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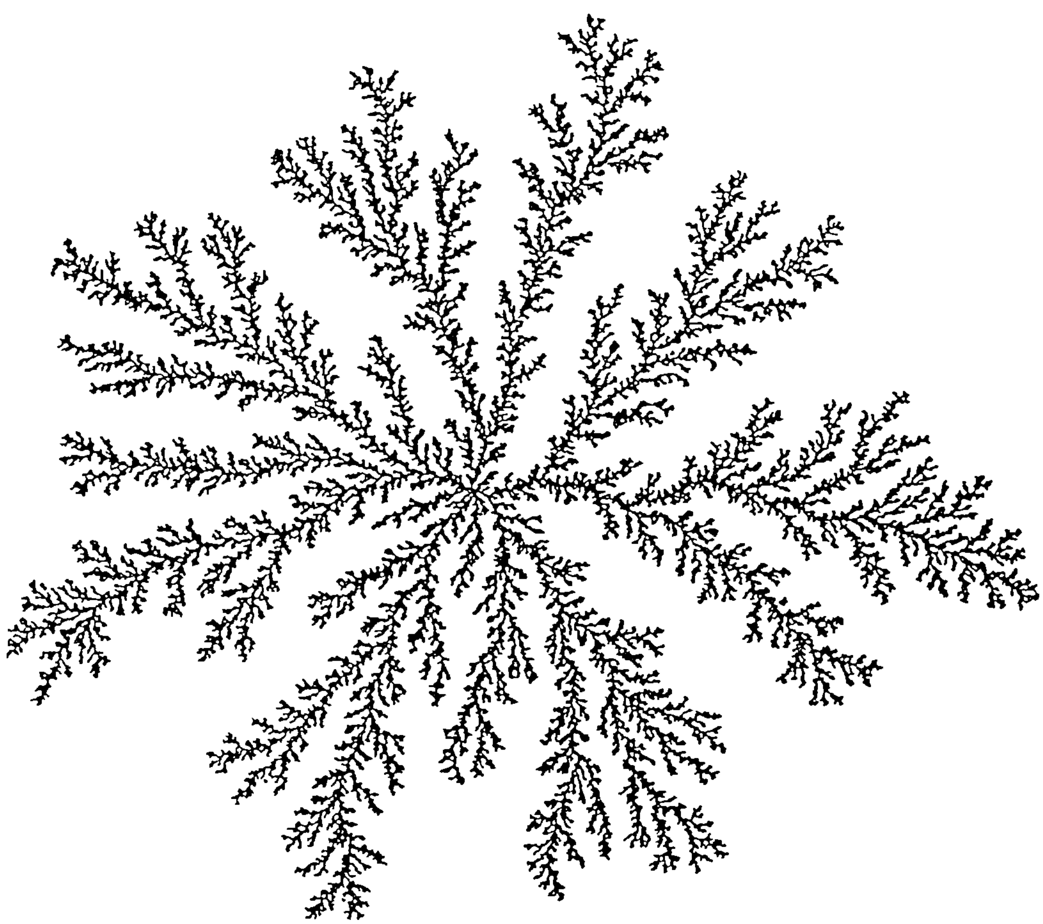

DLA fractal cluster
of 10^6 particles

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TOPOLOGY OF THE UNIVERSE

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Abstract: *The unexpected possibility of a multiply connected character of the space of the Universe is demonstrated for the nonspecialists by using simple examples. Indications for the realization of this possibility in the actual Universe are briefly discussed and its cosmological implications mentioned.*

FROM TOPOLOGY TO CRYSTAL STRUCTURE

When speaking about the 'topology of the Universe' one may be inclined to think of the connectivity properties of material structures within the Universe, which – according to our present knowledge – may be 'filamentary', 'bubbly', 'frothy', 'sponge like' etc. (Melott, 1990). There is, however, a possibility for a deeper kind of a 'topology of the Universe', when the space itself (even a possible emptiness alone) does have its own nontrivial connectivity properties. E.g., going always straight ahead in the space one may find himself at his starting point after having covered some characteristic distance depending on the direction of the journey (cf. Ellis, 1971; Wolf, 1967). Recent astronomical observations – and also quantum-cosmological considerations – have made this possibility worth of special attention (Paál, 1971; Paál et al., 1991; Fang and Sato, 1984; Fang and Mo, 1987).

