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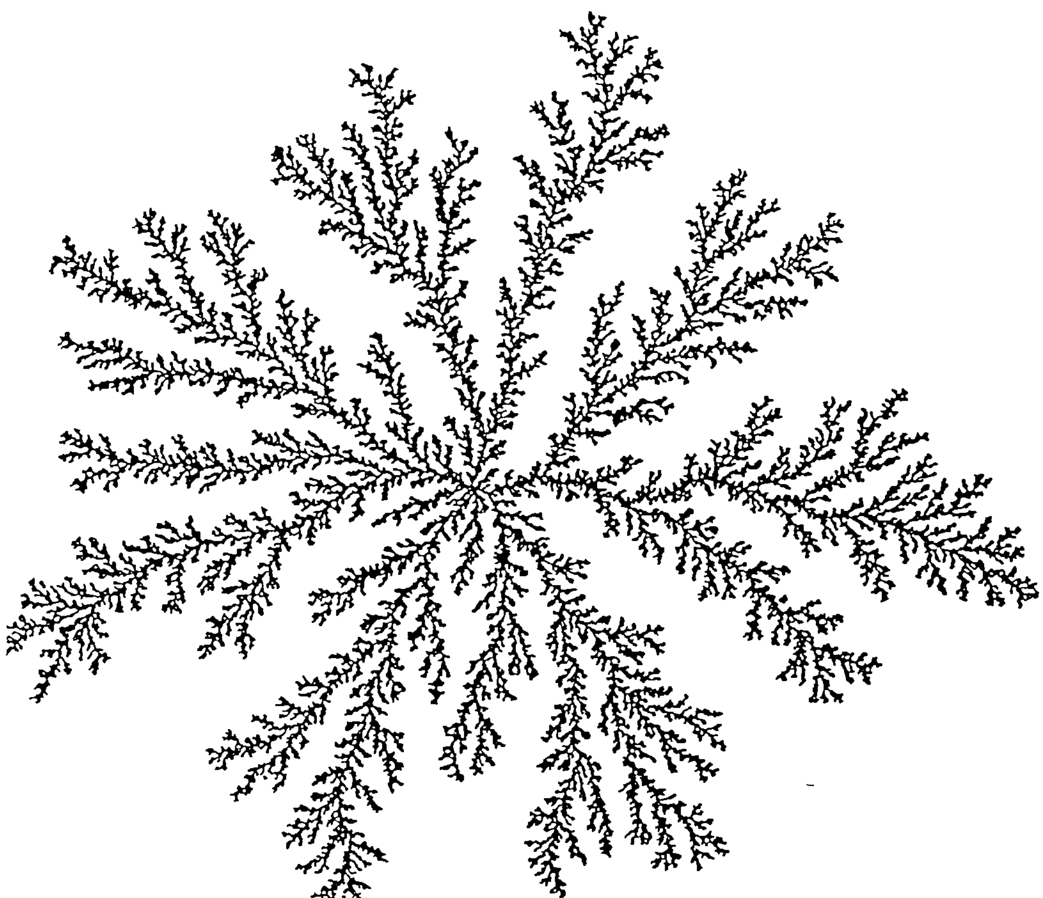

DLA fractal cluster
of 10^6 particles

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SYMMETRY IN EDUCATION

SYMMETRY IN ART, SCIENCE, AND TECHNOLOGY: INTERDISCIPLINARY UNIVERSITY TEXTBOOKS

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In an earlier note we remarked that there are some interdisciplinary courses on symmetry at various universities, but to assign or recommend appropriate reading materials is usually a great problem, despite the existence of a large number of books on symmetry (cf., Vol. 1, No. 1, p. 103). This reviewer faced such problems at three different places – in Hungary (1985), in Arizona (1988), and in Fiji (1989) – when conducting symmetry-related interdisciplinary courses. Indeed, in the case of the course entitled *Symmetry in Science, Art, and Technology* given at Arizona State University (Tempe, Arizona), we used different readings for arts-humanities students and for science-engineering students. Now we are happy to report that there exists a real university textbook on symmetry in art, science, and technology by Szaniszló Bérczi. Of course we should be more precise by emphasizing that it is the first broadly interdisciplinary textbook. Before that, let us note some other books on symmetry that are useful in higher education.

