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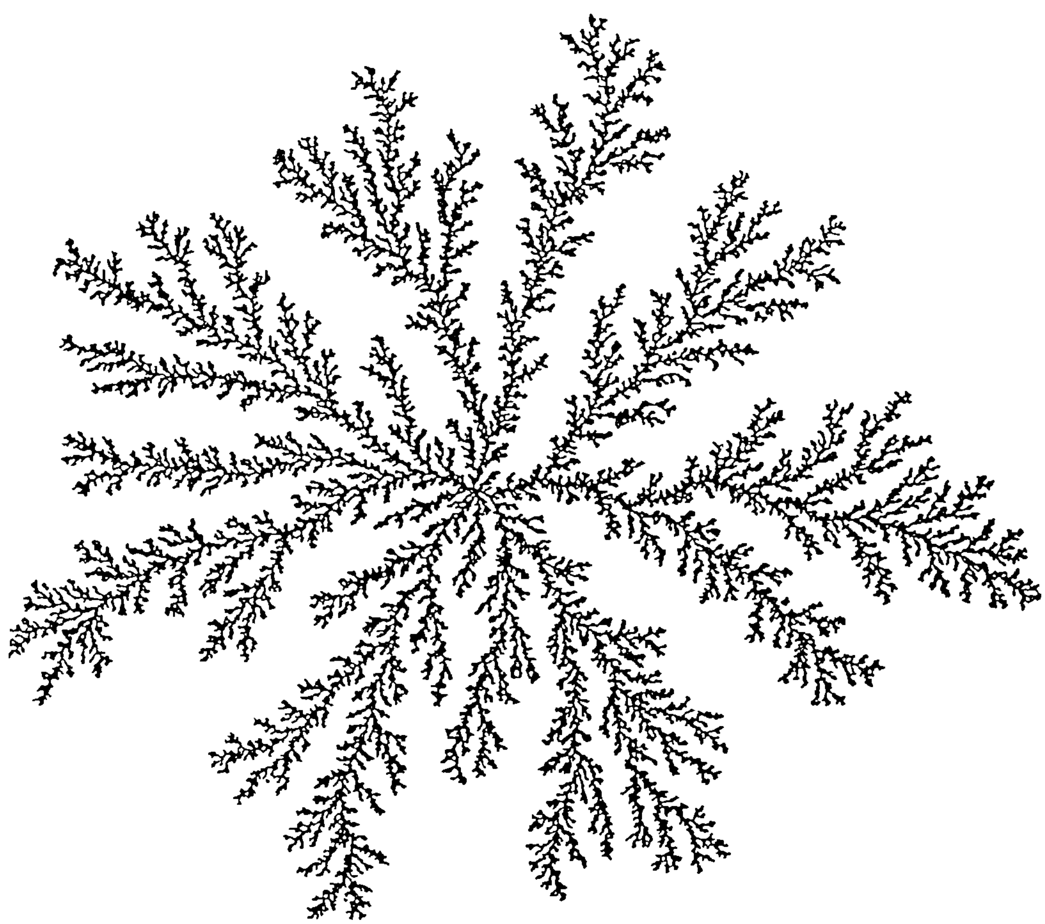

DLA fractal cluster
of 10^6 particles

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THE EVOLUTION OF COSMIC SYMMETRIES

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Abstract: *We discuss the changes of symmetries (both spatial and internal) of the Universe from the beginning to the present age. For this one must clarify what is a 'beginning'; what do initial conditions mean for the total and unique Universe and if any then what may be the correct initial conditions. While one cannot settle down this question with a finality, we discuss the problem in details and try to give a probable answer. The most probable scenario contains many subsequent spontaneous symmetry breakings, sometimes with a partial compensation between deteriorating internal symmetries and improving spatial ones.*

1. WHAT IS COSMOLOGY?

Cosmology is a pseudo-Greek term of the same structure as geology, meteorology etc., so roughly it is the science of the Cosmos or Universe. Originally it was distinguished from *cosmogony*, a discipline describing the genesis of the Universe. However, in General Relativity the matter governs the geometry of the space-time, and, except for very special cases, the matter of the whole Universe cannot remain in equilibrium. Therefore any valid theoretical description of the Universe will automatically contain its evolution as well, and then there is no need for a separate cosmogony.

Now the question is if there is an independent science of cosmology at all. The answer is no if

1) there is no such definite entity as Universe; or
2) there are no specific laws of the Universe.

And for the present lecture: in the first case we cannot speak about the symmetries of the Universe; in the second we can but it will have a limited physical relevance.

To explain this statement let us see what may be the Universe. If the Universe is simply an incoherent sum of entities then it has no definite global structure including symmetries, and the local structure randomly changes from point to point. . If the Universe has a global structure but all its laws are consequences of known local laws, then the actual symmetries and their evolution belong to the purely descriptive part of science, and therefore one cannot learn too much from it, although the story may still be interesting.

These questions are rather fundamental and we cannot be expected to be able to solve them here. However a very brief recapitulation of earlier ideas is edifying. Not claiming completeness,

1) the Universe of the ancient Greek science is a single finite unit, with its own laws, and with spherical symmetry.

