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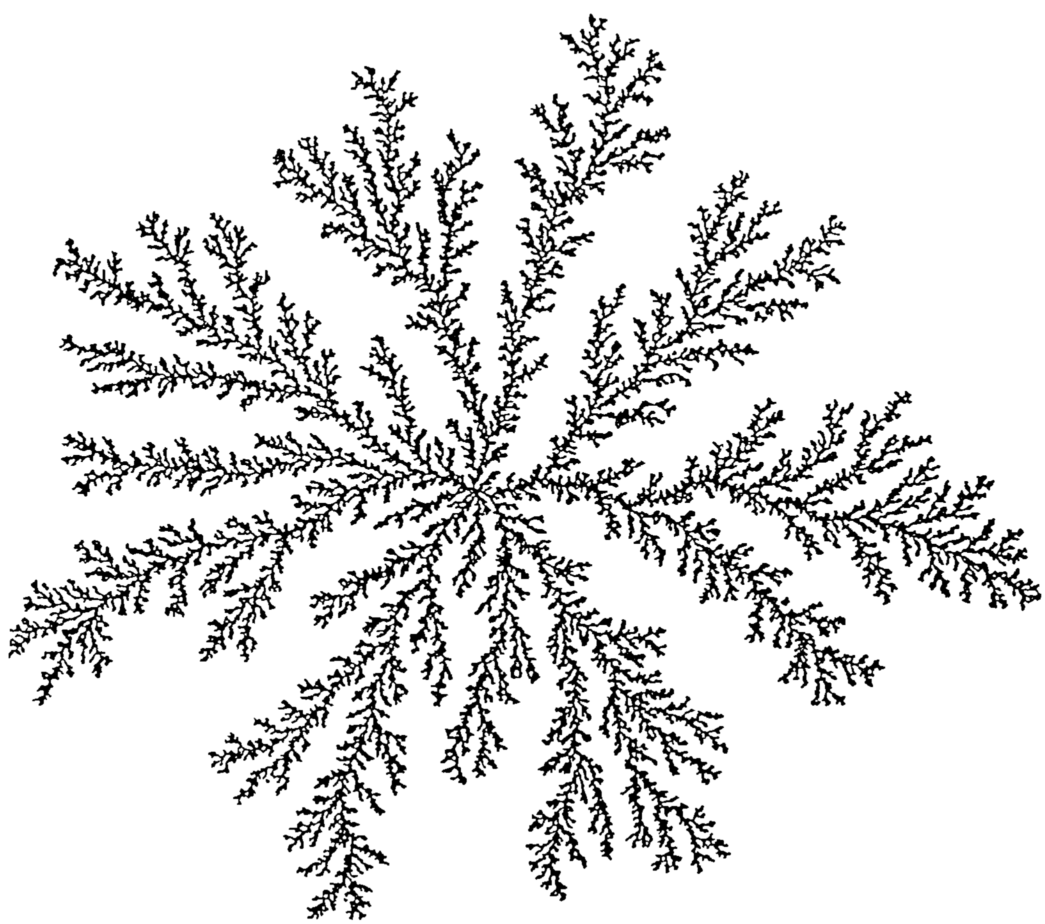

DLA fractal cluster
of 10^6 particles

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ON GEOMETRIC SYMMETRIES AND TOPOLOGIES OF FORMS

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ABSTRACT: *A brief bird's eye overview is given about possible symmetries and topologic structures of bodies in the 3 dimensional Euclidean space. This is done for basis of understanding some further papers of this issue.*

1. INTRODUCTION

The notion of *symmetry* is widely used for characterizing patterns of biological objects. However, it is often used purely qualitatively. It is not so in mineralogy and crystallography, but their symmetries are almost exclusively *discrete*. Crystallographic analogons are very useful in many cases (e.g., for the radial symmetry of *Ctenophora*, radial pseudosymmetry of *Echinodermata*, left-right symmetry of higher evolved animals, etc.). Still, *continuous* symmetries are also possible and they need another approach.

There are more complicated cases as well. Is an *Ammonita* symmetric? Its chambers grow along a spiral line in a regular manner. There is no left-right symmetry, neither radial symmetry, nor rotational symmetry. Still a regularity is clearly seen. Is it a symmetry, and if it is, in what sense?

The question cannot be fully answered. Any well-defined transformation may be called a symmetry. However, one knows what is a *geometrical* symmetry. The present paper applies the formalism to situations relevant for biological objects.

Section 2 is a brief recapitulation of the Killing vector technique of Riemann spaces. Section 3 applies the equations in the 3 dimensional Euclidean space. First all the Killing vectors (including conformal ones) of this space are calculated and then, now excluding the conformal ones, all possible cases are listed when the actual symmetry of the matter distribution is smaller. Section 4 contains some remarks for discrete symmetries.

Section 5 illustrates the possible subgroups of continuous symmetries on examples, while Section 6 mentions some principles of topologic classifications of objects.

Some explanations and commenting remarks, which might also properly be footnotes as well, are included into the text, but for showing the specific rôle in CAPITALS.

2. ON KILLING SYMMETRIES

This Chapter is a brief recapitulation of the theory of continuous symmetry transformations for pedestrians. For more details see (Eisenhardt, 1933) and citations therein.

Consider a Riemann (pseudo-Riemann) space of n dimensions. There is a coordinate system $\{x^i\}$ on the manifold. Because of the Riemannian structure *any* regular recoordination is permitted (Eisenhardt, 1950), but this possibility will be mainly ignored in this paper.

A domain of the space is filled with matter. This matter is characterized by a set of fields $\{\Phi_{\alpha}(x^i)\}$. This set contains the relevant data of the matter, say Φ_1 may be the density, Φ_2 the colour, Φ_3 classifies the tissue at x^i as liver, muscle, kidney, etc. The domain is generally finite, but we restrict ourselves to the interior and neglect the boundaries. For the present we ignore the discontinuous nature of the matter in biological objects. Then the cellular structure is averaged away. This is satisfactory if the number of cells is $>> 1$.