1. INTERDISCIPLINARY TEXTBOOKS AND TEACHING MATERIALS

William S. Huff published a five-part series of booklets (1967-77) entitled *Symmetry: An Appreciation of its Presence in Man's Consciousness* (altogether 93 pages), which presents a very impressive visual introduction to symmetry. Although it is not stated explicitly, obviously Huff developed this series in connection with his very influential Basic Design Course for students of architecture at Carnegie-Mellon University, Pittsburgh, and later at the State University of New York at Buffalo. These booklets can be used as recommended literature for much wider audiences, because they give a systematic introduction to the subject with carefully selected figures and related brief notes. Thus, Part 2 discusses the isometric symmetry operations, or, using the author's term, "isomorphic coverage operations" (cf., the German term *Deckung*, coverage, which refers to the fact that a symmetric figure by definition can be transformed by appropriate operations into a position such that the new one covers the original one). The six operations discussed, using the author's terminology (and adding in parentheses further names used in mathematics or crystallography, if any), are the following: mirror reflection, rotation, translation, translation and mirror reflection (glide reflection), rotation and translation (twist or screw rotation), rotation and mirror reflection (rotatory reflection, mirror-rotation, or rotoinversion). The first four can be defined in both 2- and 3-dimensional spaces, while the last two are 3-dimensional operations. Incidentally, in the 3-dimensional case the mathematical-crystallographical literature usually lists a further basic operation, the central inversion, or in other terms, reflection through a point (although it can be replaced by a rotatory reflection where the rotation is a halfturn). Part 3 deals with the seven "homoeomorhic coverage operations", i.e., similarity symmetries. The author follows the German chemist Karl Lothar Wolf's terminology and classification (cf., Wolf and Wolff, 1956), which is not identical with the usual approach by mathematicians. In a note, however, the author makes clear this difference. Part 4 gives an historic survey on geometric models used by astronomers, Part 5 presents a survey of symmetries in nature, including crystals, biological shapes, chemical structures, while the final Part 6 focuses on the symmetries of the human body and architecture. Note that this series was privately published, and the very first part never appeared. Luckily the author sent copies of the existing Parts 2-6 to many university libraries in North America. The rich set of figures, which gives the main stream of the series, includes many rare and unique illustrations. Thus, some are from Huff's Studio, including works by his students; others were researched by him in various libraries and rare book collections. The scientific illustrations are also very fortunate. For example, Huff presents the shape of a snowflake from Diderot's 18th century encyclopedia, also pointing out that the actual image displays some degree of fantasy (p. 4.7). The densest packing of equal balls is explained by an old instruction for cannon ball stacking, adapted from a book on the history of weaponry (p. 4.7). The dodecahedron, a regular polyhedron formed by 12 pentagons, is illustrated not only by Kepler's drawings (pp. 4.4-4.5), but also by an exciting stellar version of this, where all of the pentagons are replaced by pentagrams to present a skeleton with stars (p. 4.12; the original drawing is at The Percier and Fontaine Collection, Burnham Library, Art Institute of Chicago). When the author needs pictures on left-handed and right-handed crystal-forms, he presents the polyhedral models made by Pasteur to demonstrate his 1848 discovery of handedness in the world of molecules (p. 5.12). There are also contributions by Huff himself. Among others, he designed a very impressive tabular diagram to illustrate Mendel's law of heredity (p. 5.13). The reference style is also exceptional: the visual approach is not interrupted by long remarks about the origins of figures, but at the end of each booklet there are notes about all of them, mostly with further information about the actual problem and detailed bibliographical data. We strongly recommend reprinting these rare booklets; unfortunately the format of our journal is too small for this purpose. (Note that Parts 4-6 were reprinted by an architectural journal, but even those issues are not easily available.)

There is a very remarkable Russian series on symmetry and its generalizations. P. L. Dubov, V. A. Frank-Kamenetskii, and I. I. Shafranovskii published a four-part series of brochures (1984-87) for a special course given to students of crystallography, mineralogy, petrography, and geochemistry at the Leningrad (now St. Petersburg) State University (altogether 270 pages of small size). This series, however, can also be recommended for students of other fields, including mathematics and physics; moreover, some parts are also interesting to biology students. After surveying the classical symmetry groups of crystallography in the first part, the authors discuss their generalizations in the second part, including both colored symmetries and non-isometric symmetries, while the third part is devoted to a detailed study of antisymmetry (black-and-white symmetry). The final part presents tables of symmetry groups for individual student work. Specifically, the space groups of the cubic system and the 80 two-sided wallpaper groups are discussed: all of them are represented by graphical illustrations. Note that the study of various non-isometric symmetries (similarity symmetry, conformal symmetry, curvilinear symmetry, etc.) in the second brochure provides much important information beyond the usual scope of crystallography and mineralogy textbooks; moreover Dubov's chapter on generalized local symmetries presents some totally new results (Chap. 2.6). We think that this is a special advantage: a good textbook goes beyond surveying the known results and, at an advanced level, may demonstrate the latest research and even present open problems. Such sections may significantly encourage the interest of the research-oriented students. On the other hand, the brochures include simpler questions after each chapter. This is again very important. There are very many problem books on well-established university subjects, but we need something similar in the field of interdisciplinary symmetry studies. The brochures are very well illustrated, although the quality of printing is poor. This series is in some sense the counterpart of Huff's. Indeed, the series by Huff does not require a mathematical background and has no intention – beyond a rather intuitive and visual approach – of developing such skills, while the series by Dubov and his coauthors clearly needs a solid background in crystallography and mathematics. Consequently, the first one cannot be recommended as a basic text for science students, while the second one would be too difficult for students of arts. On the other hand both series can be used 'inversely' as additional material to provide a broader scope: mathematics and science students may enjoy the impressive visual representations of scientific ideas and their design-related applications in Huff's series, while arts students may profit from the series of Dubov and his coauthors, especially from the part on generalized symmetries where there are some exciting figures.

There are two other interdisciplinary textbooks published in Russia. *Simmetriya v tekhnike* [Symmetry in Technics, in Russian] by R. P. Povileiko (1970) deals with design and human engineering (129 pp.). This is the reason that in the English translation of the title we do not use the word "technology", but rather "technics", i.e., an expression that was preferred by the noted historian Lewis Mumford (cf., his book *Art and Technics*, New York, 1952). Povileiko's book was written for a course at the Novosibirsk Institute of Electrical Engineering. Consequently, the author could use more mathematics than Huff, but his book still remains understandable at the level of most design students. On the other hand, it does not cover as many topics as the series by Huff. The other textbook entitled *Sistemno-simmetriinyi analiz ob"ektov prirody* [Systemic-Symmetric Analysis of Objects of Nature, in Russian] by E. M. Khakimov (1986) was published for a course at the Chelyabinsk Teachers' Training College (95 pp.). This book was written for science students with a philosophical interest. Most examples of the author are based on crystallography, mineralogy, and biochemistry. On the other hand, the author does not use the conventional approach, but rather presents his own method emphasizing the aspects of hierarchy and modelling.

We should also mention an American booklet entitled *Symmetry, Rigid Motions, and Patterns* by Donald W. Crowe (1986), which not only provides a systematic study of geometric symmetries in the plane, but also demonstrates their importance for pattern analysis of ornamental arts (viii + 36 pp. and 20 unnumbered plates with worksheets and transparency materials). It appeared in the framework of the High School Mathematics and its Applications Project (HiMAP), which is designed to provide materials for secondary school mathematics teachers. We think, however, that this booklet is also useful for undergraduate students of mathematics, as well as for students of science education. Some parts may be recommended even for anthropology students specializing in material culture. Indeed, the author, a professor of mathematics, participated in excavations in Ghana. Thus he can provide first hand information on both geometry and anthropology. The main purpose is, however, rather mathematical.