2) Newton's Universe is more or less the infinite absolute space + the matter filling it; the empty absolute space would have an $E(3)$ symmetry but the matter may or may not share this. This Universe, which is only a *nomen collectivum*, has no law of its own.

3) in General Relativity the Universe may or may not exist as a definite object. General Relativity can describe the Universe as an object; it may possess some definite symmetry, for example. However no specific laws of the Universe appear in General Relativity.

4) the so called standard cosmological model contains some cosmological *principles* which may be consequences of some unknown cosmological laws.

Methodical and consequent thinking in a model may point to some problematical points where the model is incomplete, self-contradictory or impossible. In the present case the result is as follows.

1) Aristotle's cosmology was free of paradoxes (and this was the reason to live for two millenia), although finally it has been proven incorrect.

2) Newton's cosmology had paradoxes, some of which was known from the beginning. E.g., gravity causes instability in an infinite Universe (Newton, 1756) known later as Seeliger's paradox.

3) General Relativity removes the old paradoxes. E.g., the Universe may be finite or compact (Paál, 1993); even if it is infinite, gravitational instabilities are not necessary because of the nonlinearity of gravitational laws; it may be of finite age etc.

4) The standard model is of maximal spatial symmetry (for simplicity but also suggested by observations). Applying the known laws the Universe possesses some constants of integration. Now, these constants can be read off from observations, and changing them even moderately the present Universe would be *qualitatively* different from the actual one (Carr and Rees, 1979). So the actual Universe seems 'improbable'. Of course, this term does not have any meaning here, because there is only a single Universe, so one cannot study a set of them with different initial conditions. However, if one eliminates the probability problem by

the unicity of the Universe, then he must tacitly accept that something is behind the only values of the constants, so they are prescribed *somehow*. Then we may expect unknown laws of the total Universe. We stop here, not discussing the question if some of the laws may be given by the unknown unification of general relativity and quantum field theory.

2. PART vs. WHOLE

In any case the present Universe consists of a lot of parts as galaxy clusters, galaxies, stars and particles. It is obvious to classify entities as *primary* (no constituents), *secondary* (with primary constituents), etc. However, this classification is insufficient for the present purpose, not telling anything about the relations between the 'higher' entity and its parts. Since we are interested in the problem of specific laws, it is worthwhile to list the 3 qualitative different possible relations between a higher entity and its parts. Social sciences were long ago confronted to this question (laws of associations, constitutions of states, etc.) and from their results here we take the following cases (names in the canonical German) (Marx, 1964). The discussion and classification will be useful also for a later lecture in this Volume (Lukács, 1993b).

1) *Einheit*. Constitutional analogy is the centralised state. The Whole exists in its own right with its own laws, the Parts have no independent existence or laws.

2) *Einigung*. Constitutional analogy is the federation. Both the Whole, and the Parts exist in their own rights with their own laws.

3) *Vereinigung*. Constitutional analogy is the confederation. The independent entities are the Parts with their own laws; the Whole is their sum with interactions, so can be derived from them.

Obviously any further transitional stages could be identified, but these three will suffice now for us.

The present Universe is clearly not an *Einheit*. Namely, its parts (e.g., stars) can quite satisfactorily be described or explained without referring to the total Universe. There remain the other two possibilities; we could choose between them according to the success or failure of explanatory models (see later). However this was not necessarily true for the primordial Universe, which probably did not have separate parts of permanent identity (cf. later).

3. BUILDING TOGETHER OR TAKING APART?

Consider the hierarchy of objects in the present Universe. The observable part contains more than 10^4 galaxy clusters, of which each contains roughly 10^4 galaxies. Our galaxy contains $\sim 10^{11}$ stars and an average star contains $\sim 10^{57}$ nucleons and similar number of other particles. Our Sun is 4.6×10^9 ys old (Novotny, 1973), the oldest stellar clusters in our Galaxy have ages $(10\text{-}20)\times10^9$ ys, and extrapolating back the recession of galaxies the objects of the presently observable volume were packed with a density higher by orders of magnitude than the present one at $\approx 15\times10^9$ ys ago. So the formation of any presently existing structure should be explained on a scale 10^{10} y or shorter.

Then galaxies cannot build up by random encounters of 10^{11} independent stars (instead of detailed formulae remember that the solar system cannot have had a close encounter in the last 4 billion years, therefore such processes are too infrequent to collect 10^{11} stars in 15 billion years). Also, it is improbable that thousands of independent galaxies could have congregated to form a galaxy cluster in 15 billion years. This suggests a *fragmentation* from protoclusters to stars.

To be sure, star formation is fairly explained by gravitational contraction. However the above probability considerations suggest formation *inside* an existing galaxy; so we may try with the idea that first the galaxy is formed, then it fragments into smaller matter elements, and finally these 'droplets' contract into stars.

If so, then it is obvious to go one step farther by investigating the possibility that proto-galaxy-clusters dropped out from a primordial unity. Indeed, this is the standard explanation in this years; we shall see why.