In order to make the somewhat mystic or incredible spatial recurrences understandable for the specialists of other fields of sciences it is customary to consider first the case of two-dimensional spaces (surfaces) inhabited by flat beings ('observers' or 'travellers') and start with the simple example of an infinite cylinder. Here there is clearly only one single direction in which the flat traveller may return to his original place, if the route chosen by him is the straightest possible one while crawling on the surface (without superfluous turns aside) – but such a return is impossible in all other directions. In the usual Euclidean plane there is no possibility for a linear return at all. The transition from one of these cases to the other seems fairly easy. One may always roll up a flat (uncurved) plane into a cylinder so making a 'multiply connected' surface out of a 'simply connected' one. – Obviously in the simply connected plane there is only one straight line connecting any *two* given points, while in the multiply connected cylinder there may be infinitely many of them.

This simple example is relatively easy to reformulate in such a mathematical form as to permit generalization to any dimension and space curvature. Imagine first a picture just painted by fresh paint on the surface of a cylinder, which is then sent rolling quickly on a plane sheet of paper before the lapse of time needed for the paint to dry. Clearly repeated images of the picture will appear painted on the paper in excactly periodical distances. One may say in a somewhat more abstract language that an infinite set of equidistant pictures on a plane precisely 'represents' a single picture on a cylinder, or that by identifying all the points corresponding to each other according to a 'group of parallel translations' in a plane (n times a given length d) an equivalent representation of a cylinder is obtained from a two-dimensional Euclidean space (plane). The plane is called the simply connected 'covering space' and the translations generate a 'compactified' multiply connected space, the cylinder. The short range 'local' properties of these spaces coincide, but the long range 'global' ones (e.g., long range returns) differ. The plane and the cylinder are said to represent topologically different 'space forms' of the flat (Euclidean) space.

In a more general language one may say that any 'space form' of constant curvature can be derived from a simply connected spherical, flat or hyperbolic universal covering space (of positive, zero or negative curvature) by introducing into the latter a discontinuous group of fixed point free isometric transformations and identifying its points which correspond to each other under the transformations of the group. This identification leads to the multiply connected 'quotient space'. A set of infinitely many points of the original space (which are connected by the transformation group) plays the role of a 'single point' in the new space.

According to this scheme there are five topologically different space forms of the two-dimensional Euclidean space. The first is the trivial *plane* with no compactification (recurrences, identifications), the second is the (infinitely long) *cylinder* obtained by a single sequence of translations, i.e., identifications by equidistant steps along a single direction, the third is the (finite) torus obtained by two algebraically independent sequences of translations, i.e., identifications by steps along two different directions, the forth is the (infintely long) *Möbius band* obtained by one sequence of translations combined with a reflection, while the fifth is the *Klein bottle* obtained by two sequences of translations, one of them combined with a reflection. It is easy to see that if one tried to use two translations combined with two reflections, then a fixed point would emerge whose local properties would differ from those of other points of the space. So a two-dimensional Euclidean space may have 3 space forms with infinite extent and 2 space forms without infinite extent! – Note that this finiteness is essentially different from the much better known finiteness of the surface of the simple sphere (i.e., two-dimensional space of positive curvature). The latter is isotropic, i.e., the distance of returns is the same in all directions, while the former is necessarily anisotropic.

In the same spirit but somewhat more generally one can construct a three-dimensional 'cylinder' by introducing one sequence of translations in the three-dimensional Euclidean space. In this case not infintely long two-dimensional strips, but infintely long and wide three-dimensional layers would 'repeate themselves' (would be identified) in the original three-dimensional covering space. Without enumerating all the 18 possibilities for the three-dimensional Euclidean space forms we only mention here the *three-dimensional torus* (designated by T^3) generated by three independent sequences of translations (without reflections or rotations) which is the simplest case when the three-dimensional *Euclidean space* can be fully 'compact' (finite, closed) and *can have finite total volume.*

For visualization a compactified space may also be thought of as a system of conguent cells in a crystal lattice with some definite rule of alignment of the cells. In general the cells may have different forms and the alignment may happen according to different combined translations, rotations and reflections as well. Only specific transformations lead to meaningful compactifications. This also applies to spaces of nonzero curvature, although the meaning of 'translation' or 'rotation' is somewhat more obscure in these cases. For details see Ellis (1971), Wolf (1967).