There are some further textbooks on symmetry in various disciplines. We mention some of them here that may have some importance for interdisciplinary studies, despite the focus on specialized subjects. The Springer Series of Undergraduate Texts in Mathematics includes two symmetry related items: *Transformation Geometry: An Introduction to Symmetry* by George E. Martin (1982) and *Groups and Symmetry* by Mark A. Armstrong (1988). Both books can be recommended not only to mathematics students, but also to those physics and chemistry students who are specializing in solid state physics or structural chemistry, respectively. A similar German series of Springer entitled *Hochschultext*, i.e., texts for higher education, also includes a book that should be mentioned: *Symmetrien von Ornamenten und Kristallen* [Symmetries of Ornaments and Crystals, in German] by Michael Klemm (1982). Its level is even higher. The emphasis on ornaments in the title is somewhat misleading: there are 10 pages at the beginning where the 7 frieze and the 17 wallpaper groups are illustrated; the remaining 200 pages deal with sophisticated mathematics, without any artistic illustrations. There is another textbook in Russian about the mathematical theory of symmetry groups by A. M. Zamorzaev and A. F. Palistrant (1977). It is written for advanced students of mathematics at the Kishinev State University in Kishinev (now in Moldova). Note that this university became a major center for the study of symmetry in mathematical crystallography under the leadership of Zamorzaev. Probably there is no other university in the world where so many people earned doctorates in symmetry related topics, including antisymmetry, colored symmetry, similarity symmetry, and affine symmetry. The International Union of Crystallography also made an important step to help university education: they published a brief teaching edition of their tables of space symmetry groups (Hahn, 1985). It includes some of the introductory articles and a carefully selected section of the 'full' tables. The latter is so expensive that it is hardly affordable by students. The reason for the teaching edition was precisely to provide some basic materials in a much cheaper form. The topic of symmetry of molecules also attracted authors of textbooks at various universities. The textbook in English by Hollas (1972) was translated into German in 1975. Then in the same year of 1979 further textbooks were published at three different places: a Croatian book by L. Klasinc, Z. Maksić, and N. Trinajstić (1979) and two Russian textbooks by Yu. G. Papulov (1979) at the Kalinin State University in Kalinin (now Tver') and by P. M.

Zorkii and N. N. Afonina (1979) at the Moscow State University, respectively. Also see the more recent textbook by P. M. Zorkii (1986). These books, especially the last two ones, are useful not only for chemistry students, but also for those who study other fields of materials science. A booklet in Danish by J. Sløk *et. al.* (1975) focuses on symmetry in science. Last, but not least we should mention two art-related books: there is a textbook in Japanese about patterns and symmetry written by S. Kumagai and Y. Sawada (1983) at the Kanazawa Technical University, while a book in Spanish about the symmetry-properties of the embroideries and laces in the region of Castilla and Leon was authored by F. Rull Perez (1987) at the Institute of Education of the University of Valladolid, Spain. Note that we do not mention here physics textbooks because those can hardly be used outside of the discipline.

2. INTERDISCIPLINARY MONOGRAPHS

We should list, however, some monographs on symmetry that can be used either as required texts or as recommended readings. The books entitled *O symetrii w zdobnictwie i przyrodzie* [On Symmetry in Decorative Art and Nature, in Polish] by Stanisław Jaśkowski (1952) and *Symmetry Discovered* by Joe Rosen (1975) can be used immediately as textbooks. Obviously these authors, a Polish mathematician and an Israeli physicist, were thinking about students when they produced these monographs, although, according to our knowledge, these did not accompany specific courses. Both of them include exercises and problems, as well as a detailed list of further readings. We should also speak here about one of the most comprehensive works in the field of symmetry: the monumental volume entitled *Symmetry in Science and Art* by A. V. Shubnikov and V. A. Koptsik (1972), written in Russian, but very quickly translated into English. This book has many functions together: considering just the rich set of illustrations and the related explanations, it is a popular-scientific book for all interested people; going through systematically Chapters 1-9 and the conclusion, it is a textbook for advanced students; while considering the remaining Chapters 10-12, it is a monograph for specialists with many novelties (some of them still not available in other books). We are very sorry to report that this book is totally out of print. We even contacted the American publisher, Plenum Press of New York, and recommended reprinting it. There are, however, various difficulties with this: they would prefer not a simple reprint, but rather an updated version; on the other hand, the original form of the book with color plates would be much more expensive today. We are aware that V. A. Koptsik would be glad to revise this book and/or to write a second part. Unfortunately, the first author, A. V. Shubnikov, a very distinguished crystallographer, passed away long ago, during the preparation of the original Russian edition of this joint work, which was based on his earlier popular-scientific booklet (Shubnikov, 1940). There is a very impressive German book, too, which is unfortunately less well known (although the previously mentioned series by Huff contributed to its popularization, see Parts 2 and 3 in the series). It is the two-volume book by Karl Lothar Wolf and Robert Wolff (1956) entitled *Symmetrie* [Symmetry, in German]. There is a truly unique feature of this book: it includes 'symmetrically' about 50 per cent text (Vol. 1) and 50 per cent illustrations (Vol. 2). This rich set of illustrations is useful not only for people interested in science, but also for those who focus on arts. Another richly illustrated monograph was released more recently by Dorothy K.

Washburn and Donald W. Crowe (1988). It can be recommended to students of anthropology and archaeology. In addition to this, the book is also suitable for any course where the 2-dimensional patterns are discussed in detail. From the point of view of exercises, it is very useful to consider the Dutch artist M. C. Escher's periodic drawings, which were collected and discussed for educational purposes by Caroline H. MacGillavry (1965). The pioneering work on symmetry analysis in anthropology was published by Anna O. Shepard (1948). It has a great historic importance, and in this sense it can be recommended to interested students for survey. Indeed, we think that a course on symmetry should also deal with the history of the subject. In this sense, it is desirable to keep some historic books on reserve in the university libraries and to request their study. There are various ways of working with them. One of the possibilities in the case of a course at an advanced level, is to ask students to prepare seminar reports on the basis of such books. Here we very strongly recommend a rarely mentioned, but really unique, book by the Dutch chemist F. M. Jaeger (1917) entitled *Lectures on the Principle of Symmetry and its Applications in All Natural Sciences*. This work gained a second augmented edition in 1920, as well as being translated into French in 1925. Jaeger's monograph includes almost all progressive elements of the new wave of interdisciplinary books on symmetry: from the reference to Greek authors to the analysis of ornamental works, from snow crystal images to the adaptation of Haeckel's biological drawings. In addition to this, the bibliographic footnotes present a gold mine of information.