4. ON STANDARD COSMOLOGIES

The simplest nontrivial and reasonable cosmological model is a space-time with full *spatial* symmetry (i.e., constant curvature $k = 0$ or ±1 on constant time hypersurfaces). Then (Robertson and Noonan, 1969)

$$ds^2 = dt^2 - R^2(t)\{dx^2 + S^2(x)d\Omega^2\} \tag{4.1}$$

where

$$S(x) = \begin{cases} \sin x & \text{for } k = +1 \\ x & k = 0 \\ \operatorname{sh} x & k = -1 \end{cases} \tag{4.2}$$

Then the distance of 'naturally moving' objects change proportionally to R, so densities are $\sim R^{-3}$. This scale function R is governed by the Einstein equations which (without cosmological constant) read as

$$\dot{R}^2 = (8\pi/3)G\rho R^2 - k \tag{4.3}$$

$$\ddot{R} = -(4\pi/3)G(\rho + 3P/c^2)R \tag{4.4}$$

where G is the Cavendish constant of gravity, ρ is the mass density and P is the (dynamical) pressure. With an additional equation of state $P = P(\rho)$ (or of similar form) these equations can be integrated for a given initial condition (discussed later).

Assume full thermodynamic equilibrium and blackbody radiation for the equation of state

$$P = N(\pi^2/90)T^4/(\hbar c)^3 \tag{4.5}$$

where T is the temperature and N is the number of independent helicity states, roughly the number of different kinds of particles, ~ 100. Then $\rho = 3P/c^2$, and for $k = 0$ eqs. (4.3-4) can be analytically solved. By comparing the derivative of (4.3) with (4.4) one gets

$$T = T_0 R_0 / R \tag{4.6}$$

where $R_0 = R(t_0)$, $T_0 = T(t_0)$ and t_0 is an arbitrary convenient time moment. Then substituting to (4.3) one gets

$$R = (32\pi^3 N/90)^{1/4}(Gc^{-2}/(\hbar c)^3)^{-1/4} T_0 R_0 \sqrt{t-t'} \tag{4.7}$$

where t' is a constant of integration. However,

$$T = (32\pi^3 N/90)^{1/4}(Gc^{-2}/(\hbar c)^3)^{-1/4}/\sqrt{t-t'} \tag{4.8}$$

Observe that $T = \infty$ at $t = t'$. This time moment is the natural zero point of the time counting, and in this convention *no free constant appears in the thermal evolution.*

Eq. (4.7) deserves some discussion but this will be postponed till the next Chapter. Now we turn to (4.8). It gives a continuously decreasing temperature. Above $T \sim 10000\ K \sim 1$ eV the photons would have destroyed all the atoms, so a completely ionised plasma was present. Photons are substantially coupled to free electric charges, so before t-t' ~ 300000 ys they destroyed any virtual gravitationally bound structure. When T goes below 1 $eV \sim 10^{-12}$ erg, the gas becomes fairly transparent for photons and structures of sufficiently large masses become bound. At this temperature

$$\epsilon = \rho c^2 \sim 10^{3}\text{erg/cm}^{3} \tag{4.9}$$

For the smallest bound object

$$E_{\text{therm}} \sim PR^3 \sim E_{\text{grav}} \sim GM^2/R;\ M \sim \rho R^3 \tag{4.10}$$

Hence the first appearing bound objects have the mass $\sim 10^{51}$ g $\sim 10^{18}$ solar mass. Since this value is roughly that of the largest galaxy cluster, the standard cosmology can explain the appearance of galaxy clusters. Therefore it seems that

1) before 300000 ys from t' there were no identifiable structures above elementary particles in the Universe; and

2) among the objects there was a *fragmentation* chain Universe $\to$ protoclusters $\to$ protogalaxies $\to$ matter for protostellar contraction.

However at 300000 ys the matter probably contained the known elementary particles and in the standard model it is so from the beginning. Thus we still have to discuss the initial conditions, since elementary particles can stand up in different configurations.

5. ON INITIAL CONDITIONS

Now we return to eq. (4.7). It is valid only for $k = 0$, but for early times (high temperatures) it is a good approximation also for $k = \pm 1$. Eq. (4.1) shows that a constant factor is undefined for $k = 0$, but it is not so for the other two cases. No clear evidence for k is seen in the observable part $\sim 10^{28}$ cm, therefore now

$$R_{pr} \sim q \times 10^{28} \text{ cm}, \; q > 1 \tag{5.1}$$

From the blackbody radiation

$$T_{pr} = 3\,K \tag{5.2}$$

Therefore in eqs. (4.6-7)

$$T_o R_o \approx q \times 10^{12} \text{ ergcm} \sim q \times 10^{28} \text{ cmgrad} \tag{5.3}$$

From mass counting the particle number of the observable part is $\sim 10^{78}$. For the size R it multiplies with q^3.

