Symmetry: Culture and Science

Symmetry and Topology in Evolution

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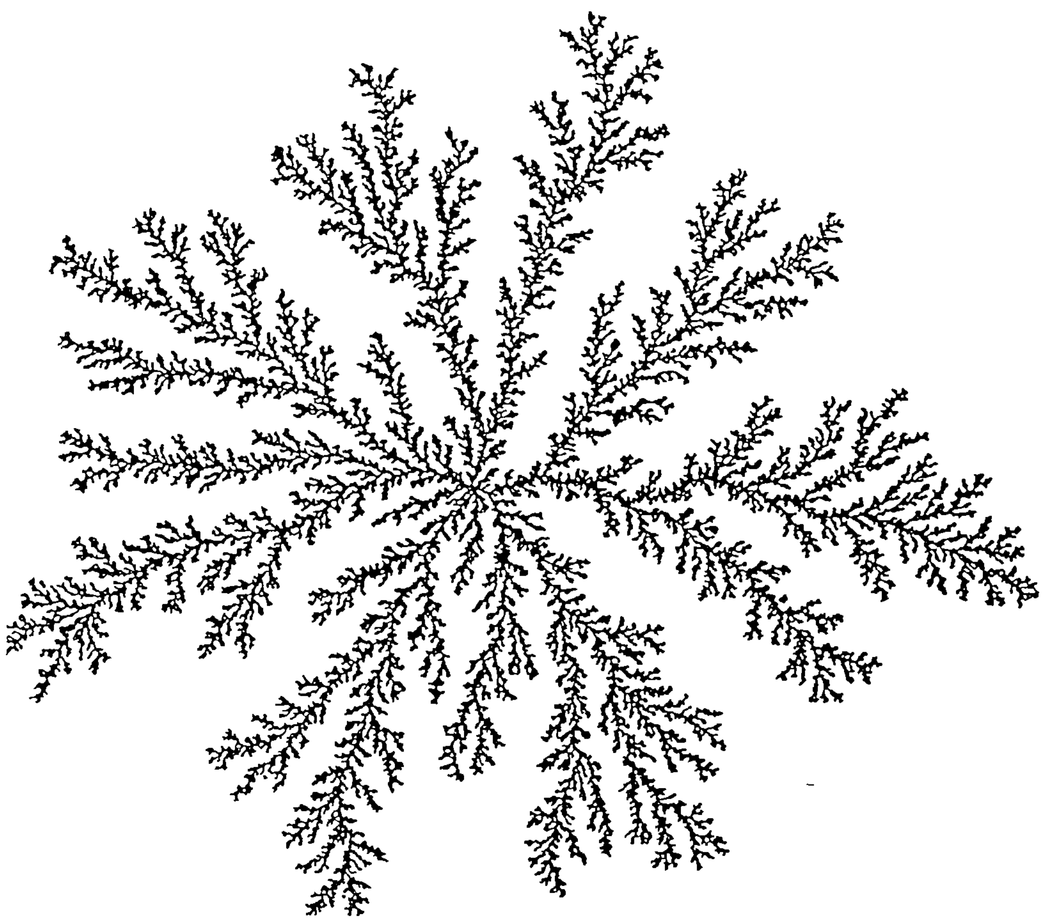

DLA fractal cluster
of 10^6 particles

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SYMMETRIC NEWS

Conference on

THE PRESENT STATUS OF THE PHILOSOPHY OF MATHEMATICS

The *Symmetrion – The Institute for Advanced Symmetry Studies* hosted the 1993 annual meeting of the *International Academy of the Philosophy of Sciences* in Budapest 19 - 22 May, 1993. The meeting composed part of a scientific conference, on *The Present Status of the Philosophy of Mathematics* with the participation and contributions of the members of the Academy and invited specialists of the theme. The conference was opened by *Evandro Agazzi*, President of the Academy, *György Darvas*, Director of the Symmetrion, and *Ernő Pungor*, Minister for Technological Development of the Hungarian Government. It was sponsored by the International Academy of the Philosophy of Sciences, the Hungarian National Committee for Technological Development, the Ministry of Culture and Education of Hungary, the Hungarian National Committee for UNESCO, the Hungarian Academy of Sciences, the George Lukács Foundation, the Europe Institute, Budapest, and the Eötvös Loránd University, Faculty of Humanities, Budapest.