There are many popular-scientific books on symmetry that reach university level material. The best known item in this category is the booklet *Symmetry* by Hermann Weyl (1952), which was obviously influenced by Jaeger's cited book (see Weyl's reference to it in his preface and p. 29). Currently Weyl's book is available in at least 11 languages: in addition to the original English edition and its authorized German version, the book was translated into Bulgarian, French, Italian, Japanese, Hungarian, Polish, Romanian, Russian, Spanish (and possibly others). Note that the popular scientific works on symmetry that also cover actual research results were championed by Russian authors, including the very first comprehensive book on the subject by G. V. Vul'f [Wulff] (1908) and later the books by A. V. Shubnikov (1940) and I. I. Shafranovskii (1968). This tradition was continued by interdisciplinary monographs where there is a stronger emphasis on particular fields. For example, the book by Urmantsev (1974) may be used as a recommended reading in special courses where the aspects of system approaches and biological-philosophical questions are emphasized, while the monograph by S. V. Petukhov (1981) may be suggested to graduate students of biology and mechanical engineering. Returning to popular-scientific works reaching an advanced level, we should note that relevant books were written in Romanian (Roman, 1963), in Bulgarian (Sheikov, 1977, it is also available in German and Hungarian), as well as in Hungarian (Hargittai, 1983, an extended version is available in English and Russian). We may also mention a book published in Russian by a Mongolian publisher on philosophy and symmetry (Sodnomgombo and Khvan, 1981). From the late 1970s there is broad set of interdisciplinary monographs with a lighter or heavier emphasis on particular disciplines: we list some of them in the bibliography accompanying this article. Let us turn after this survey to the work mentioned at the beginning of this article.

3. BÉRCZI'S TEXTBOOK: *SZIMMETRIA ÉS STRUKTÚRAÉPÍTÉS* [SYMMETRY AND STRUCTURE-BUILDING, IN HUNGARIAN], BUDAPEST: TANKÖNYVKIADÓ, 1990, 260 pp.

The author of this textbook, Szaniszló Bérczi, a Hungarian physicist-astronomer, is clearly an interdisciplinary personality: his publications, according to the references, appeared in anthropological, artistic, biological, geological, mathematical, and physical journals or proceedings, as well as in general scientific-technical publications; all together 13 items by him are listed in the bibliography at the end of the book. One of his earlier works, a booklet on symmetry in ornamental arts, was reviewed in this journal (Bérczi, 1986-87; see SR 1.2 – 1 in Vol. 1, No. 2, pp. 219-220) and he is also the co-editor of and a contributor to a two-part special issue of this quarterly (Vol. 4, Nos. 1 and 2, i.e., this issue and the previous one).

The textbook by Bérczi, written for his course at the Eötvös Loránd University, Budapest, Hungary, is a remarkable revitalization of the best traditions of 'natural philosophy', an approach which is unfortunately very rare in our age. Recent specialization has forced most researchers to 'withdraw' into various laboratories. Bérczi, however, remained a fan of the largest laboratory, the whole of nature. He takes his students to collect minerals and stones in the framework of learning basic technologies, which lead to a petrographical discovery (pp. 148-149). He carefully sketches biological objects and ornamental works with artistic skill (see many illustrations in the book), and develops intuitive mathematical theories to describe their structures. These skills are very useful for a broad interdisciplinary approach to symmetry, as was demonstrated by some great pioneers, including Ernst Haeckel and D'Arcy Thompson.

Bérczi starts his textbook with a survey of dualities, including order-disorder, and then he turns concretely to order, or orderliness, focusing on symmetry (Chaps. 1 and 2). The book has a very important feature: the concept of symmetry is discussed not only in the context of art and science, as most interdisciplinary works on the subject do, but the technological aspects also have a special emphasis. Thus, most publications discuss frieze patterns as a crystallographical-ornamental topic, while Bérczi extends it with weaving and other craft forms (Chap. 3). Moreover, the approach is based partly on his own archaeological-anthropological studies. In the next chapter, the discussions of the theory of regular and semiregular tilings and polyhedra include a section on geodesic domes and shell structures used in architecture (Chap. 4). The book also deals in detail with crystal-structures and quasicrystals (Chap. 5), and gives a very useful table of structural hierarchy (Fig. 5.10, pp. 151-154). Another part of the book connects the topic of plant-structures and fractals (Chap. 6). The author, after surveying his own research on Fibonacci numbers in botany, presents a new cellular automaton model. Finally the last part entitled "From ornamental arts to the fourth dimension" deals with the intuitive artistic-technical development of figure-systems (Chap. 7). The author discusses here not only the communal art of various nations, but also describes some interesting examples in the arts of M. C. Escher, Salvador Dali, and a contemporary Hungarian artist, Tamás F. Farkas (cf., his works in this quarterly, Vol. 3, No. 1, "Symmetric gallery", pp. 83-100, as well as in this issue). The book includes an appendix on the Fibonacci series.

The selection of illustrations is exceptionally good throughout the book. This is a very important advantage of Bérzi's book, because the literature on symmetry has, unfortunately, too many 'invariant' illustrations. Bérczi, however, does not hesitate to draw his own illustrations or new versions of others. May we add here a note in connection with one of the illustrations. Hermann Weyl (1952, pp. 65-66) in his above mentioned classical book *Symmetry* pointed out that pentagons are very rare in architecture and he knows, beyond the Pentagon near Washington, D.C., only one inconspicuous example in Venice. (We should add that pentagons are frequent in mediaeval military architecture, but Weyl's statement is valid concerning civil architecture.) Bérczi presents the ground-plan of a further example: a Calvinist church in Szeged, Hungary (Fig. 1).