With this data in mind go back to eqs. (4.3-4). They can be rewritten as

$$\tfrac{1}{2}\dot{R}^2 - GM/R = -\tfrac{1}{2}k \tag{5.4}$$

$$dE + PdV = 0; \; E = (4\pi/3)R^3\epsilon; \; M = E/c^2 \tag{5.5}$$

Eq. (5.4) can be read as 'energy conservation' with 'kinetic' and 'potential' energies; the next equation shows that the changes are adiabatic (no exterior) and M changes. However going backwards to the past both terms on the left hand side of (5.4) are growing in absolute value as $1/t$. We cannot exactly use (4.7-8) now because they would give $k \equiv 0$, but it can be seen that for early times the 'initial conditions' R_o and T_o must be more and more fine tuned to get the constant difference. Eqs. (4.3-4) of course preserve the difference, but now we try to think according to causality.

At present all the 3 terms in (5.4) are in the same order of magnitude. Going back the needed tuning is $\sim (t/t_{pr})$. If anything prescribed R at a t_o it must have prescribed with this accuracy. If the initial conditions were determined in an age T ~ 1 GeV, e.g., then the tuning must have been cca 10^{-30}.

IF THE INITIAL CONDITIONS WERE RANDOM FOR THE UNIVERSE THEN THE PRESENT UNIVERSE IS ABSURDLY IMPROBABLE.

Now, it is difficult to interpret this statement. First, *when* should the description start with an initial condition? In a ballistic problem the proper moment for the initial conditions is when the stone is leaving the hand. Until that moment the equations of motion are not those of the ballistic problem, and at that moment various initial velocities can and used to be prepared (in a reasonable range). Then the proper moment for setting the initial conditions for the Universe would be the moment when the evolution was switching from unknown previous ones to the

Einstein equations (4.3-4) ('creation of the Universe'?). But we do not know anything about previous, qualitatively different, eras of evolution (with the exception of a slight guess, see the next Chapter). In addition, there is only a *single* Universe. We do not know if it even has *any* meaning to imagine a set of differently prepared Universes. It is quite possible that one could not apply the usual dichotomy of (arbitrary) initial conditions + (fixed) equations of motion on the (total) Universe. But practically all branches of physics use this dichotomic description to evolutionary situations. So it is possible that the Universe would need a new type of physics or at least additional cosmological laws prescribing, e.g., the unique 'initial conditions'. This was the reason to permit the possibility that the *present* Universe might be an Einigung, having its *own* laws as well as its parts have theirs. It is *possible* that the Universe cannot be totally understood from its parts.

However we do not want to be involved in theology or similar disciplines. Therefore no philosophical questions of creation of the Universe, the will of any Creator to prepare special initial conditions etc. will be discussed. Here we formulate two questions in the usual physical language.

1) Was any *special* time moment t_0 in the past which was a 'beginning' to impose initial conditions?

2) If there is any room for initial conditions, then what are the quantities for which initial values are to be imposed?

For Question 1) we of course cannot give a final answer. However we list, without the claim for completeness, the possibilities for the kinds of such t_0's. A special time moment might have been the starting point of the Einstein evolution. Another possibility would have been the minimum of the radius $R(t)$. The third possibility is the maximum of the temperature or energy density or any related quantity. Obviously the above blackbody model does not contain any of these three special moments: in it the Einstein equation always governs the evolution, the minimal radius is 0, which is a singular point unsuitable to prescribe data, and the maximal temperature is ∞, again in a singular state. We return to Question 1) in the next Chapter.

For Question 2), in contrast, the answer is simple. In the above simple model of blackbody radiation there are two independent quantities, R and T. So an initial condition is the prescription of R_0 and T_0. In more complicated models some further quantities may appear, e.g., for the total numbers or densities of different charges or particle.

6. ON THE 'BEGINNING'

The fact that we cannot go beyond General Relativity is not a proof of its ultimate validity. The Einstein equation is the simplest equation governing the curvature which is conform to present observations as e.g., planetary motions. However it is quite possible that the equation possesses extra terms unseen in the present observations. To be definite, the standard form of the Einstein equation is

$$R_{ik} - \tfrac{1}{2}g_{ik}R^r{}_r - \lambda g_{ik} = -(8\pi G/c^4)T_{ik} \tag{6.1}$$

Here g_{ik} is the metric tensor, R_{ik} is the Ricci tensor which contains g_{ik} together with its first and second derivatives in a covariant manner, linearly in the second ones, λ is the so called cosmological constant, while T_{ik} is the energy-momentum tensor of the matter (Robertson and Noonan, 1969). Above λ was neglected, but the present observations give upper bounds for $|\lambda|$, and its effects can be felt at *low* densities, so λ is irrelevant for early times. Now, the simplest possibility for a different evolution is the presence of higher neglected terms (nonlinear or higher in derivatives).

Of course the neglected term may be of any form and may appear just in the next measurement or 20 orders of magnitude above. However there is a *natural* way to generate some higher terms.