Now identify a number of points in the matter; 3 will be sufficient, in an infinitesimal neighbourhood

$$x_I{}^i = x^i + \mathrm{d}x_I{}^i$$

In a Riemann space the distances of infinitesimally close points can be written as

$$\mathrm{d}s_{IK}{}^2 = g_{rs}(x^i)\mathrm{d}x_{IK}{}^r\mathrm{d}x_{IK}{}^s; \qquad \mathrm{d}x_{IK} = \mathrm{d}x_I - \mathrm{d}x_K \tag{2.1}$$

(NOTE THE EINSTEIN CONVENTION (Eisenhardt, 1950): THERE IS AUTOMATIC SUMMATION FOR INDICES OCCURRING TWICE IF BOTH ABOVE AND BELOW.) In a Riemann space g_{ik} is positive definite; in a pseudo-Riemann one this does not hold. Then in a pseudo-Riemann space (as the spacetime around us) 0 distance is possible between points which are not neighbours. Then in such a space the metric generates a so called light-cone structure: the lines of 0 distance build up a kind of connection. Here we will not have pseudo-Riemann spaces, but still the problem of neighbours is to be discussed, since the metric tensor does not show the *topology*. (E.g., the same metric tensor is valid for a plane, a mantle of a cylinder and that of a cone.) This problem is relegated to Section 6, and until that we regard the topology as known.

So we have our selected infinitesimal triangle around x^i. Let us try with a *transformation*. Introduce a vector field $K^i(x)$, and shift all the points along this vector field in the manner

$$x^{i'} = x^i + \epsilon K^i(x) \tag{2.2}$$

with the *same* ϵ for all the points. Then, depending on the vector field, there are three possibilities:

1) The new triangle is geometrically *identical* with the original one, i.e., the three angles have remained unchanged, and so for the lengths of the sides. Then we call this shift a *Killing symmetry*.

2) The new triangle is *similar*: the angles are unchanged and the lengths proportional. Then the shift is a *conformal Killing symmetry*.

3) Neither is true. Then the shift is not a symmetry. (To be sure, weaker symmetries can be defined and sometimes are used (Katzin *et al.*, 1969). However somewhere one must draw a line, and we stop at conformal Killing symmetry.) In addition, for a symmetry we require that at the new points the matter remain the same (symmetry) or at least similar (conformal symmetry), i.e., that Φ_α show some scaling.

Now we are going to discuss the geometry in details: then some transformations are symmetries from geometric viewpoint, and later the consumer can check if Φ_α shows scaling or not.

We start with infinitesimal shifts, and this will be enough, because one can repeat the transformation in any times. So now $|\epsilon|$ is infinitesimal, and we can calculate up to first order in it. Note that

$$dx^{i'} = dx^i + \epsilon K^i{}_{,r}dx^r \tag{2.3}$$

where the comma stands for partial derivative. (The partial derivative generally destroys the definite tensorial structure in Riemann spaces, so formulae of relevance must be reformulable in terms of covariant derivatives, denoted by semicolon. For the definition of the latter derivative see the standard literature, e.g., (Eisenhardt, 1950). All our final formulae could be written into such a form.) Our previous conditions for a symmetry can be written as

$$ds'^2 \equiv g_{rs}(x')dx^{r'}dx^{s'} = e^{\epsilon\Omega}ds^2 \tag{2.4}$$

where $\Omega=0$ if the symmetry is not conformal.

Now, eqs. (2.2-4), in first order, lead to

$$g_{ir}K^r{}_{,k}+g_{kr}K^r{}_{,i}+g_{ik,r}K^r = \Omega g_{ik} \tag{2.5}$$

By means of covariant derivatives this equation can be rewritten as

$$K_{i;k} + K_{k;i} = \Omega g_{ik} \tag{2.5'}$$

which is the *Killing equation* (Eisenhardt, 1933); conformal if Ω differs from 0. In terms of Lie derivatives (Eisenhardt, 1933) the equation gets the even more compact form

$$L_K g_{ik} = \Omega g_{ik} \tag{2.5''}$$

expressing the fact that the change of the metric tensor is proportional to itself along the vector field **K** (0 if the symmetry is not conformal). This is just the reason for the similarity between the triangles.

If the metric of the space is given (which we assume henceforth), then eq. (2.5) is a system of linear partial differential equations, which can be solved somehow. Combinations with constant coefficients are again solutions, so the general solution is

$$K^i(x) = q^\tau K_\tau{}^i(x) \tag{2.6}$$

where the q's are constant; provided that we have got all the independent solutions $K_\tau{}^i(x)$. For conformal symmetries the corresponding Ω's add up in a similar manner.

For more than one Killing vectors the corresponding transformations form a group, since a sequence of symmetry transformations is a symmetry transformation by its result. Then one can calculate the commutator of two infinitesimal transformations by using eq. (2.3); the result reads as

$$[K_\alpha, K_\beta]^i = K_\alpha{}^r K_\beta{}^i{}_{,r} - K_\beta{}^r K_\alpha{}^i{}_{,r} \tag{2.7}$$

(WHERE ALL THE COMMAS COULD BE SUBSTITUTED BY SEMICOLONS AS WELL BECAUSE THE DIFFERENCE CANCELS DUE TO THE ANTISYMMETRIC COMBINATION). Since the commutator again would generate a symmetry transformation,

$$[K_\alpha, K_\beta]^i = c_{\alpha\beta}{}^\tau K_\tau \tag{2.8}$$

where the c's are the *structure constants* of the group.

3. CONTINUOUS SYMMETRY TRANSFORMATIONS IN THE 3 DIMENSIONAL EUCLIDEAN SPACE

Biological objects live around us in a 3 dimensional Euclidean space. There

$$g_{ik} = \delta_{ik} \tag{3.1}$$

This is a zero curvature space. For constant curvature the number of independent Killing vectors is $\frac{1}{2}n(n+1)$; including conformal ones $\frac{1}{2}(n+1)(n+2)$ (Eisenhardt, 1933). So in our case the number of possible independent symmetries is 10, of which 6 keep distances, while 4 are only similarities. The complete list is as follows:

$$P = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \begin{pmatrix}1\\0\\1\end{pmatrix}; \qquad \Omega_P = 0$$

$$J = \begin{pmatrix} 0 \\ +z \\ -y \end{pmatrix} \quad \begin{pmatrix} -z \\ 0 \\ +x \end{pmatrix} \quad \begin{pmatrix} +y \\ -x \\ 0 \end{pmatrix}; \quad \Omega_J = 0$$

$$D = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \qquad \Omega_D = 2 \tag{3.2}$$

$$Q = \begin{pmatrix} x^2-\frac{1}{2}r^2 \\ xy \\ xz \end{pmatrix} \quad \begin{pmatrix} yx \\ y^2-\frac{1}{2}r^2 \\ yz \end{pmatrix} \quad \begin{pmatrix} zx \\ zy \\ z^2-\frac{1}{2}r^2 \end{pmatrix}; \qquad \Omega_Q = x$$

with

$$r^2 \equiv x^2 + y^2 + z^2 \tag{3.3}$$

where boldfaces stand for 3-component entities colloquially called vectors of axial vectors. Generally **P** is called *translation*, **J** *rotation*, D *dilatation* and sometimes **Q** *inversion*, being in some connection with the inversions to the unit circle. The most general vector field for a symmetry transformation is built up from the above ones with 10 constant coefficients.