Although we are aware of a few other examples of pentagonal ground-plans in civil architecture, this topic has no good documentation. Thus Bérczi's data are helpful for a new survey of this topic. Note that further documentation of the same church is given in English in a more recent article by Bérczi and Papp (1992). (Interestingly, the latter source gives the date of design as 1940, while the textbook refers to 1942; perhaps 1940 is the year of the actual design, while the building was completed in 1942.)

Of course we do not claim that all details of this book will be completely clear to arts students, and all artistic sections are challenging to science students. We claim, however, that the basic ideas can be followed by all students, perhaps omitting a few small sections. Note that Bérczi conducted his course with mostly science and technology students, but it was also attended by arts students. How can Bérczi deal with both sides together? First of all, he does not go into too much technical detail, but rather uses an intuitive, sometimes even fuzzy, approach. In some cases, mathematicians may even claim that there are concepts that are not defined exactly. It is worth, however, to try to speak in a more intuitive way for a broader audience (we will return to this question later). Bérczi also likes to associate distant subjects and to explain their common properties. For example, he speaks at the same time to science students about 2-dimensional crystallographic groups, to technology students about weaving techniques, and to arts students about the ornaments of Finno-Ugric nations in Siberia. All of them will find, without too much difficulty, the important information for themselves. In addition to this, Bérczi obviously follows the old Chinese proverb that a picture is worth a thousand words, and we have already praised his illustrations. (Incidentally, he has mottos from Lao-Tzu at the beginning of each chapter.)

3.1 Teaching of polyhedra

We illustrate here Bérczi's method in the case of the theory of regular (Platonic) and semiregular (Archimedean) polyhedra, which is a sophisticated geometrical subject. Probably most students of arts have no desire to read mathematical textbooks on this topic, despite the fact that the importance of polyhedra can be seen in many works by famous artists from Leonardo da Vinci to Salvador Dali, from Albrecht Dürer to M. C. Escher. Thus, some authors provided fine picture-books on this subject (e.g., Critchlow, Holden, Loeb, Pearce and Pearce, Pugh, Williams, and Wenninger). Bérczi continues this tradition, making a new step: in his case there is no need to leaf through many pages, because he developed a "periodic

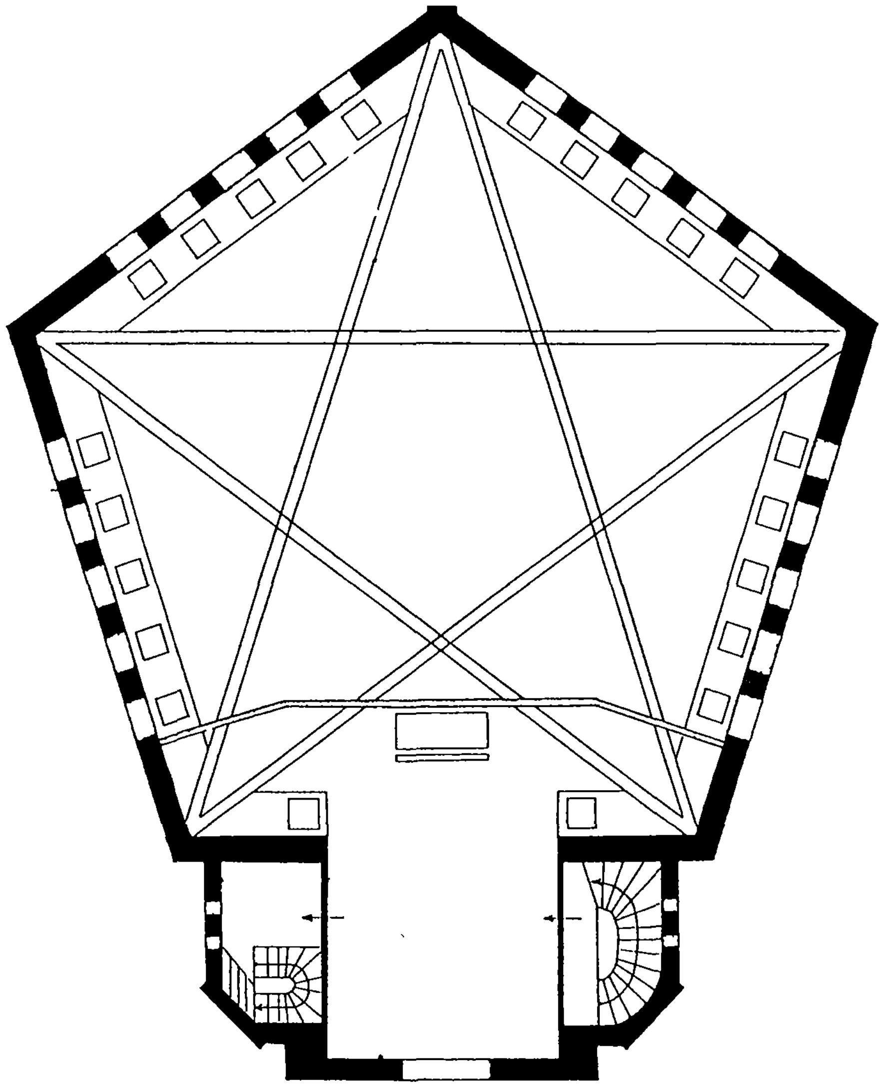

Figure 1: Ground-plan of the Honvéd Square Calvinist Church, Szeged, Hungary, designed by József Borsos in 1942. The pentagram in the plan shows the five arched rib-consoles that hold the ceiling. The rectangle, which can be seen under the pentagram, however, does not belong to the structural design: it shows the place of the harmonium. After Bérczi's Figure 2.2b (p. 38).