The Einstein equation (6.1) can be obtained from a variation principle whose Lagrangian is

$$L = L_{\text{matter}} + L_{\text{grav}} \tag{6.2}$$

where

$$L_{\text{grav}} = \mathrm{f}(R) = (c^4/8\pi G)(R+2\lambda);\ R \equiv R^r{}_r \tag{6.3}$$

Now, assume that $\mathrm{f}(R)$ is nonlinear. Indeed, Lánczos (1972) pointed out that a purely quadratic Lagrangian would lead to a *dimensionless* action integral, a pure number, which, therefore, seems to be something 'fundamental', good for asymptotic behaviour. Writing

$$\mathrm{f} = \sigma R^2 + (c^4/8\pi G)(R+2\lambda) \tag{6.4}$$

the Einstein equation, *up to first order* in σ, changes to

$$R_{ik} - \tfrac{1}{2} g_{ik} R^r{}_r - \lambda g_{ik} + \sigma(8\pi G/c^4)(R_{;ik} - R_{;r}^{\;r}\, g_{ik}) = -(8\pi G/c^4)T_{ik} \tag{6.5}$$

The new coefficient σ has the dimension $\mathrm{gcm^5/s^4}$. Do we have any guess for the value of such a quantity?

Yes, certainly. Contemporary physics knows 3 quite fundamental and general phenomena, each with its own single and unique characteristic constant. They are *(i) gravity* with the Cavendish constant $G = 6.67\times10^{-8}\ \mathrm{cm^3/gs^2}$, *(ii) relativity* with the velocity of light $c = 3.00\times10^{10}$ cm/s, and *(iii) quantization* with the Planck constant $\hbar = 1.05\times10^{-27}\ \mathrm{gcm^2/s}$. Partially unified theories do exist but the final triadic unification ('Relativistic Quantum Gravity') is not at hand. However, its fundamental scales must be set by G, c and $\hbar$. Now, the obvious form for σ of the above dimension is

$$\sigma = (\text{fundamental number}) \times \hbar c^3 \tag{6.6}$$

So extra terms of (6.5) type may be expected from any quantum extension of General Relativity. As an approximation, let us start from (4.1-8), and calculate the extra terms. They become comparable with the old ones at

$$T \sim T_{Pl} = \sqrt{\hbar c^5/G} = 1.22\times10^{19}\ \text{GeV} \sim 10^{15}\ \text{erg} \tag{6.7}$$

which is out of any possibility for observation. But this means that in the unified theory of all present theories this may be the point where Geometry emerges from the sea of foaming quantum fluctuations, or the corresponding temperature may be a maximal temperature.

The simplest *model* anticipating such phenomena is General Relativity + a Hawking radiation from the change of the geometry. Then the past history of the Universe is geodetically incomplete. It appears with infinite temperature but with finite energy density $\epsilon \sim \epsilon_{Pl} \equiv T_{Pl}{}^4/(\hbar c)^3$, and with a finite radius $R \sim R_{Pl}$,

$$R_{Pl} \equiv \sqrt{\hbar G/c^3} \sim 10^{-33}\,\mathrm{cm} \tag{6.8}$$

(Diósi *et al.*, 1986). According to the uncertainty principle a quantum fluctuation from $\epsilon = 0$ to $\epsilon = \epsilon_{Pl}$ in a volume $R_{Pl}{}^3$ survives till $t_{Pl} \equiv R_{Pl}/c$. However, if energy is *produced* (which is so for $P < 0$, and also if Hawking radiation of this type is present (Diósi *et al.*, 1986)) then during that time the original energy of the fluctuation can be reproduced and then the Universe can remain for later use.

This is a very primitive 'model'. However it contains a 'beginning' (of the evolution governed by the Einstein equation). There the initial conditions are

$$R_0 \sim R_{Pl},\ \epsilon_0 \sim \epsilon_{Pl} \tag{6.9}$$

i.e., completely prescribed by number constants (number of helicity states, π, etc.) and by the 3 fundamental constants. In this scenario no freedom appears in the initial conditions of our single and unique Universe, which is hopeful.

Now let us extrapolate back the standard Universe (4.7-8). There is a state with $T \sim T_{Pl}$ and $\epsilon \sim \epsilon_{Pl}$. However there $R \sim 0.1 \times q$ cm $\sim 10^{32} \times q \times R_{Pl}$. This is a rather 'unnatural' initial condition which cannot be expected from any 'Relativistic Quantum Gravity Theory'. This high factor is reflected in the mentioned 'fine tuning problem'.

AGAIN, THE PRESENT UNIVERSE SEEMS IMPROBABLE IN THE LIGHT OF FUNDAMENTAL CONSTANTS.

Let us note that the present elementary particles seem improbable as well. Namely, the massive ones are *below* the only natural combination for rest energy $\sim T_{Pl}$ by 20 orders of magnitude, and some possess sizes 20 orders of magnitude *above* R_{Pl}. While these masses and radii are in natural relations with each other, the masses themselves remain unexplained from (still unknown) fundamental theories. The fundamental elementary objects in any 'Relativistic Quantum Gravity' would have $r \sim R_{Pl}$, $m \sim T_{Pl}/c^2$ (and maybe a lifetime $\sim t_{Pl}$), so protons, electrons etc. cannot be the elementary objects of 'the fundamental' theory.

WE HAVE REASONS FOR DOUBTS IF THE KNOWN ELEMENTARY PARTICLES HAD BEEN PRESENT AT 'THE BEGINNING'.

