interval of variations from 1.19 to 1.28). The records were shown to contain also an 11-years periodic component corresponding to the well-known periodic element in the activity of the Sun, and also subharmonics of this period. For the annual means the presence of a periodic element makes the picture somewhat more complicated, but does not damage the *R/S* diagrams. In the case of e.g.,

monthly mean surface air temperatures, cyclic components (the main one of which is the yearly wave) yield a R/S diagram with a much reduced slope.

Below we present the results of an R/S analysis of surface air temperature change estimated from meteorological station records for the 1880-1985 years. Surface air temperature has been measured at a large number of meteorological stations for the past century. Hansen and Lebedeff (1987) have combined observational data from more than 2500 stations in a way which is desired to provide accurate long-term variations. They find that meaningful temperature change can be obtained for the past century, despite the fact that the meteorological stations are confined mainly to continental and island locations. Figure 2 shows the surface air temperature change for the global earth and the northern and the southern hemispheres as constructed by the Hansen and Lebedeff (1987) data. Data indicate common warming in the past century (0.6 K and 0.4 K for the northern and the southern hemispheres respectively, and 0.5 K global warming). This linear trend was eliminated from data before R/S analysis. A detailed description of the analysis procedure is presented in Bodri (1993). Figure 3 illustrates the variations of R/S values versus time span. The slope H was estimated from the time spans of 5, 7, 10, 20, 40 and 70 years. It equals 0.79 ± 0.03 for global data set, 1.11 ± 0.03 for the northern hemisphere, and 0.75 ± 0.06 for the southern hemisphere. The fractal dimensions are 1.21, 0.89, and 1.25, respectively.

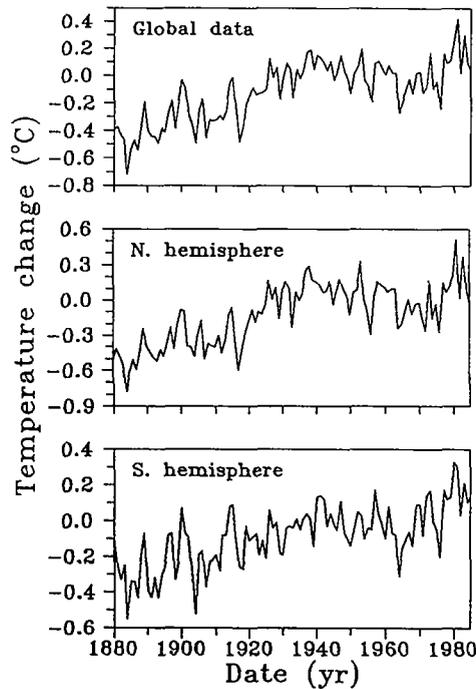


Figure 2: Surface air temperature change estimated from meteorological station records (by data from Hansen and Lebedeff, 1989).

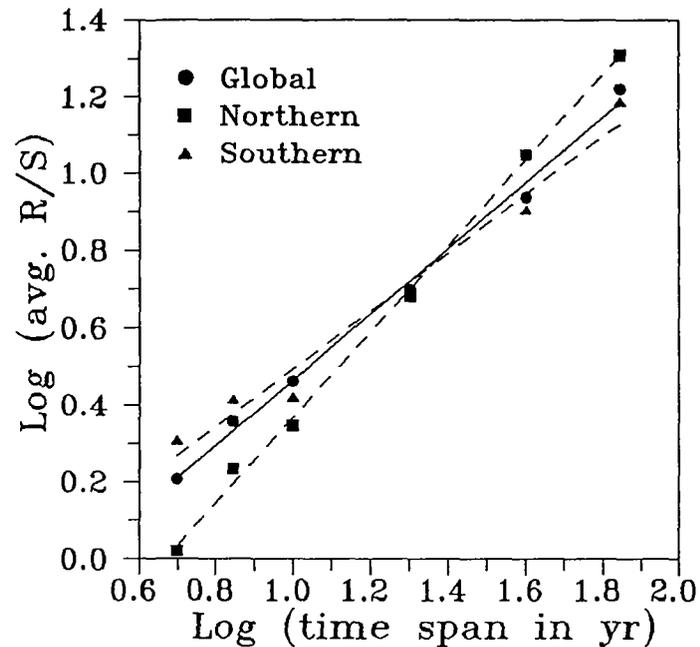


Figure 3: Plot of average rescaled range (R/S) values versus time spans for surface air temperature.

3. CONCLUSIONS

Analysis of different climatic records shows that despite the pronounced peaks of spectra in the 'cosmic' frequencies (orbital forcing, solar activity) their actual behaviour is highly non periodic and obeys fractal statistics.

Table 1 presents a listing of D -values of different climatic records that have been analysed to date from the point of view of their fractal geometric features. Important observation is the good agreement of fractal dimensions from the time scales 1-100 years to scales 5,000 to 2,000,000 years, which suggests the inference that the irregularity characteristics of a variety of climatic records are very similar. The fractal dimension for surface air temperature change at the northern hemisphere makes an exception. A natural explanation for such a deviation may be antropogen activity which has disturbed the intrinsic variability of climatic system. Such explanation seems even more feasible in the light that fractal dimension for the southern hemisphere, which could be affected by the man activity to a less extent than the northern one, shows practically the same value as the other climatic records.

The values obtained for the D -parameter suggest that climatic changes are characterized by long-time persistence (e.g., Mandelbrot, 1982). If extensive further studies of the different weather-paleoclimatic phenomena will support this conclusion, one may propose the obvious hypothesis that this similarity reflects the uniform interaction of the climate-forming processes and the materials being affected. The

reason for this general relationship can only be speculated upon. Nevertheless, such investigations will provide insight into the systematic nature of climatic records at different time scales. Unique fractal geometry of climatic records would permit interpolation of climatic changes from one scale to the next. It would appear realistic to reconstruct the entire power spectrum of the climatic changes from discrete data sets and with some additional information.

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