Obviously the observable properties of a multiply connected universe are identical with those of the corresponding 'universal covering model universe' populated by equal configurations in strictly congruent cells and all the known formulae of observational cosmology hold in this crystal-like covering. These models naturally show some kinds of periodicities according to spatial distance. This is how they can be discovered astronomically. The question is then, whether there are indeed such discoveries.

FROM CRYSTAL STRUCTURE TO WORLD MODEL

Hints to actual periodicities in the space distribution of astronomical objects first appered 20 years ago in connection with *quasars* (Paál, 1970; Paál, 1971; Fang and Sato, 1984; Fang and Mo, 1987 and references therein), but have failed to pass the proper statistical significance tests. Really significant periodicity has first been found in the space distribution of *galaxies* by Broadhurst et al. (1990). It turned out that the typical distances between galaxy pairs are distributed according to a non-random periodic pattern whose regularity depends on the world model, more particularly on the forces acting on the expansion of the Universe (Paál et al. 1992).

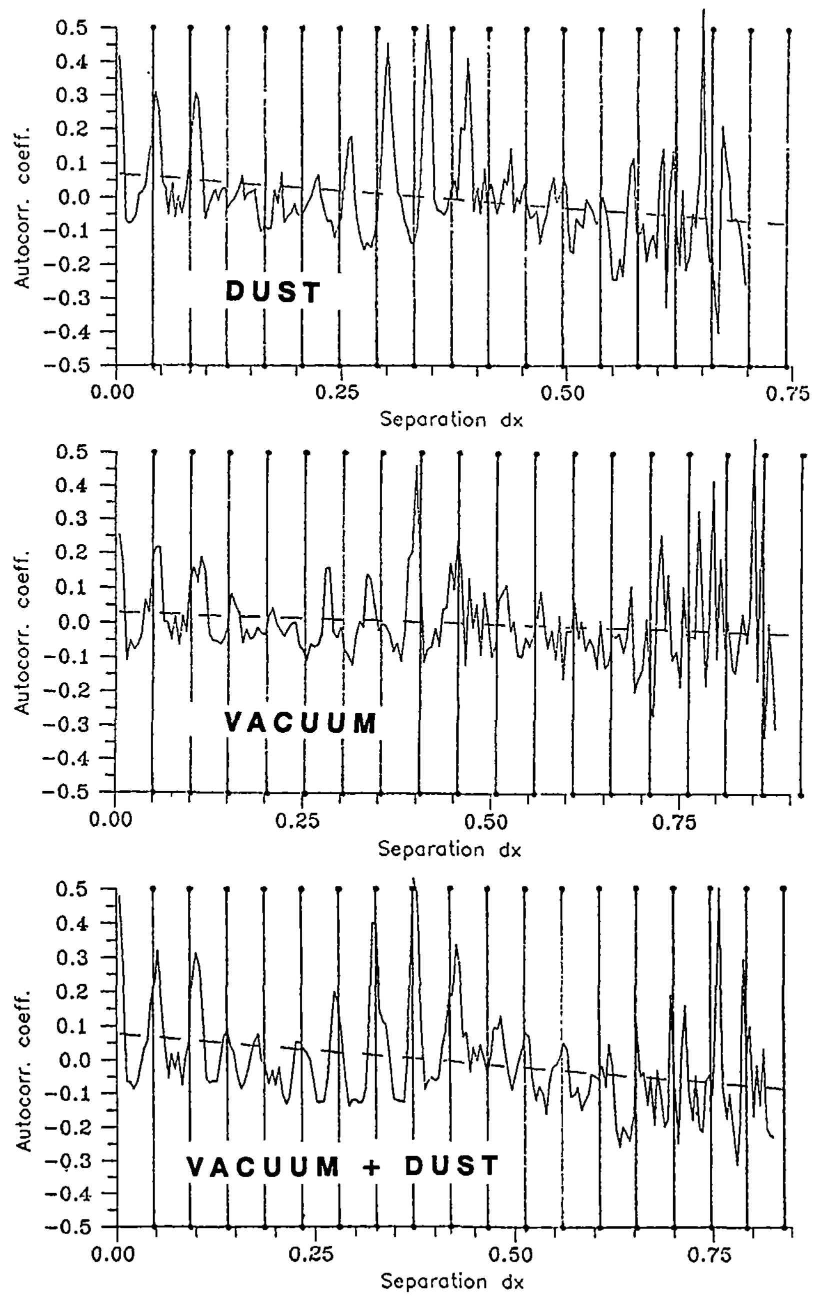
The so called autocorrelation coefficient of galaxies indicates the relative excess of galaxy pairs with a given spatial separation. This spatial distance of galaxy pairs is, however, not a directly observable quantity, it can only be calculated from the measured redshifts of spectral lines in the light of the galaxies. This calculation in turn yields different results depending on whether the universe is filled simply by the usual cosmic matter ('dust'), which – according to Einstein's General Theory of Relativity – is gravitationally attracting, or it is filled mostly by the modern quantumtheoretical 'vacuum' (a background sea of energy without particles of identifiable form), which is gravitationally repulsive (Paál and Lukács, 1990). In either extreme case of dust or vacuum dominance the calculated periodicity of the space distribution of galaxies turns out to be relatively poor, see Figure 1 and Figure 2 (although some 15 partially irregular periods may be distinguished even in these cases). However for a vacuum/dust ratio equal to 2/1 one finds 17 fairly regular periods (Fig. 3). The superiority of the latter model becomes even more prominent, if one calculates the periods also by an independent method (i.e., by minimizing the squared deviations of the places of maxima of the galaxy distribution function from an equidistant set of distances) and indicates the multiples of this period on the diagram of the autocorrelation function (equidistant vertical lines on the Figures). The periods defined in these two different ways coincide only for the model of Figure 3, while in Figures 1 and 2 the two kinds of periods differ so strongly that the phase differences accumulate to a full period. Consequently this tuning of periods singles out the best cosmological model, unless one is willing to accept that prominent periodicities appear just by mere chance without any deeper physical reason.