table" of all of them. Let us see, however, from the beginning the corresponding section (Chap. 4). First of all, he illustrates the topic of polyhedra with artistic examples. (A minor critical remark: he uses the symbol (5, 5, 5) to refer to a dodecahedron drawn by Leonardo in the caption of Fig. 4.1, p. 80, although this notation is not introduced until p. 87.) Then Bérczi gives the definition of both regular and semiregular polyhedra and tilings (tessellations) together in a table (p. 87):

Polyhedron or tiling	Faces	Vertices
regular	one kind of regular polygons	equal and regular
semiregular	many kinds of regular polygons	equal and non-regular

A mathematician may claim that it is a bit fuzzy, because the table does not refer to the fact that the polyhedra should be convex (consequently, the regular star polyhedra are not excluded). One may also claim that instead of referring to "equal vertices" the term "equivalent vertex-figures under a group" should be used to exclude the so-called pseudo-rhombicuboctahedron (or Miller-Bilinski-Ashkinuze polyhedron) from the list of semiregular polyhedra. Others may say that the strict definitions of the concepts "regular" and "non-regular" vertices are missing. We should note, however, that Bérczi's simple language is more easily understood than a very sophisticated and totally precise definition. Moreover, he refers briefly to convexity prior the table (p. 86), discusses the questioned pseudo-rhombicuboctahedron afterwards, using the simple name "scalped polyhedron" (a part of the rhombicuboctahedron is cut and put back after rotating it, pp. 100-101), and finally the concepts of regular versus non-regular vertices are clear from the illustrations. This part is followed by a rather technical discussion on the number of regular polyhedra; probably this sub-chapter should be skipped by arts students. (Note that this reviewer, investigating Euclid's original text on polyhedra and pointing out some problems there, proposed a much simpler proof of the fact that are exactly five regular polyhedra, without using the Euler's formula or any theorem beyond high school mathematics, cf., Nagy, 1988, pp. 2-3). After this, Bérczi, referring to some preliminaries, introduces the concept of "truncating sequence" of polyhedra. Let us consider, for example, an octahedron (double pyramid) shaped with eight regular triangles (Fig. 2, see the polyhedron at the left). At each vertex four triangles meet.

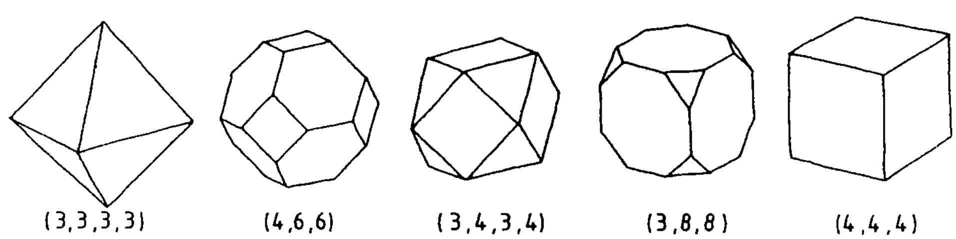

Figure 2: A truncating sequence: from the octahedron (at the left) to the cube (at the right). A part of Bérczi's Figure 4.7 (p. 93).

We may denote this polyhedron by the symbol (3, 3, 3, 3), referring to the fact that going around at any vertex, we see triangles, 3-sided polygons, four times (i.e., we list the number of sides of all polygons at a vertex in a cyclic order). Now let us truncate all 'corners' of this octahedron in a 'regular way'. If we perform this operation carefully, we will get squares at the former corners, while the triangles are cut into hexagons. Obviously, this operation can be done in such way that finally all of the hexagons have equal sides, i.e., they are regular (Fig. 2, second from left). This means that we obtain a semiregular polyhedron formed using two kinds of regular polygons: squares and hexagons. Considering the faces at any vertex, this polyhedron can be denoted as (4, 6, 6), i.e., at each vertex a square and two hexagons meet. If we continue the truncation at the vertices, in each case cutting along a plane parallel to the first cut, we get bigger squares, while at one point the hexagons will be reduced to regular triangles (Fig. 2, third from left). This is another semiregular polyhedron formed by regular triangles and squares. Going around at any vertex starting with a triangle, we have after this a square, again a triangle, and finally again a square: (3, 4, 3, 4). In the case of the next step of truncation, the triangles become smaller, while the former squares intersect each other and form regular octagons (Fig. 2, fourth from left). Using the conventional notation, it is (3, 8, 8). At the very last step, the triangles are reduced to points, while the other faces become squares: it is a cube (Fig. 2, at the right). Note that Figure 2 does not show that the polyhedra became smaller and smaller, all of them are enlarged to a size where the circumspheres (the spheres through the vertices of the individual polyhedra) are approximately the same. The reason for this convention is very simple: the truncation sequence can also be 'read' from right to left, therefore there is no sense in emphasizing one direction. Indeed, starting with the cube (Fig. 2, at right), and truncating its corners, we obtain the (3, 8, 8) semiregular polyhedron (second from right), and we may continue this process until we obtain finally an octahedron (at left).

Similar truncation sequences connect the tetrahedron with the tetrahedron (starting with a tetrahedron and finishing with a smaller tetrahedron), and the icosahedron with the dodecahedron. Moreover this method also can be used in the case of regular tilings by cutting a new polygon at each vertex and thus making other regular or semiregular tilings. Bérczi, in addition to the described "simple truncation sequences", introduces two more complicated types, and finally provides his periodic table (Fig. 3). It includes all the possible regular and semiregular polyhedra and tilings, with the exception of the regular prisms and antiprisms, as well as the above mentioned "scalped" polyhedron; these are discussed by Bérczi separately. We think that the middle section of the table, where the simple truncations are demonstrated in case of polyhedra and tilings, is easy to follow for all interested people. The left and right wings, where there are more complicated truncations, need more experience. The same is true in the case of the hyperbolic tilings in the last row. It can be claimed, however, that even in these more complicated cases, all readers will have some visual insight into the theory of these figures. Consequently, this table has a wide variety of applications in education: it is useful for mathematics and some science students to understand all details and to memorize all cases, but it is also challenging to other students to have a general view of the subject. Of course it is a further exercise for mathematics students to prove that the list, with the above mentioned exceptions, is complete, as well as to study some later sections of the same chapter on the 4- and 5-dimensional cases. On the other hand, art and technology students will find material on the "weaving" of polyhedra, some analogies in Escher's art, and the importance of polyhedra in architecture (the last part of Chap. 4).