By evaluating the commutators (2.7) one gets

$$\begin{aligned}
&[P_\alpha, P_\beta] = 0 \\
&[P_\alpha, J_\beta] = \epsilon_{\alpha\beta\tau}\delta^{\tau\sigma}P_\sigma \\
&[P_\alpha, D] = P_\alpha \\
&[P_\alpha, Q_\beta] = \delta_{\alpha\beta}D + \epsilon_{\alpha\beta\tau}\,\delta^{\tau\sigma}J_\sigma \\
&[J_\alpha, J_\beta] = \epsilon_{\alpha\beta\tau}\delta^{\tau\sigma}J_\sigma \\
&[J_\alpha, D] = 0 \\
&[J_\alpha, Q_\beta] = \epsilon_{\alpha\beta\tau}\delta^{\tau\sigma}Q_\sigma \\
&[D, Q_\alpha] = Q_\alpha \\
&[Q_\alpha, Q_\beta] = 0
\end{aligned} \tag{3.4}$$

This is the *conformal group of 3-space*. Ignoring the conformal symmetries we get the symmetry group of the Euclidean space in stricter sense, $\{P, J\}$, which is the $E(3)$ group of 3 translations and 3 rotations.

Now we are ready with the widest possible group of continuous symmetry transformations for biological objects. Then comes the second condition that in the new points the matter be the same (or similar) as in the original ones. So a continuous conformal symmetry transformation will have the form (2.2), where K^i is as in (2.6), with the particular K's from (3.2), so that the material characteristics (density, colour, composition, etc.) show a scaling as well; a continuous strict symmetry will contain only $\boldsymbol{P}$ and $\boldsymbol{J}$ from (3.2) and the material characteristics will be strictly unchanged. (It is possible to be more general. One may define transformation for the material data as well, say changes in colour, and then a combined shift and colour change may be a symmetry. This construction is known in some branches of physics, as e.g., the supersymmetry of particle physics (Gol'fand and Lihtman, 1974) but this will not be done here.) In what follows we assume that there is no problem to decide if the new point is materially identical with (or similar to) the original one.

Then by observation one can select the actually existing subgroup. The theoretical possibilities are limited because *generally* the commutator relations (2.7) cease to be closed if we omit symmetries from a group. To the end of this Section we ignore the *conformal* symmetries.

Now I try to list all the possibilities. (Of course the possible subgroups of E(3) are listed in the standard literature (Petrov, 1966). However, that is not the complete answer for the present question, as will be seen.) I hope that no case is forgotten, but if the Gentle Reader were to discover a missing one then he would be honoured. To begin with, observe that we have 3 independent 'directions' (say, $i = x, y, z$). Now, select, e.g., x. We have P_x and J_x in this direction. There are then 5 possibilities:

1) Both P_x and J_x are symmetries.

2) Only P_x is a symmetry.

3) Only J_x is a symmetry.

4) Neither P_x nor J_x is a symmetry, but a special combination of theirs $H_x = aP_x + bJ_x$ is.

5) No combination of P_x and J_x is a symmetry.

Here H_x is a helical symmetry, a combined rotation + translation along the rotational axis. Now let us start from the maximal symmetry downwards. Observe that the numbering 1, 2, 3 of the components is arbitrary.

6 symmetries:

$(\boldsymbol{P},\boldsymbol{J})$. The matter is homogeneous and isotropic; no preferred point or direction.

5 symmetries:

ϕ. This class is empty, according to the Fubini lemma (Petrov, 1966).

4 symmetries:

$(\boldsymbol{P}, J_1)$. Homogeneity, with a preferred direction, which is an axis of rotation.

3 symmetries:

a) (P_1, P_2, P_3). Homogeneity without isotropy.

b) (P_1, P_2, J_3). Planar symmetry, with a rotational symmetry around an axis orthogonal to the planes.

c) (P_1, P_2, H_3). Planar symmetry, with helically rotated but otherwise identical planes.

d) (J_1, J_2, J_3). Spherical symmetry (rotations around 3 orthogonal axes).

e) (H_1, H_2, J_3). Rotational symmetry around the z axis with helical symmetries along any axis orthogonally intersecting the z one.

2 symmetries:

a) (P_1, P_2). Sequence of planes with homogeneity within each plane.

b) (P_1, J_1). Cylindrical symmetry.

1 symmetry:

a) (P_1). Translational symmetry in one direction.

b) (J_1). Rotational symmetry around one axis.

c) (H_1). Helical symmetry along one axis.

These are 12 different possibilities for non-conformal continuous symmetries of living organisms; in the cases when H's appear, there is a scale constant connecting translation and rotation. Including the 4 conformal symmetries the construction would go likewise, but this will not be done here.

4. ON THE DISCRETE SYMMETRIES

As mentioned earlier, the discrete symmetries in 3 dimensions are extensively listed in crystallographic literature. (See e.g., (Kittel, 1961) and further citations therein.) So we do not go into details, only classify the possible discrete symmetries into two classes. Namely

1) In addition to the $E(3)$ group the Euclidean space possesses 3 independent discrete symmetries, which can be chosen as reflexions to 3 orthogonal planes.

2) Any possible continuous symmetry, which is actually absent, can still appear at discrete steps. E.g., it is possible that P_1 is not a symmetry with arbitrary transformation parameter ϵ, but still it is a symmetry in steps $N\Delta$.

Again, a lot of combinations may exist. E.g. rotations and reflections to planes may be combined to reflections to sequences of rotated planes. Now, the discrete version of this is the familiar radial symmetry.

5. EXAMPLES

Up to now we restricted ourselves to the Cartesian coordinates x, y, z. Now, for explicit examples, it is useful to use other special coordinates, but we do not want to go into the details of coordinate transformations (for which see e.g., (Eisenhardt, 1950)). Only let us note that by performing the coordinate transformation $x^{i'} = x^{i'}(x^k)$ a vector is transformed as

$$K^{i'} = K^r(\delta x^{i'}/\delta x^r) \tag{5.1}$$

Two special coordinate systems will be mentioned. The first is the spherical polar system (r,θ,Φ):

$$\begin{aligned} x &= r\sin\theta\cos\Phi \\ y &= r\sin\theta\sin\Phi \\ z &= r\cos\theta \end{aligned} \tag{5.2}$$

with the corresponding inverses. The second is the cylindric system (ρ,φ,ζ):

$$\begin{aligned} x &= \rho\cos\varphi \\ y &= \rho\sin\varphi \\ z &= \zeta \end{aligned} \tag{5.3}$$

Now let us see some symmetries. First we return to the list of possible continuous strict symmetries.