We do know that some 'elementary particles' are composite objects and were created at a definite stage of the evolution of the Universe. However, e.g., the electron seems point-like, so really elementary. Still, its mass does not belong to 'fundamental unified physics'. (Its electric charge does, since $e^2 \sim \hbar c$, the numerical factor being as moderate as 1/137.)

For charges or particle numbers the natural initial conditions are simple enough. A particle number is dimensionless, and charges may be defined likewise. Then the fundamental constants G, c, $\hbar$ yield nothing for them; the natural initial condition is either $N_\alpha \sim 1$ or $N_\alpha = 0$. The same is true for the total entropy S. In contrast, extrapolating back from the present Universe, $N_{\mathrm{baryon}} \sim q^3 \times 10^{78}$, and $S \sim 10^{87}$ (Guth, 1981).

7. ON PHASE TRANSITIONS

The above 'unnatural' numbers seem to point into a common direction, discovered by Guth (1981):

THERE SEEM TO HAVE BEEN SUBSTANTIAL ENERGY AND ENTROPY PRODUCING PROCESSES IN THE EARLY PAST OF THE UNIVERSE.

As shown by eq. (5.5), this is possible if $P < 0$. Negative (dynamical) pressures are unfamiliar, but may appear e.g., in scenarios when a phase transition cannot start because of any barrier inducing supercooling of the high temperature phase. Then the energy is higher than the equilibrium value, therefore the pressure is lower. Since *now* $P > 0$, it was >0 in the past in any not unstable or metastable state, but for transient periods, followed by reheating, it may have been even negative.

To illustrate this we show the simplest nontrivial example; for details see (Kämpfer, Lukács, and Paál, 1990). Consider a system without particle numbers. Omitting the details we have an equation of state $p(T)$, and

$$s = p_{,T} \tag{7.1}$$

$$\epsilon = Ts - p = Tp_{,T} - p \tag{7.2}$$

Consider a model system of coupled scalar and vector bosons. The scalar Higgs bosons possess a quartic self-potential $V(\Phi)$, mentioned in a parallel paper (Lukács, 1993a). Φ is a multicomponent quantity for a set of scalar bosons. If a particular Higgs has a nonzero expectation value $<\Phi>$ (in a side minimum of the quartic potential), then some coupled vector bosons get masses.

At a given temperature T all the vector bosons with $m << T$ simulate a blackbody radiation, while those with $m >> T$ have negligible contribution to the pressure. So in the roughest approximation

$$p \approx (N^* \pi^2/90) T^4/(hc)^3 - V(\Phi)/(hc)^3 \tag{7.3}$$

where N^* is the number of helicity states for the particles $m<T$, and V is to be taken at the actual equilibrium state, $\Phi = \Phi_0$ or $\Phi = \Phi_\pm$. Then, according to eq. (7.2),

$$\epsilon = (N^* \pi^2/30) T^4/(hc)^3 + \; + V(\Phi)/(hc)^3 \tag{7.4}$$

Recapitulate the Figures of (Lukács, 1993a). The quartic potential possesses a central peak. Until the average energy (roughly the temperature) is above this peak, $<\Phi> = 0$. For lower energies the state sits down to one of $\Phi_{\pm}$. There is a difference ΔV between the two cases. In addition, in the spontaneous symmetry breaking the vector bosons coupled to Φ get masses proportional to $<\Phi>$. Let us assume that at least some of these masses are $>T$. Then we have *two different* equations of state, one for the high temperature phase in which the state of Φ is mirror-symmetric ($<\Phi> = 0$):

$$p_o(T) = (N_o\pi^2/90)T^4/(hc)^3 - \Delta V/(hc)^3 \tag{7.5}$$

and one for the low temperature phase where Φ sits in an asymmetric minimum

$$p_+ = ((N_o\text{-}\Delta N)\pi^2/90)T^4/(hc)^3 \tag{7.6}$$

The two phases are in equilibrium at T_{eq} for which

$$p_o(T_{eq}) = p_+(T_{eq}) \tag{7.7}$$

whence

$$T_{eq} = (90\Delta V/\pi^2\Delta N)^{1/4} \tag{7.8}$$

If T is decreasing, there is a symmetry breaking at T_{eq}.

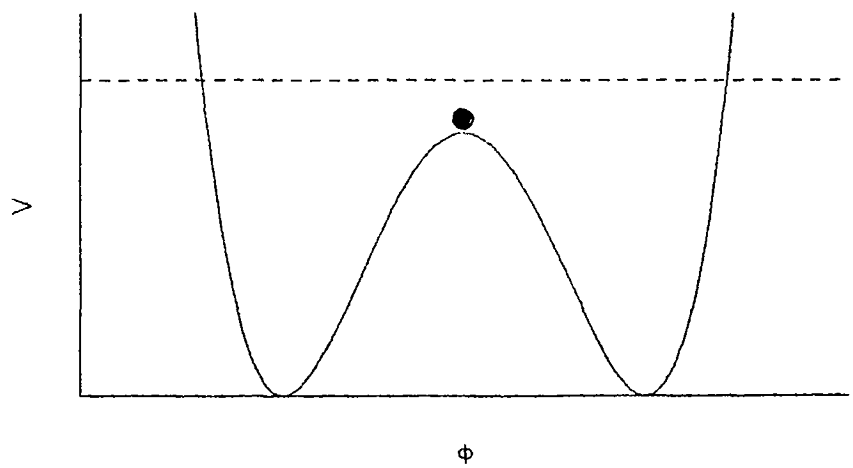

Figure 1: Just above T_{eq}.