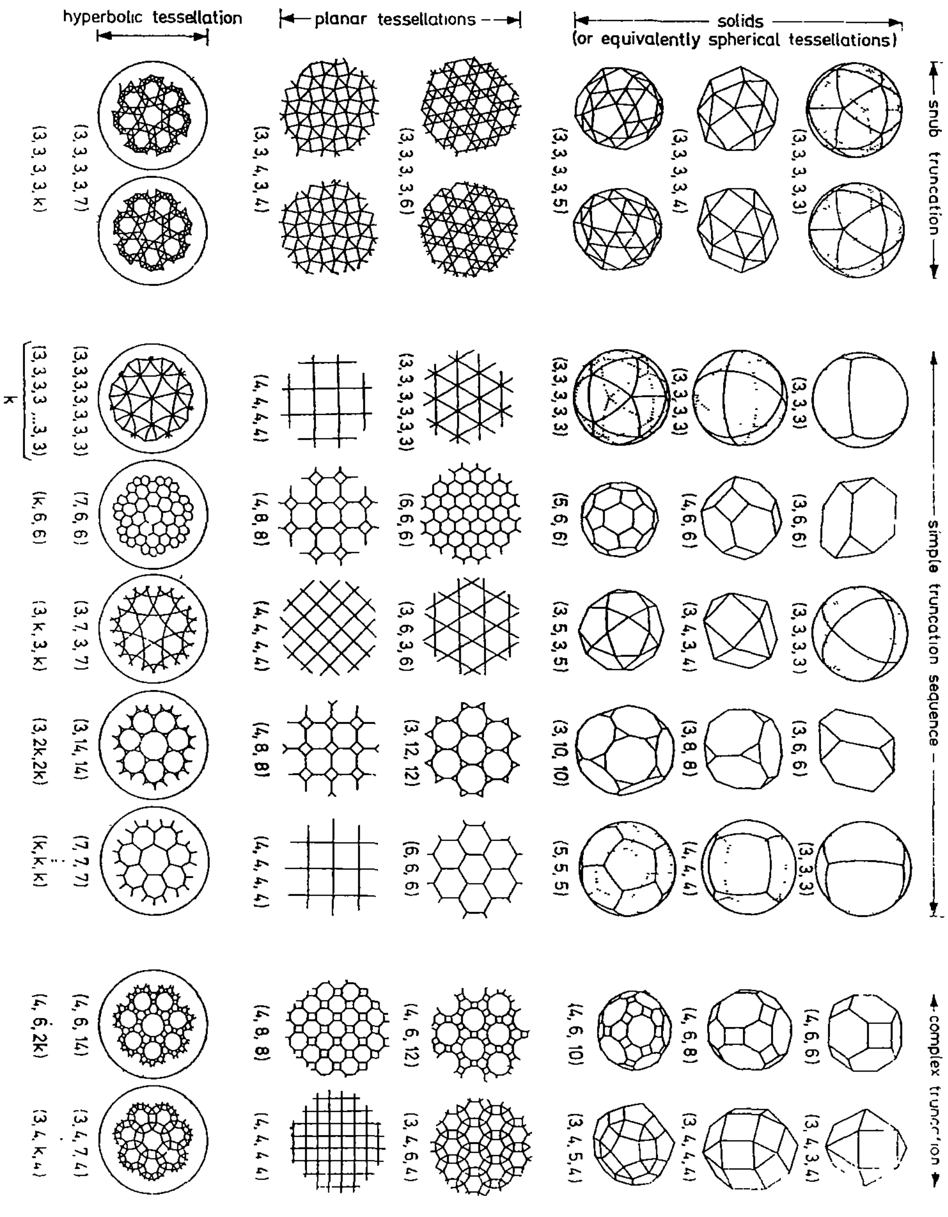

Figure 3: Bérczi's periodic table of regular and semiregular polyhedra and tilings, made by truncation operations. The table is extended by a sequence of hyperbolic tilings (last row). Note that the table gives all regular polyhedra with their spherical versions to distinguish them (i.e., these are projected onto their circumspheres from the centers). The table here is the English version of the periodic table in an earlier publication by Bérczi (1980, p. 187); cf., Figure 4.8 (pp. 95-98) in the textbook.

We suggest some minor corrections to the table. Although Bérczi's idea is clear that he would like to distinguish the regular polyhedra from the semiregular ones by using their spherical versions, we suggest using a different distinction: to draw the regular polyhedra with bolder lines. The reason for this suggestion is very simple: the spherical versions of polyhedra are less well known by students, and the section where these are introduced in the book is a bit too technical, thus some students may skip it. Moreover, the distinction using bolder lines can be also used in case of the regular tilings, which are in the present form not marked. We also suggest being more careful with the position of polyhedra: e.g., in case of the previously discussed truncation sequence (Fig. 2), which also features in the periodic table (Fig. 3, middle section, second row), the polyhedron (3, 8, 8) is a bit twisted in comparison with the left side polyhedron (3, 4, 3, 4). Specifically, the octagon of (3, 8, 8) and the square of (3, 4, 3, 4) facing in the direction of the viewer should be in parallel positions. Finally, there are some excessively deformed drawings, especially (3, 4, 5, 4), at the very right column, third from the top, which should be replaced with better ones. This table, which was nicknamed by a Hungarian mathematician as the "winged altar" of polyhedra and tilings, is so useful that we should try to make it even better. We are confident that it has a special value in education.