3d, spherical symmetry.

By transforming all the 3 J's in (3.2) into spherical polar coordinates (not given here) one obtains that the transformations act on r = const. surfaces. Any point on such a surface can be rotated into any other. Therefore the symmetry exists if

$$\Phi_\alpha = \Phi_\alpha(r)$$

By other words, any material date can change only with the radial distance from a center.

3b, planar symmetry with rotation.

It is the most convenient now to use cylindrical coordinates. The rotation axis is taken in the φ direction. In the planes orthogonal to this axis there act two translations and one rotation, which is the $E(2)$ group. Such a plane is then maximally symmetric. Therefore nothing depends on the coordinates ρ, φ. Consequently the symmetry exists if

$$\Phi_\alpha = \Phi_\alpha(\zeta)$$

2b, cylindrical symmetry.

The symmetry direction is taken as z. Then, by transforming P_z and J_z one gets

$$P_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \qquad J_z = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Consequently there remains

$$\Phi_\alpha = \Phi_\alpha(\rho)$$

1c, helical symmetry.

As mentioned, $H_1 = P_1 + qJ_1$. The symmetry direction is taken as z. Then in cylindric coordinates

$$H_1 = \begin{pmatrix} 0 \\ q \\ 1 \end{pmatrix}$$

The condition that a scalar obey the symmetry is, for the analogy of eq. (2.5")

$$L_K \Phi_\alpha = 0$$

which reads as

$$\Phi_{\alpha,r} K^r = 0$$

(Eisenhardt, 1950). Substituting the actual K^i and solving the partial differential equation, the result is

$$\Phi_\alpha = \Phi_\alpha(\rho, \varphi \quad - q\zeta)$$

Finally, consider a case when conformal Killing vector is also included. Let the *only* symmetry be

$$J_1 + qD$$

We take the symmetry direction z. Then in cylindric coordinates

$$(J_1 + D)^i = \begin{pmatrix} q\rho \\ 1 \\ q\zeta \end{pmatrix}$$

Again, requiring this as a *strict* symmetry on the material fields, from the vanishing of the Lie derivative one obtains

$$\Phi_\alpha = \Phi_\alpha(\rho e^{-q\varphi}, \ \zeta e^{-q\varphi})$$

By other words, in one 360° rotation the *equivalent* ρ and ζ values increase by a factor e^q. Consequently the structure is spirallic, with proportionally growing elements. *Similar* structures, but of course only with discrete symmetries, are well known (e.g., *Ammonites*). However, if this symmetry exists, chamber sizes, suture lines etc. must obey a very definite growth law with a single scaling constant. Such a scaling can and ought to be checked.

6. TOPOLOGY

As told earlier, the metric g_{ik} does *not* define the neighbours on a manifold. As an example, consider the mantle of a cylinder of radius ρ_0. Points ($\varphi = 2\pi\text{-}\epsilon$, ζ_o) and ($\varphi = \epsilon$, ζ_o) are very near to each other if $\epsilon << 1$. On the other hand, performing the transformation (5.3) the two points get at a distance $(2\pi\text{-}2\epsilon)\rho_o$, because the mantle is cut just between the neighbouring points and they are rolled apart. So the coordinates and the metric are not enough; some extra information is needed about the possible "compactification" of the surface.

In the 3 dimensional Euclidean space we do have this information. However, possible configurations still are to be classified according to neighbourhood relations (connectivity). The problem has an extended literature, so here only a few simple examples will be mentioned; for the details cf. e.g., (Patterson, 1956).

Consider a compact *2 dimensional* entity (for such case the complete classification is known), e.g., the unit disc $\rho \leq 1$, $0 \leq \varphi < 2\pi$. This disc is *simply connected*. This term means that any closed curve on it can be reduced to any point or can be transformed into any other closed curve by *continuous* distortion (see the notion of homotopy groups (Patterson, 1956)). This is shown on Figure 1.

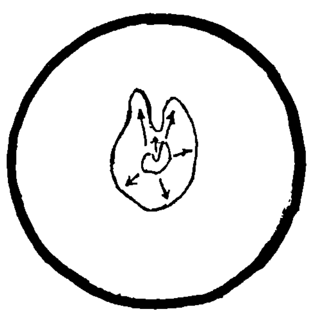

Figure 1

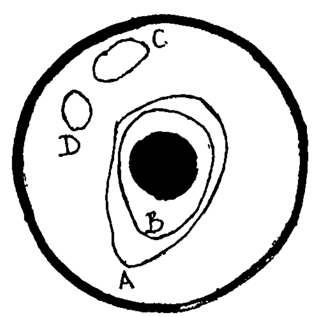

Figure 2

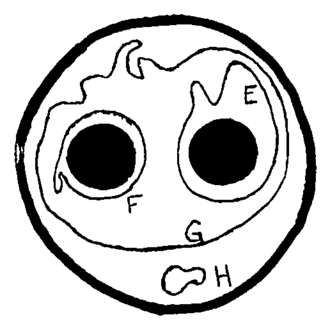

Figure 3

Now, punch a small hole around the center. The punctured disc is no more simply connected (Fig. 2.). The closed curves now are classified into 2 disjoint classes. Curves around the hole (say A and B) can be distorted into any other such curve but cannot be distorted into curves not surrounding the hole (*C* or *D*) and vice versa. In addition, a disc with a central hole can be distorted into another disc with eccentric hole. (THE DISTORTION, ON THE OTHER HAND, IS DETECTED BY THE METRIC STRUCTURE, BECAUSE DISTANCES BETWEEN POINT PAIRS ARE CHANGING. NEIGHBOURHOOD AND METRIC RELATIONS TOGETHER ARE ENOUGH FOR COMPLETE DESCRIPTION.) So for *topology* all the discs with one hole are equivalent.

Punch a second hole (Fig. 3). This disc you can deform into any other disc *with two holes*. Again, on it closed curves classify into four classes: surrounding the first hole (*E*), the second hole (*F*), both holes (*G*) or neither of them (*H*). And so on. These objects are inequivalent with each other.

Things go similarly for *bodies*. Consider a sphere. Each internal closed curve can be deformed into any other. This will not change if one digs a pit into it. However, driving a shaft completely through the interior is no more simply connected: curves surrounding the tunnel cannot be deformed into curves not surrounding it. At this point we deliberately stop.

It may obviously be important the possible ways of internal connections for the structure and operation of an organism.

7. CLOSING REMARK

This paper is by no means complete, and in addition at this point may seem rather pointless. However, it contains just the necessary mathematical background for the rest of the articles in the next issue. Our present purpose was not to enjoy pure mathematics but to help the understanding of some following articles.