Now consider a situation when T is just passing T_{eq} very rapidly. The scenario is sketched on Figures 1-3. Just above T_{eq} the symmetric state is stable. Just after passing the state starts to roll towards a side minimum, but if the cooling is very fast then there are states similar to Figure 2. That snapshot is a state in which T has substantially decreased but still $<\Phi>$ is moderate and ΔV is still almost the original. This is the supercooled symmetric state. Assume that it supercools to

$T_{eq}/3$. Then the thermal part of p has decreased by two orders of magnitude, so is practically negligible compared to ΔV. In this case, from (7.1-2)

$$\epsilon \approx -p \approx \Delta V \tag{7.9}$$

Then for $k = 0$ eqs. (4.3-4) give an exponential expansion

$$R = R_1 e^{t/\gamma} \; ; \; \gamma^2 = (3/8\pi)((\hbar c)^3 c^2/G\Delta V) \tag{7.10}$$

called *inflation*. During this inflation ϵ is roughly constant, while the volume increases, so energy is produced, while the total entropy is roughly constant because of the adiabatic change.

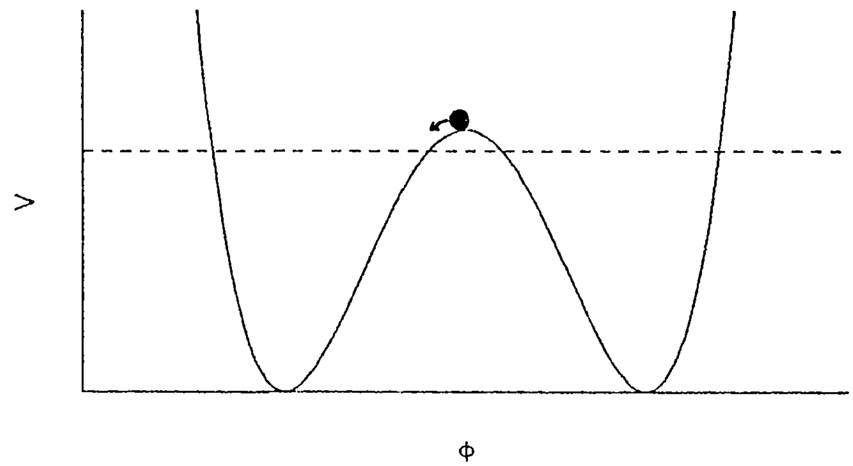

Figure 2: Just below T_{eq}. The state starts to roll down.

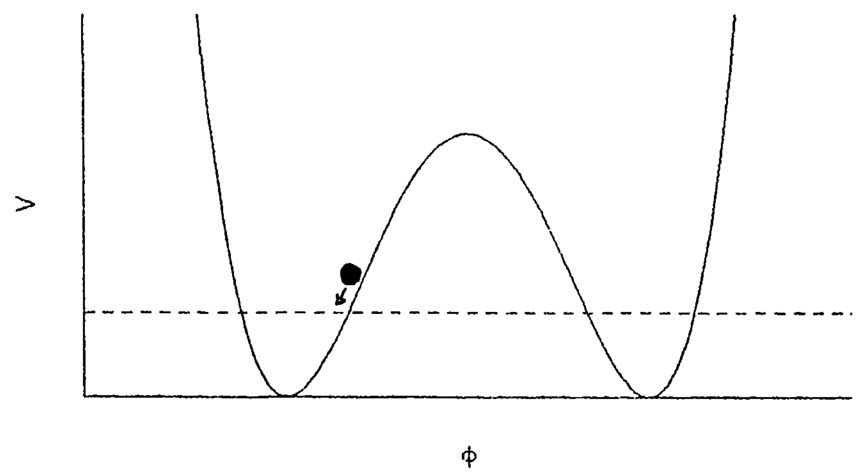

Figure 3: Well below T_{eq}. The state starts to settle down. Reheating will follow.

The supercooling ends after some Δt needed to reach the side minimum. Then the asymmetric phase is established. Comparing (7.4) and (7.6), the energy of the ground state becomes lower, so some energy must go again into thermal degrees of freedom, which is the reheating. This is a nonequilibrium process, producing an entropy $\Delta S \sim \Delta E/T_{eq}$, where ΔE is the energy produced in the inflation. If $\Delta t/\gamma \sim o(10^{1} - 10^{2})$, then the inflation can produce almost *any* increase in energy or entropy, e.g., the factor $\Delta S/S \sim 10^{87}$ needed to eliminate the fine tuning problem in (Guth, 1981).

This is only the simplest possible scenario, but for the present goal it is enough. Let us stop at this moment and summarize the effects of a spontaneous symmetry breaking preceded by a supercooling.