Symmetrion, an institution which aims to bridge different disciplines, different cultures, as well as sciences and arts was chosen to organise this meeting not by chance. The Symmetrion is striving to lend a certain *generality* to the supported activities, in some sense similar to philosophy. The general umbrella, in this case, is the *concept of symmetry*, which is practically present in most sciences and arts, in any – theoretically unsplit, but unfortunately for a long time split – culture. To play with the words, the philosophical problems of mathematics are very close to the philosophy of the Symmetrion, due to the roots of this concept and phenomenon (cf. the "Aims and scope" of the *Symmetry: Culture and Science* on p. 224). This was the reason why the institute was happy to host this event.

The roots of symmetry are partly in *ancient* mathematics, partly in *ancient* philosophical thinking, (among others) where science and philosophy, arts and crafts, crafts and science were not yet separated. Symmetry formed a part of the picture of the known world of the Greek, Indian, Chinese philosophers (cf. the symbols: Platonic perfect solids, mandala, yin-yang), their picture of the Universe, as well as the known part of the Earth (cf. Herodotos) and used to be a basic principle in measuring ($\sigma\upsilon\mu$ + $\mu\varepsilon\tau\rho\iota\alpha$ = common measure), geometry (Euclid), and played a central role not only in ornamental art, but in architectural composition (Vitruvius) as well. It was not by chance that symmetry as a thought-forming concept and phenomenon returned in the centre of universal thinking in the *Renaissance*, to disappear again with the *Renaissance* universal intellectuals (the representatives of which type were e.g., Leonardo, Dürer), and to return in the scientific thinking at the turn of the *twentieth century*, when the development of different disciplines and new movements in the arts made apparent the need to search for common features, common laws, and common methods in different disciplines and arts (cf., classification in crystallography, the Noether theorem, quantum physics, quantum chemistry); and symmetry, now in a broader sense (cf., invariance, rhythm, repetition, tiling) played again the role of catalysing the thoughts of different intellectual activities (cf. the collaboration of mathematicians and physicists, philosophical problems of the astronomical interpretation of the Universe, the Bauhaus movement – comprising artists, scientists, and craftsmen (technê)). It turned out that the lack of symmetry (symmetry breaking) is one of the main compasses in the hands of scientists to discover new laws and phenomena (cf. P. Curie: "Dissymmetry makes the phenomenon") that brought together again physicists, crystallographers, chemists, and biologists, not to speak of mathematicians, and the representatives of all related disciplines. Symmetry- or dissymmetry-related considerations of scientists inspired artists, and those of artists fertilised the brains of scientists (e.g., Escher, Penrose, B. Fuller, and cf., the birth of quasicrystals, the theory of fractals, fullerenes), and the mutual influence open earlier unbelievable vistas in the functional brain asymmetry (lateralisation) and its consequences.