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ANNOUNCEMENTS

FIRST CIRCULAR

CALL FOR PAPERS, WORKSHOP TOPICS, AND EXHIBITION ITEMS

SYMMETRY: NATURAL AND ARTIFICIAL

Third Interdisciplinary Symmetry Congress and Exhibition of the

INTERNATIONAL SOCIETY FOR THE INTERDISCIPLINARY STUDY OF SYMMETRY (ISIS-SYMMETRY)

August 14 - 20, 1995
Old Town Alexandria (near Washington, D.C.) U.S.A.

FIELDS OF INTEREST

SYMMETRY: NATURAL AND ARTIFICIAL

The congress and exhibition present a broad interdisciplinary forum where the representatives of various fields in art, science, and technology may discuss and enrich their experiences. The concept *symmetry*, having *roots in both art and science*, helps to provide a 'common language' for this purpose. The new 'bridges' between disciplines could *inspire further ideas* in the original fields of participants, as well as *facilitate the adaptation of existing ideas and methods from one field to another.*

The title of the congress emphasizes the presence of symmetry (dissymmetry, broken symmetry) both *in nature* and *in the objects created by artists, scientists, and engineers.*

Exhibition: Ars Scientifica

There have been several exhibitions representing the specific impact of certain fields of science and technology on art, but ISIS-Symmetry has initiated a *regular forum* for a broader *interface of art and science*. The exhibition will consist of two parts: a professional exhibition and an informal one, based on the objects illustrating the lectures given by the participants. Some workshops will be conducted in the exhibition rooms. SPECIAL INTERESTS of *Ars Scientifica* are, among others: kaleidoscopes, polyhedra, model designs, new media.

CALL FOR PAPERS

A lecture proposal should include a maximum 4-page *extended abstract* in a *camera ready* version. Keeping in mind the interdisciplinary goals of the congress and the composition of the participants, please try to help the readers outside of your main discipline e.g., by explaining some special concepts, using intuitive approaches, or giving comprehensive tables and illustrations. The extended abstracts should either (*a*) describe concrete interdisciplinary 'bridges' between different fields of art, science, and technology using the concept of symmetry; or (*b*) survey the importance of symmetry in a concrete field with an emphasis on possible 'bridges' to other fields. Note, please, that the central topic of the present congress *Symmetry: Natural and Artificial* opens a wider door towards technological applications. Papers discussing links between any form of symmetry-asymmetry phenomenon or law in nature on the one side, and artistic, technical achievements on the other, are preferred. Please consider that the meetings of ISIS-Symmetry are informal and do not substitute for the disciplinary conferences, only supplement them with a broader perspective.

The extended abstracts should be submitted in 2 copies, mailed 1 each to G. Darvas and D. Nagy, on A4 or letter size pages, printed on one side of each sheet, with at least 2.5 cm (1 inch) margins both sides, top and bottom, double spaced, 12-point characters.

Sample:

TITLE WITH CAPITAL LETTERS

[two line-spaces]

Joe Symmetrist and Josephine Asymmetrist
Department of Dissymmetry, Fibonacci University
San Symmetrino, SY 12358, Symmetryland
E-mail: symmetrist@fibonacci.edu

[two line-spaces]

The text should be printed in one column. Figures (black-and-white only) may interrupt the text. Please avoid using any other heading (e.g., 'Extended Abstract', 'submitted to ...'). Page numbers should be marked with pencil.

References [at the end of the abstract]:
Alphabetical order, full bibliographic description.

For more details refer to the "Instructions for contributors" on pp. 110-111.

CALL FOR EXHIBITION ITEMS

Items for the exhibitions should be introduced in the same form as lecture-abstracts on *A4* or letter size sheets, in black-and-white camera ready, reproducible form. Please mark with pencil at the top of the sheet: (EXHIBITION). A short description and/or explanation of the items, as well as the connection to the main theme of the congress and exhibition, is preferred. Please give the dimensions of each item. Art works, models, demonstration materials, etc. are welcome, e.g., in the following sections: Kaleidoscopes, Polyhedral symmetry, The beauty of molecules, Aesthetics of man-made constructions, Mechanical structures inspired by nature: Artificial and natural structures, Design principles, New Media. Proposals for further sections are encouraged.

CALL FOR WORKSHOP TOPICS

Please give the approximate title, short description (how do you plan to organize the workshop), other expected/proposed contributors, etc. Proposals emphasizing interactions, mediated by symmetry, between different disciplines; science, art, and technology; cultural origins, and relying upon the interest of participants with different backgrounds, are preferred.

CALL FOR PROPOSALS FOR EVENING ACTIVITIES OR PERFORMANCES

Music, dance, video, laser, etc. programs are welcome. Please submit your proposals, similar to the lecture abstracts, with descriptions of the feasibility and the technical requirements. Please mark with pencil at the top of the sheet: (PERFORMANCE), (VIDEO), etc., respectively. (Formal requirements are the same as above for papers.)

DEADLINES

for application and short description of contribution and other proposals:
December 15, 1994;

for submitting final (camera ready) versions of the extended abstracts:
March 31, 1995.

THE FORMAT OF THE CONGRESS AND EXHIBITION

The tradition, initiated by ISIS-Symmetry, to facilitate interdisciplinary dialogues among scientists, engineers, and artists will be continued. There will be no parallel sections (which would lead to disciplinary separation of the participants), but each *morning* there will be *plenary sessions*, while the main ideas will be discussed and developed in *afternoon workshops*. For the *evenings* there are scheduled performances and informal meetings, including recreational, and *ars scientifica* programs.

The *working language* of the congress is *English*.

The Scientific Advisory Committee of the Congress and Exhibition is the Board of ISIS-Symmetry (see inside front and back covers).

CONTACT PERSONS (for the Congress and Exhibition)

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ISIS-SYMMETRY

ISIS-Symmetry organized its first congress and exhibition "Symmetry of Structure" in Budapest, Hungary (August 1989), while the second one "Symmetry of Patterns" was held in Hiroshima, Japan (August 1992). This forthcoming triennial *congress* and *exhibition* will be hosted on a third continent.

ISIS-Symmetry events demonstrate the emphasis on internationality and interdisciplinarity. Indeed, the Society has members in 41 countries, on all continents, and its main purpose is to *bridge* art and science (different *disciplines*), East and West (different *cultures*). (Cf. the "Aims and Scope" on p. 112.) Backgrounds of the members of the Society represent various *fields of science, art, and technology*. Their activity is linked by the concept of symmetry. *Application of symmetry* is also a general tool and method influencing and fertilizing the creative thinking of each other.

APPLICATION FORM

Name: ....

Affiliation: ....

Mailing Address: ....