3.2 Some suggestions

Returning to the early question in connection with the priorities of Bérczi's book: it is – according to our best knowledge – really the first textbook that is written for a course attended by all art, science, and technology students. As we saw, this is not the first interdisciplinary textbook on symmetry in general, but the other books have more emphasis on either art or science. It seems to us that Bérczi was not aware of the previously discussed books by Huff (1967-77), Dubov, Frank-Kamenetskii, and Shafranovskii (1984-87), Povileiko (1970), Khakimov (1986) at all, while he refers to some other works by Crowe, but not to the mentioned teaching materials (Crowe, 1986). Thus Bérczi's book is an independent achievement. Moreover, it includes more material than the other works. Unfortunately all of these books are difficult to purchase: some of them are totally out of print, while others have only a limited circulation. Incidentally, the only places where Bérczi's textbook is available are some university bookstores in Hungary. It would be important to somehow help the circulation of this book. This problem is not easy, because most university textbooks are still subsidized in Hungary, consequently the publisher prints them just for students. A possible option would be to produce another version of this work for broader distribution, which would have a higher price.

Of course the first edition of Bérczi's book is not without mistakes, but many of these are just minor errors that are easy to correct in case of a later revision. Thus, Eschet (p. 31) is obviously Escher; Fejes-Tóth (p. 91) and even Fjes-Tóth (p. 253) should be Fejes Tóth, without the hyphen; the co-discoverer of quasicrystal is not Schachtman (p. 130), but Shechtman; MacKay (pp. 150, 155, 254) spells his name with lower case "k"; D'Arcy Thompson, M. C. Escher, and Owen Jones in the references (pp. 252 and 254) are alphabetized under D, M, and O, instead of T, E, and J, respectively; etc. We suggest revising the reference technique in general. The book includes a remarkable five-page list of references, but many of these items are not referred to in the text. On the other hand, the main text includes references which are not listed at the end, see, e.g., (Frank, 1952), (Barlow, 1901) on pp. 137 and 139, there are quotes without any indication of the source, e.g., by the writer László Németh (p. 212). Note that the so-called Barlow theorem and its proof, which was popularized by Coxeter's textbook *Introduction to Geometry* (New York, 1961, pp. 60-61), was known by Bravais in mid 19th century, long before Barlow. In other cases, the full bibliographic descriptions of books are given in the text, although these items are listed in the references, see, e.g., on p. 91 the monograph by Fejes Tóth, and on p. 218 the books by Artamonov and Forman, Rugyenko [Rudenko]. Obviously, a 'symmetrization' is needed between the references in the text and in the list at the end. Note that currently the latter is only in a quasi-alphabetical order: e.g., Czank is before Crowe, Dódony is before Dienes, etc., and the chronological order is also often violated in the case of individual authors, e.g., Bérczi (1989) is followed by Bérczi (1987), later we have Bérczi (1979). There is a misprint in the Contents: Chapter 6 starts on p. 159, not on p. 150. Turning to the figures, note that the appendix on Fibonacci series is interrupted with eight figures that do not belong to this topic, but to other subjects (unfortunately there are no appropriate references there, see pp. 240-247). It would be better to collect these illustrations separately, as well as to add captions. The same statement is also valid for the very last figure on p. 260, which is a schematic summary of the whole topic, but needs more explanation. The book is so rich in data and illustrations that it requires an index and a list of figures with full credits. In addition to these, there are some sections where the text may need a bit more revision than just to correct some misprints or minor mistakes. Last, but not least, we should remark that this textbook is designed specifically for Bérczi's course and his own skills. It is not lacking some subjective views, even overstatements. This is not a problem at all when he himself uses this textbook. On the other hand, it would be a great advantage if he could prepare a version of his textbook that can be more easily adapted by other instructors.

This reviewer, however, has no illusions about how a university textbook is produced in Hungary, which costs 16.50 Forints (first edition, 1990) or 32 Forints (second printing, 1991 – it is about half a dollar, that is the price of two local telephone calls in the U.S.). Also considering these circumstances, full credit should be given to the author for his heroic work. We hope very much that a revised and reedited version of this work will find its way to the broader public of readers of English.

... and we also hope that there will be more textbooks and courses on symmetry. More recently some universities decided to offer interdisciplinary courses at various levels. These courses should give a broader outlook to students and also supplement their knowledge in the main subject. These are offered in the framework of various interdisciplinary programs, see, e.g., General Studies (GEN), History and Philosophy of Science (HPS), New Liberal Arts (NLA), Science, Technology, and Society (STS). The name of the program may vary from university to university. We think that symmetry may have a good place in these interdisciplinary programs, because it has a direct 'umbilical cord' to the 'mother-subject' of each student. On the other hand, an interdisciplinary symmetry course may attract students with various backgrounds, which has an additional benefit. The situation that, let us say, mathematics and anthropology students analyze ornamental patterns together, mechanical engineering and architecture students discuss the stability of geodesic domes, biology and electrical engineering students design a robot based on biosymmetries, computing students suggest new transformations to multimedia projects of art students, and many similar connections may inspire new cooperation among students, who otherwise never meet and work together in the classroom. In this metaphoric sense, too, university education needs more 'symmetry' in art, science, and technology.

REFERENCES

The data of the referred books, together with many other ones that are applicable as recommended readings for university courses, are available in the section "Bibliography" of *Symmetro-graphy* of this issue (Nagy, 1993). We do not list here monographs on polyhedra, because we also plan to publish a bibliography on that topic.

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Bérczi, S. and Papp, L. (1992) A unique fivefold symmetrical building: A Calvinist church in Szeged, Hungary, In: Hargittai, I., ed., *Fivefold Symmetry*, Singapore: World Scientific, 235-243.

Nagy, D. (1988) Ideal and fuzzy symmetries: From the hard approach to the soft one, In: Nagy, D., ed., *Symmetry in a Cultural Context 2: Proceedings*, Tempe, Ariz.: Arizona State University, 1-6.

Nagy, D. (1993) Bibliography of textbooks and monographs recommended for interdisciplinary courses on symmetry at the university level, Section *Symmetro-graphy*, *Symmetry: Culture and Science*, 4, 2, 211-219. [It includes all of the referred books and many other items].