All these developments did not leave untouched either philosophy and philosophers, or mathematics and mathematicians, as several issues, overlapping the sciences, being general or particular ones, raised questions of philosophical character. The *International Academy of the Philosophy of Sciences* realised in due time the importance of the topic. As early as in 1967, long before the last mentioned discoveries, it devoted its annual colloquium to the topic of *The symmetry: As a heuristic principle in the different sciences*. Indeed, the papers at that meeting represented a variety of the sciences, where symmetry did (and where it did not?) play a heuristic role. Our predecessors and teachers in the late sixties realised the wider heuristic role played by symmetry in distant disciplines, they demonstrated that it is not a narrow mathematical product, but is applied widely, a concept and phenomenon that may connect virtually separated fields. Symmetry led the attention of the Academy from the philosophical problems of mathematics to sciences, and now leads it back at a higher level of abstraction. (Only M. Bunge contributed to both events.) Soon after, in 1970, another meeting was organised in the course of the Interdisciplinary Seminars of Venice, the proceedings of which were edited by one of the organisers of this conference (E. Agazzi (ed.): *La Simmetria*, Bologna: Societa Editrice il Mulino, 1973, 452 pp.). The process started and accelerated. The eighties saw a boom in the number of symmetry-related interdisciplinary discoveries and also interdisciplinary symmetry meetings; one can notice the increase in symmetry-related publications in this decade. All these led, as it is well known for the readers of *Symmetry: Culture and Science,* to the formation of the *International Society for the Interdisciplinary Study of Symmetry (ISIS-Symmetry)* in 1989, which later founded the Symmetrion, and launched this periodical. Without overemphasising the role of symmetry, it remained one of the tools linking sciences and philosophy. This tool is partly heuristic, partly methodological, partly pure mathematical (applied by the representatives of the individual sciences). It worth mentioning what *philosophical questions* can be formulated, without the demand of completeness, in connection with the role of symmetry in *mathematics*. Let's see only a few non-traditional examples:

One can quote several examples when mathematical models are created to describe already existing objects. How to judge the opposite situation, the predictive role of mathematics: when the mathematical model precedes the object, e.g., an artificial chemical object (compound, structure)? The so-called Bucky-ball (created by the architect Buckminster Fuller) had been for a long time no more than a mathematical toy, or object of art. Suddenly it became a central 'actor' in modern chemistry, after the achievements of Krätschmer, as well as Kroto and others, having synthesised the fullerene C molecule. Similar situations can be observed in the history of all quasicrystal research. The problems of quasicrystals are wider than the realisation of artificial compounds: it puts the question of the *reality* of the fourth and higher dimensions models. What is the relation of the real, quasi-real, and artificial objects in this background?

The *Bauhaus*, and subsequently greater emphasis, the *Hochschule für Gestaltung in Ulm*, declared that philosophy *and* mathematics should be involved in art education. (Cf. the lectures of K-L. Wolf (mathematics-chemistry-crystallography-art-philosophy), the seminars of T. Maldonado, and as guest-lecturers M. Heidegger, N. Wiener, and M. Bense (philosophy of mathematics)). They completed this program and, disregarding the closing of their school long ago, philosophy and mathematics can no longer be erased from art curricula.

The role of computers in mathematical proofs is as frequently questioned as applied. In many respects it is left to the philosophers to decide to what extent and on what conditions computer based algorithmic (cf. repetition) proofs are acceptable as equivalent to the traditionally gained evidence. (Cf. the debates about the proof of the *four colour problem* by Appel and Haken using an algorithm of H. Heesch, a honorary member of ISIS-Symmetry.)

The study of symmetry in the development of the matter shows that the whole process, from the so-called big bang through the formation of particles and higher level material (physical, chemical, organic) structures, is a series of permanent losses of symmetries: the 'more developed' a material structure, the less symmetry it possesses. (Certain papers in this issue demonstrate also this statement.) What role is played by accidentals in the series of these spontaneous symmetry-breakings? How does a structure potentially exist in a 'lower level' material system?

So we reach successively to the level of the human brain, which owns a functional asymmetry. The consequences of the lateralisation of functions include the different ability of the two hemispheres for mathematical functions. There are (partially) separated the abilities for logical-intuitive, detail oriented-holistic or impressionist, discrete-continuous, digital-analogous, differential-integral, algebraic-geometrical, sequentional-simultaneous, temporal-spatial, element-set, ordinal-cardinal operations. Realising these asymmetric functions one can understand the different ways of thinking of different people, different affinities for different mathematical tools (cf. e.g., the two – analytical or wave mechanics and digital or matrix mechanics – formulations of quantum mechanics). One must not neglect the consequences in teaching mathematics starting from the basic notions of cardinal and ordinal numbers, synthetic or analytical reading, the use of music and visual demonstration in the education, etc., up to the understanding of different cultures (including ethnomathematics). These wide interdisciplinary concerns call for revising our concepts on the role, structure, and functions of mathematics. These metamathematical considerations lead out of the frameworks of mathematics.