City: .... State/Country: ....

Fax: .... Phone: .... E-mail: ....

I intend to: O attend the Congress O submit a paper O exhibit

Tentative title of my contribution: ....

OBITUARY

GYÖRGY PAÁL

1934 – 1992

Our colleague, one of the organisers of the 1991 symposium, from whose material this issue has been selected, György Paál, astronomer, long since member of the Geonomical Scientific Society of the Hungarian Academy of Sciences, died in March, 1992 after 5 years of ailment.

His scientific career started at the end of the fifties, with observations for galaxy clusters and quasars. In this study he formulated a hypothesis – from quasar aggregations at some red-shifts – about the non simply connected topology of the Universe, in 1970. Such a topology, quite possible according to the General Relativity, would result in regular, although non-trivial, recurrences of multiple pictures, and, for very simple expansion laws of Universe, in a rather crystal-like symmetric visual appearance of the large-scale picture of the Universe. Then, for a while, he turned to "internal" symmetries: regularities about fundamental constants, patterns in the data of celestial objects and observed characteristic quantities of the Universe. In the eighties he concentrated on the cosmological consequences of the particle physical theory of Grand Unification. This theory claims that the 3 fundamental interactions (electromagnetism, weak interaction, and strong interaction), whose combined symmetry seems to be $U(1) \times SU(2) \times SU(3)$, are rather three projections of a higher scheme of a higher symmetry at least $SU(5)$, broken spontaneously at some energies.

In 1987 his serious illness started, and physicians were very pessimistic about his future. He chose to fight, not only for his life but rather for the possibility to continue his scientific work. He doubled his professional activity. In 1991 the newest cosmological observations about the existence of repeated "Great Walls" separated by hundred millions of light years suggested him to return to non-trivial topologies; several papers were published about this possibility, and the present issue contains one brief report on this. He died during this work, after several years of slow but continuous worsening of his health.

His co-authors are continuing this work. At this point it is impossible to decide which is the exact symmetry of the Universe from among the 3 possible ones, and which is its topology from amongst the 10 possible ones for the symmetry $E(3)$ or from amongst the infinite or uncounted possibilities of the cases $SO(4)$ or $SO(3, 1)$. For any case his strong ability for a global viewpoint is seriously lacked.

But his works remain, and we remember his thinking pattern and working style as examples for our later work.

Béla Lukács

Contributions to *SYMMETRY: CULTURE AND SCIENCE* are welcomed from the broadest international circles and from representatives of all scholarly and artistic fields where symmetry considerations play an important role. The papers should have an interdisciplinary character, dealing with symmetry in a concrete (not only metaphorical!) sense, as discussed in 'Aims and Scope' on p. 336. The quarterly has a special interest in how distant fields of art, science, and technology may influence each other in the framework of symmetry (symmetrology). The papers should be addressed to a broad non-specialist public in a form which would encourage the dialogue between disciplines.

Manuscripts may be submitted directly to the editors, or through members of the Board of ISIS-Symmetry.

Contributors should note the following:

■ All papers and notes are published in English and they should be submitted in that language. The quarterly reviews and annotates, however, non-English publications as well.

■ In the case of complicated scientific concepts or theories, the intuitive approach is recommended, thereby minimizing the technical details. New associations and speculative remarks can be included, but their tentative nature should be emphasized. The use of well-known quotations and illustrations should be limited, while rarely mentioned sources, new connections, and hidden dimensions are welcomed.

■ The papers should be submitted either by electronic mail to both editors, or on computer diskettes (5 ¼" or 3.5") to György Darvas as text files (IBM PC compatible or Apple Macintosh); that is, conventional characters should be used (ASCII) without italics or other formatting commands. Of course typewritten texts will not be rejected, but the preparation of these items takes longer. For any method of submission (e-mail, diskette, or typescript), four hard-copies of the text are also required, where all the necessary editing is marked in red (inserting non-ASCII characters, underlining words to be italicized, etc.). Three hard-copies, including the master copy and the original illustrations, should be forwarded to György Darvas, while the fourth copy should be sent to Dénes Nagy. No manuscripts, diskettes, or figures will be returned, unless by special arrangement.

■ The papers are accepted for publication on the understanding that the copyright is assigned to ISIS-Symmetry. The Society, however, aiming to encourage the cooperation, will allow all reasonable requests to photocopy articles or to reuse published materials. Each author will receive a complimentary copy of the issue where his/her article appeared.

■ Papers should begin with the title, the proposed running head (abbreviated form of the title of less than 35 characters), the proposed section of the quarterly where the article should appear (see the list in the note 'Aims and Scope'), the name of the author(s), the mailing address (office or home), the electronic mail address (if any), and an abstract of between 10 and 15 lines. A recent black-and-white photo, the biographic data, and the list of symmetry-related publications of (each) author should be enclosed; see the sample at the end.

■ Only black-and-white, camera-ready illustrations (photos or drawings) can be used. The required (approximate) location of the figures and tables should be indicated in the main text by typing their numbers and captions (Figure 1: [text], Figure 2: [text], Table 1: [text], etc.), as new paragraphs. The figures, which will be slightly reduced in printing, should be enclosed on separate sheets. The tables may be given inside the text or enclosed separately.

■ It is the author's responsibility to obtain written permission to reproduce copyright materials.

■ Either the British or the American spelling may be used, but the same convention should be followed throughout the paper. *The Chicago Manual of Style* is recommended in case of any stylistic problem.

■ Subtitles (numbered as 1, 2, 3, etc.) and subsidiary subtitles (1.1, 1.1.1, 1.1.2, 1.2, etc.) can be used, without over-organizing the text. Footnotes should be avoided; parenthetic inserts within the text are preferred.

■ The use of references is recommended. The citations in the text should give the name, year, and, if necessary, page, chapter, or other number(s) in one of the following forms: ... Weyl (1952, pp. 10-12) has shown...; or ... as shown by some authors (Coxeter et al., 1986, p. 9; Shubnikov and Koptsik 1974, chap. 2; Smith, 1981a, chaps. 3-4; Smith, 1981b, sec. 2.12; Smith, forthcoming). The full bibliographic description of the references should be collected at the end of the paper in alphabetical order by authors' names; see the sample. This section should be entitled *References*.

Sample of heading (Apologies for the strange names and addresses)

SYMMETRY IN AFRICAN ORNAMENTAL ART
BLACK-AND-WHITE PATTERNS IN CENTRAL AFRICA
Running head: Symmetry in African Art
Section: Symmetry: Culture & Science

Susanne Z. Dissymmetrist and Warren M. Symmetrist
8 Phyllotaxis Street
Sunflower City, CA 11235, U.S.A.