Thus if one looks for a *periodicity*, which may be a signal for the *topologically non-trivial character of the space* of the Universe, then one finds a particular *world model* filled with twice as much vacuum as dust. This model also has further important advantages for cosmology.

Astronomical observations seem to show that the attracting dust gives about 1/3 part of the critical density needed to make the space of the Universe almost flat (uncurved, Euclidean). Now we may have found the remaining 2/3 part in the form of vacuum and so the old 'flatness problem' may have disappeared. In other words a kind of 'missing mass' seems to have been found. – On the other hand in these moderately vacuum dominated models the total expansion time measured from the Big Bang to the present epoch is essentially longer then in the dust models and so it is much easier to find time in them for the oldest stellar systems, i.e., a 'missing time problem' is also easier to solve in this case.

FROM WORLD MODEL TO "WORLD'S END"

However the new model may have terrifying consequences as well. The obtained density of vacuum 10000 times surpasses that of the thermal cosmic background radiation. This implies that – according to the Stefan-Boltzmann law – our vacuum is already 10 times supercooled and so may explode in any moment. When in the very early Universe a supercooled vacuum was destroyed by a phase transition, then some bosons mediating interactions acquired rest energy and mass (out of the released vacuum energy). If the present supercooled vacuum gave mass to the remaining massless bosons, then those bosons could be e.g., the photons mediating the electromagnetic interaction, implying that the range of electric force would fall from infinity to about a millimeter or so (Paál, Horváth, and Lukács, 1992). If gluons got masses, then the stability of nuclei and the nuclear fusion processes might change (e.g., in stars).


DUST
VACUUM
VACUUM + DUST
Autocorr. coeff.
Separation dx
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
0.00
0.25
0.50
0.75

Figures 1-3