One needs a new mathematics in the era of the use of computers and understanding the functioning of the human brain! One needs to face (and answer) the principal problems (questions) arising with this new maths! The new problems go far beyond the boundaries of classical mathematics: they are knocking on the door of philosophy.

Of course, a conference gives not enough space to discuss the whole spectrum of the possible topics, that may be inferred from the general title. The contributions concentrated on a few issues, and discussed them from several directions. This helped the discussion not to 'diffuse'. Let the reader forgive the reviewer for being inspired by the spirit of the place, and placing subjectively more emphasise on the role of symmetry. On the one hand, let's notice: the term symmetry was used in a very wide sense, not only as a mathematical notion, but as something bridging disciplines, cultures, sciences and arts, as general as philosophy is placed 'over' the sciences (many do not consider it as a science, but another quality); on the other hand, one will not encounter all the philosophical consequences raised by mathematics. We have filtered the possible issues, and on this occasion the filter was given. After any excursion into the 'lands' of sciences, one returns home repeatedly to the (methodological?) foundations: to mathematics and to philosophy. So do the investigators of symmetry as well. The meeting provided forum for these discussions.

Of course different filters may filter different dimensions of the topics. The same papers may be judged from different aspects: from the point of view of history, use of logical calculus, use of central concepts, e.g, structure, category, completeness, syntax, coding, *or*, symmetry, etc.; most of them may be found in most of the papers. The inexhaustibility of science provides us the opportunity to find new and new values through different filters in the really valuable works. These papers held values, indeed, thanks to the excellent team of authors.

The papers presented at the conference failed to cover all the possible philosophical questions put forward by modern mathematics. They concentrated on a few of them, what provided the advantage of surveying the topics from different points of view.

Some of the papers can be characterised as *self-reflections of mathematics*, or in this case, as *metamathematics* in the Aristotelian sense. They were partly applying the methods of logic, which leads us to the second large group of papers, having been discussed the *traditional* and *modern logical theories* in their relation both to mathematics and the foundations of philosophy. Some of them treated the relation of the *categorical foundations* and *structuralism*, others have a more *historical* character, while further papers concentrated on the problems of *coding*. Finally, there were papers dealing with the *applications and applicability of mathematics in the sciences* (e.g., in physics, biology), also with emphasised logical influence.

M. Bunge (Montréal) discusses *mathematical fictionism* in a wide cultural context. He considers mathematical fictionism as an alternative to the classical philosophies. Further he distinguishes mathematical and artistic fictions, as well as fiction and falsity. However all mathematical objects are fictions, and all mathematical concepts and propositions are just "as fictions as ... Escher's impossible buildings" he states; therefore mathematicians, like different artists deal in fictions. But, regarding the specifics of mathematics he concludes, that the ontological neutrality of mathematics explains why this discipline is the universal language of science, technology, and even philosophy, in other words, why it is portable from one intellectual field to the next.

A. Mercier (Bern) presents a cultural-philosophical-historical essay on the role and character of mathematics in respect to different cultures, the sciences, the arts, and ethics.

P. Martin-Löf's (Stockholm) paper discusses the relation between mathematics, Logic, and the theory of knowledge.

A. Cordero's (New York) contribution gives an analysis of ontological commitment and the mathematisation of scientific theories.

G. Darvas's paper deals with the principle of symmetry in mathematics and philosophy, discussing what role has been played by symmetry in the relationship of these two disciplines, as well as the open questions emerging on their boundaries in our age.

W. Sieg (Pittsburgh) in his metamathematical (mathematics' self-reflection) paper gives a revisionary description of Hilbert's program, and a sketch of a general reductive program. Based on Gödel's reflections on Hilbert's program, he discusses the mathematical experience from (a) a quasi-constructivist aspect and (b) a conceptional (axiomatically characterised abstract structures) aspect.