Warren M. Symmetrist
Department of Dissymmetry, University of Symmetry
69 Harmony Street, San Symmetrino, CA 69869, U.S.A.
E-mail: symmetrist@symmetry.edu

Abstract

The ornamental art of Africa is famous ...

Sample of references

In the following, note punctuation, capitalization, the use of square brackets (and the remarks in parentheses). There is always a period at the very end of a bibliographic entry (but never at other places, except in abbreviations). Brackets are used to enclose supplementary data. Those parts which should be italicized — titles of books, names of journals, etc. — should be underlined in red on the hard-copies. In the case of non-English publications both the original and the translated titles should be given (cf., Dissymmetrist, 1990).

Asymmetrist, A. Z. (or corporate author) (1981) *Book Title: Subtitle*, Series Title, No. 27, 2nd ed., City (only the first one): Publisher, vii + 619 pp.; (further data can be added, e.g.) 3rd ed., 2 vols., ibid., 1985, viii + 444 + 484 pp. with 2 computer diskettes; Reprint, ibid., 1988; German trans., *German Title*, 2 vols., City: Publisher, 1990, 986 pp.; Hungarian trans.

Asymmetrist, A. Z., Dissymmetrist, S. Z., and Symmetrist, W. M. (1980-81) Article or e-mail article title: Subtitle, Parts 1-2, *Journal Name Without Abbreviation*, [E-Journal or Discussion Group address: journal@node (if applicable)], B22 (volume number), No. 6 (issue number if each one restarts pagination), 110-119 (page numbers); B23, No. 1, 117-132 and 148 (for e-journals any appropriate data).

Dissymmetrist, S. Z. (1989a) Chapter, article, symposium paper, or abstract title, [Abstract (if applicable)], In: Editorologist, A.B. and Editorologist, C.D., eds., *Book, Special Issue, Proceedings, or Abstract Volume Title*, [Special Issue (or) Symposium organized by the Dissymmetry Society, University of Symmetry, San Symmetrino, Calif., December 11-22, 1971 (those data which are not available from the title, if applicable)], Vol. 2, City: Publisher, 19-20 (for special issues the data of the journal).

Dissymmetrist, S. Z. (1989b) *Dissertation Title*, [Ph.D. Dissertation], City: Institution, 248 pp. (Exhibition Catalogs, Manuscripts, Master's Theses, Mimeographs, Patents, Preprints, Working Papers, etc. in a similar way; Audiocassettes, Audiotapes, Compact Disks, Computer Diskettes, Computer Software, Films, Microfiches, Microfilms, Slides, Sound Disks, Videocasettes, etc. with necessary modifications, adding the appropriate technical data).

Dissymmetrist, S. Z., ed. (1990) *Dissimmetriya v nauke* (title in original, or transliterated, form), [Dissymmetry in science, in Russian with German summary], Trans. from English by Antisymmetrist, B. W., etc.

Phyllotaxist, F. B. (1899/1972) *Title of the 1972 Edition*, [Reprint, or Translation, of the 1899 ed.], etc.

[Symmetrist, W. M.] (1989) Review of *Title of the Reviewed Work*, by S. Z. Dissymmetrist, etc. (if the review has an additional title, then it should appear first; if the authorship of a work is not revealed in the publication, but known from other sources, the name should be enclosed in brackets).

In the case of lists of publications, or bibliographies submitted to *Symmetro-graphy*, the same convention should be used. The items may be annotated, beginning in a new paragraph. The annotation, a maximum of five lines, should emphasize those symmetry-related aspects and conclusions of the work which are not obvious from the title. For books, the list of (important) reviews, can also be added.

Sample of biographic entry

Name: Warren M. Symmetrist, Educator, mathematician, (b. Boston, Mass., U.S.A., 1938).

Address: Department of Dissymmetry, University of Symmetry, 69 Harmony Street, San Symmetrino, Calif. 69869, U.S.A. *E-mail:* symmetrist@symmetry.edu

Fields of interest: Geometry, mathematical crystallography (also ornamental arts, anthropology — non-professional interests in parentheses).

Awards: Symmetry Award, 1987; Dissymmetry Medal, 1989.

Publications and/or *Exhibitions*: List all the symmetry-related publications/exhibitions in chronological order, following the conventions of the references and annotations. Please mark the most important publications, not more than five items, by asterisks. This shorter list will be published together with the article, while the full list will be included in the computerized data bank of ISIS-Symmetry.

There are many disciplinary periodicals and symposia in various fields of art, science, and technology, but broad interdisciplinary forums for the connections between distant fields are very rare. Consequently, the interdisciplinary papers are dispersed in very different journals and proceedings. This fact makes the cooperation of the authors difficult, and even affects the ability to locate their papers.

In our 'split culture', there is an obvious need for interdisciplinary journals that have the basic goal of building bridges ('symmetries') between various fields of the arts and sciences. Because of the variety of topics available, the concrete, but general, concept of symmetry was selected as the focus of the journal, since it has roots in both science and art.

SYMMETRY: CULTURE AND SCIENCE is the quarterly of the *INTERNATIONAL SOCIETY FOR THE INTERDISCIPLINARY STUDY OF SYMMETRY* (abbreviation: *ISIS-Symmetry*, shorter name: *Symmetry Society*). ISIS-Symmetry was founded during the symposium *Symmetry of Structure* (*First Interdisciplinary Symmetry Symposium and Exhibition*), Budapest, August 13-19, 1989. The focus of ISIS-Symmetry is not only on the concept of symmetry, but also its associates (asymmetry, dissymmetry, antisymmetry, etc.) and related concepts (proportion, rhythm, invariance, etc.) in an interdisciplinary and intercultural context. We may refer to this broad approach to the concept as *symmetrology*. The suffix *-logy* can be associated not only with knowledge of concrete fields (cf., biology, geology, philology, psychology, sociology, etc.) and discourse or treatise (cf., methodology, chronology, etc.), but also with the Greek terminology of proportion (cf., *logos*, *analogia*, and their Latin translations *ratio*, *proportio*).

The basic goals of the *Society* are
(1) to bring together artists and scientists, educators and students devoted to, or interested in, the research and understanding of the concept and application of symmetry (asymmetry, dissymmetry);
(2) to provide regular information to the general public about events in symmetrology;
(3) to ensure a regular forum (including the organization of symposia, and the publication of a periodical) for all those interested in symmetrology.

The Society organizes the triennial *Interdisciplinary Symmetry Symposium and Exhibition* (starting with the symposium of 1989) and other workshops, meetings, and exhibitions. The forums of the Society are *informal* ones, which do not substitute for the disciplinary conferences, only supplement them with a broader perspective.

The Quarterly - a non-commercial scholarly journal, as well as the forum of ISIS-Symmetry - publishes original papers on symmetry and related questions which present new results or new connections between known results. The papers are addressed to a broad non-specialist public, without becoming too general, and have an interdisciplinary character in one of the following senses:
(1) they describe concrete interdisciplinary 'bridges' between different fields of art, science, and technology using the concept of symmetry;
(2) they survey the importance of symmetry in a concrete field with an emphasis on possible 'bridges' to other fields.

The Quarterly also has a special interest in historic and educational questions, as well as in symmetry-related recreations, games, and computer programs.

The regular sections of the Quarterly:
- **Symmetry: Culture & Science** (papers classified as humanities, but also connected with scientific questions)
- **Symmetry: Science & Culture** (papers classified as science, but also connected with the humanities)
- **Symmetry in Education** (articles on the theory and practice of education, reports on interdisciplinary projects)
- **Mosaic of Symmetry** (short papers within a discipline, but appealing to broader interest)
- **SFS: Symmetric Forum of the Society** (calendar of events, announcements of ISIS-Symmetry, news from members, announcements of projects and publications)
- **Symmetro-graphy** (biblio/disco/software/ludo/historio-graphies, reviews of books and papers, notes on anniversaries)
- **Reflections: Letters to the Editors** (comments on papers, letters of general interest)

Additional non-regular sections:
- **Symmetrospective: A Historic View** (survey articles, recollections, reprints or English translations of basic papers)
- **Symmetry: A Special Focus on ...** (round table discussions or survey articles with comments on topics of special interest)
- **Symmetry: An Interview with ...** (discussions with scholars and artists, also introducing the Honorary Members of ISIS-Symmetry)
- **Symmetry: The Interface of Art & Science** (works of both artistic and scientific interest)
- **Recreational Symmetry** (problems, puzzles, games, computer programs, descriptions of scientific toys; for example, tilings, polyhedra, and origami)

Both the lack of seasonal references and the centrosymmetric spine design emphasize the international character of the Society; to accept one or another convention would be a 'symmetry violation'. In the first part of the abbreviation *ISIS-Symmetry* all the letters are capitalized, while the centrosymmetric image iSIS! on the spine is flanked by 'Symmetry' from both directions. This convention emphasizes that ISIS-Symmetry and its quarterly have no direct connection with other organizations or journals which also use the word *Isis* or *ISIS*. There are more than twenty identical acronyms and more than ten such periodicals, many of which have already ceased to exist, representing various fields, including the history of science, mythology, natural philosophy, and oriental studies. ISIS-Symmetry has, however, some interest in the symmetry-related questions of many of these fields.

continued from inside front cover

ISIS-Symmetry
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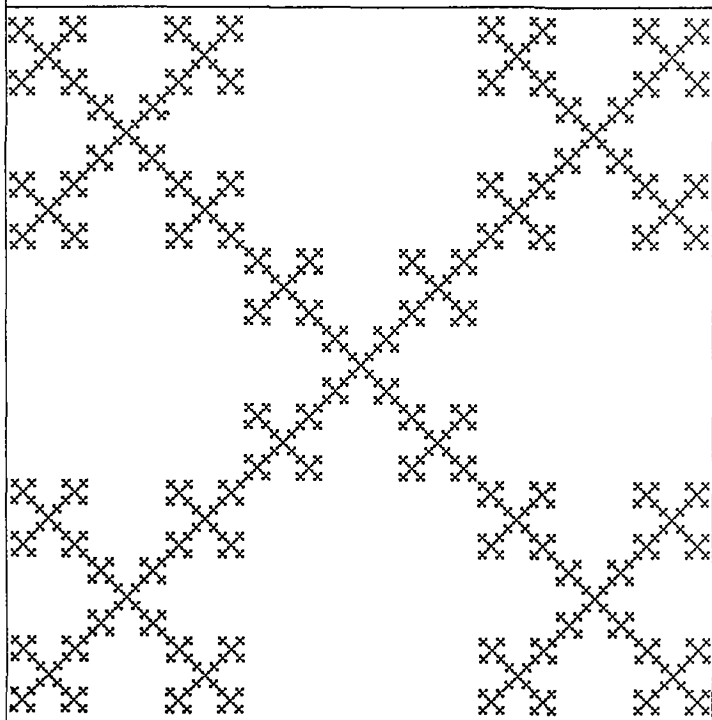
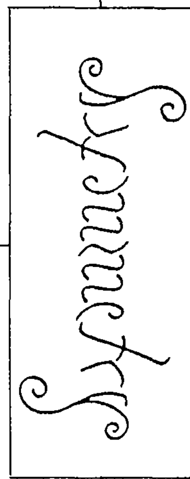
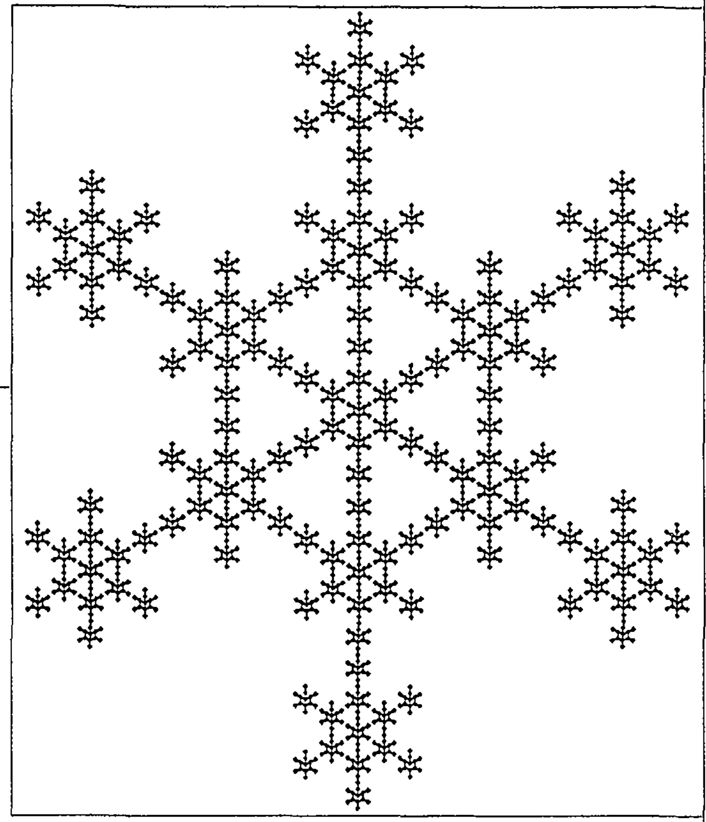
H-1361 Hungary







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