R. Thom (Paris) discusses the structure of contemporary mathematics, starting from ancient principles. He emphasises the importance of objects as against structures within mathematics. The central thesis of his article is that the classical schema of Aristotelian hylemorphism, the syntagme, *matter and form*, retains a certain validity in mathematics. But contrary to the Aristotelian dogma, "in mathematics matter is constructed through a compilation of form, in such a way, that ... every kind of matter is an 'informed' matter (materia signata)".

G. Granger (Paris), looking for "what is a profound result in mathematics", first treats the epistemological process of mathematical demonstration, then the process of mathematical revelation (of an unknown). He concludes that 'depth' in mathematics signifies the thickness of the virtualities which, once having been given a form, at once *created* and *discovered*, by the genius of the mathematician, appear as new realities. It is therefore an *ontology*, in a certain sense *tempered*, rather than a *grammar* which, with regard to the notion of depth, he has tried to sketch.

G. Heinzmann's (Nancy) mathematical self-reflection leads us by a historical analysis to the *block of logical papers*. He argues that Peirce's *theorematic reasoning* can be used to explain Poincaré's metaphorical proposition, and while for Husserl the criterion of deductive rigour was translatability in a logical language for *general* reasoning, Poincaré's purpose is to connect mathematical rigour with a *local* language relating the premises to the conclusions by means of a 'mathematical architecture'.

Several papers treat the problems of *categories and structures*. In this group, *S. Mac Lane* (Chicago) discusses the *Categorical foundations of the protean character of mathematics*. Having introduced the category of 'topos', which coordinated and unified "ideas from geometry with those from logic", he states that the notion of a topos can serve as *a* foundation of mathematics. Therefore, 'structure' will mean objects with structure together with their morphisms. The systematic study of such categories relates logical, algebraic, and geometric ideas from mathematics, and casts new light on the foundations. "Mathematics is protean, as in the case when the sheaves from geometry are also realised in set theory."

The same problem is discussed, from the aspect of syntactical considerations by *J.-P. Marquis* (Montréal). He asserts that the structuralist conception falls short of a complete and satisfactory account of mathematical objects. Mathematics comprises two irreducible dimensions: a 'quasi-concrete' or, as he qualifies it, 'combinatorial' dimension and a 'structural' or 'conceptional' dimension. The question is whether the syntactic dimension is *inherently* different from the semantics, whether one deals with a different category, in the Aristotelian sense of that expression. He claims that we now have indications that the two might not be that different from one another. Finally he concludes, that "we might be forced to articulate a 'foundation' of mathematics in which the notions of equality, isomorphism and equivalence would all play a crucial role. In general, it is hard to conceive of an *untyped* structuralist position, which is what most philosophers seem to be considering now."

Marquis is supporting the conceptions of *Ch. Parsons* (Harvard University), who, in his paper at this conference – backing structuralism – answers a specific objection to the structuralist view of mathematical objects of set theory. He says that reference to mathematical objects is always in the context of some structure and the objects involved have no more to them than can be 'expressed' in terms of the basic relations of the structure.

C. Miró Quesada (Lima) approaches the problematique of category theory and structuralism from a historical perspective. "The history of mathematics shows ... that logic, mathematics, and ontology were born tightly implicated." He discusses the relationship of category theory, and especially topos theory, to logic, as well as the relationship of logic to ontology. He also gives a critical analysis of 'noneism'.

The title of *T. Gergely*'s (Budapest) contribution is "An unic logic foundation of computer science".

Two papers deal with specific questions of historic character related to logical interpretations of concrete personalities. *A. Ishimoto* (Tokyo), defending 'conceptual realism', discusses the Frege-Russell logicism in the propositional fragment of *Lesniewski's ontology* by logical methods.

K. G. Havas (Budapest) analyses some philosophical thoughts of Hungarian mathematicians in a historical perspective, from the point of view of non-classical logic, with special regard to the competing theories and to the role of experience.

There are a few papers, in the line of the logical ones, dealing with the problems of coding. *P. Weingartner* (Salzburg) focuses on the "Language and coding dependency of results in logic and mathematics". He starts from the statement that the language of the respective scientific discipline is usually developed with the development of that discipline and is flexible enough to allow further revision on the basis of new knowledge. However there is a sense (or several senses) of language and coding-dependency which pertains to very specific problems and results in Logic and Mathematics. He justifies his position by several examples.

G. H. Müller (Heidelberg) calls the attention to two mathematical subjects which emerged in the recent century, namely *infinity* (Cantor) and a *theory of coding* (Gödel). " The relation of a code to what it is supposed to code – and many related questions – became ... treatable inside mathematics. The phenomena, results, and observations may and will lead to a better and deeper understanding of the role and the merits of weaknesses of coding in the general sense of epistemology." It is well known that infinity is an old subject of philosophy. Therefore, for him the classical distinction between potential and actual infinity was and is the starting point of probably all the more elaborated theories in this area. He develops a guideline for an epistemological motivation for set theory. Finally, he comes to the problem of understanding a higher reflection principle applied to intensionally given classes as it is needed to introduce very large cardinal numbers: " ... all of mathematics can be logically based on extensionalisation and reflection (or objectivisation)." The main purpose of his paper was to build up set theory (as far as possible) from our *conceptual ideas* and *methodological strategies*. Philosophically he concludes that Mathematical Logic has changed our outlook on the conceptual means of man.

J. Mosterin (Barcelona) paints a wide picture with examples (*The natural numbers as universal library*) on coding information in digitalised form by natural numberings from Gödel numberings, through library string codes and DNA codes, to digitalised musical sound (CD) and TV pixel codes (texts, chromosomes, pieces of music, and pictures).

D. Mundici's (Milan) paper is a bridge from the papers on *logical studies* to the *application of mathematical methods in* physics (in general: *sciences*). He uses Gödel incompleteness applied to physical descriptions, namely in the algebraic treatment of quantum physical systems (C^* algebras). He puts the question, "... why should nature allow noncommutative Gödel incomplete physical systems and their ideals ...?" and gives the answer that virtually all natural *AF* C^* algebras existing in the mathematical-physical literature seem to support the claim that there are no exact quotient structures in physics; further, identities between dimensions in these algebras are easy to compute. These algebras can often be coded by infinite-valued theories whose decision problems are computable in 'polinomial time'. Finally he states: "... nature doesn't have ideals as follows: nature can do without ideals only by repudiating Gödel incompleteness".

G. M. Prosperi (Milan) discusses how we chose the appropriate mathematical language to describe physical theories. He states that subjective experience is an important part of our understanding, and this is a key to the epistemological statute of physics, its power, and limits.

E. Scheibe (Heidelberg), treating "The mathematical overdetermination of physics", touches upon two problems. Namely: (a) the problem of which frame of logical systematisation we should use for a reconstruction of physical theories, and (b) the problem of elimination or, conversely, introduction of a piece of mathematics on the basis of one particular frame of systematisation, namely set theory. According to him, the main question that poses itself in view of a multitude of systematisation frames for physical theories is the question of their pairwise equivalence with respect to physical content. A deeper investigation of this question should look for such equivalences *within* one of those frames. He gives some examples of such equivalences for set theory.

J. and K. Ricard (Paris) deal with the problems of mathematical models used for the description of biological processes, which are like a Picasso painting (quoting L. Segel: "A painting of a goat does not look like a goat, but it embodies the main features of this animal and in a sense looks more 'goat' than a real goat"), not reality but an Idea, in the Platonist sense of the word, of that reality. They raise the following points related to the nature and the use of mathematical models of biological phenomena: (a) the anteriority of a model relative to the experimental data, (b) the reductionism, or the organicism, which is embodied in a model, (c) simplicity, aesthetics, and models, and (d) the invariance of models.

György Darvas