1) The symmetry of the actual state of some scalar bosons breaks down.
2) Some vector bosons get masses.
3) R jumps up, with substantial energy and entropy increase.
4) The inflation smoothens the existing spatial inhomogeneities, so *spatial* symmetries (uniformity) are restored.

Therefore in such phase transitions *internal* and *spatial* symmetries change oppositely: the spatial symmetry is restored on the account of the internal one.

8. ON GRAND UNIFICATION

Our particle physical measurements do not extend beyond 1000 GeV energy, except a few isolated reconstructed cosmic radiation events up to 10^{11} GeV. A very bold extrapolation, however, suggests a spontaneous symmetry breaking at $\sim 10^{15}$ GeV. Namely, the low energy particle physics seems to have 3 independent interactions

1) Electromagnetism, with symmetry group $U(1)$.
2) Weak interaction, with symmetry group $SU(2)$.
3) Quantum chromodynamics (whose peripheral effect is the strong interaction), with symmetry group $SU(3)$, *for colours*. (The earlier literature mentioned another $SU(3)$ symmetry for *flavours*, i.e., among different kinds of hadrons. This symmetry is *approximate*, and has no intimate relationship to the fundamental symmetries. It is a consequence of the fact that the hadrons are composed from quarks.)

Now, with increasing energy the 3 coupling constants seem to converge, and it is possible that all the 3 interactions belong to a common $SU(5)$ group, (the smallest one with all the 3 as subgroups) (Langacker, 1981). The resulting theory is called *Grand Unification*, in the simplest extrapolation the parameters of the theory can be calculated from low energy data, and the theory *may* be valid just above 10^{15} GeV, although up to now no predicted consequence has been observed.

In an $SU(5)$ symmetric theory there is no qualitative difference between quarks and leptons. They can continuously be transformed into each other by exchanging massless vector bosons of appropriate charges. Therefore in Grand Unification only two charges are conserved, namely

electric charge
baryon number − lepton number.

If anything new happens between 10^{15} GeV and Planck energy, we cannot guess it from here.

9. AGAIN ON INITIAL CONDITIONS

Now we can rediscuss the problem of initial conditions. We start at Planck temperature (or energy or energy density) which we take $t = 0$ (indistinguishable from $t = t_{Pl}$. A *naïve* extrapolation of the present Universe resulted in (4.13), which led to the unnatural initial condition $R = 0.1$ cm at $T = T_{Pl}$. However, as we have seen, any part of the present entropy or energy may have been produced in an inflation. So there is no evidence against

$$R_0 \sim R_{Pl}$$
$$T_0 \sim T_{Pl}$$

Hence

$$S_0 \sim 1.$$

For the charges, all observations suggest electric neutrality, so

$$Q = 0.$$

The other conserved quantity of Grand Unification is the (baryon-lepton) number. Let us count the particles in our neighbourhood. The overwhelming majority consists of a few kinds of particles as protons, neutrons, electrons, neutrinos (all with antiparticles) and photons. Photons do not carry any kind of charge. The conserved quantity Δ can be calculated presently as

$$\Delta \approx (N_p + N_n - N_e - N_\nu) - (\text{antipart.}) \tag{9.1}$$

Now, $N_p = N_e$ (neutrality; for both particles the antiparticles are negligible). Neutrons are stable only in nuclei, all the matter is practically 90% *H* and 10% *He*. Hence $N_n \approx N_p/5$. Neutrino numbers cannot be measured because low energy neutrinos practically do not interact with the measuring apparatuses. But cosmological models suggest $N_\nu \sim N_e$, and then a *slight* excess of *anti*neutrinos can compensate N_n. Therefore there is no evidence against

$$\Delta = 0.$$

In addition, for the nonconserved numbers one may assume a fully symmetric initial state.

This starting Universe cannot be distinguished from nothing at all for a period t_{Pl} due to the uncertainty principle. One may hope that these initial conditions end in our Universe; we cannot check this, because *(i)* the number factors in T_0 and R_0 are still unknown, *(ii)* we cannot calculate near T_{Pl}, *(iii)* the fine tuning problem does not exist anymore due to inflations, and *(iv)* we do not know how many phase transitions happened until now. Anyway, this is a nice initial Universe and the lest arbitrary one.

But the starting Universe was of the mass and size of a single quantum fluctuation.

THIS INITIAL 'PLANCK' UNIVERSE CANNOT HAVE CONTAINED ANY PARTICLES, OR EVEN ANY IDENTIFIABLE PARTS.

The less arbitrary initial Universe cannot have been anything else than an *Einheit*, being an undifferentiated unity.

10. A POSSIBLE SCENARIO

We do not know which kind of Grand Unification is true, or if any of them is true at all. We do not know anything even in this extent at higher energies. However, hypothetical scenarios connecting the discussed initial Universe to the present one can be drawn in a more or less qualitative manner. Steps of such scenarios were discussed in (Kämpfer, Lukács, and Paál, 1990); here we concentrate only on the changes of symmetries and those in the individuality of parts.

1) *Beginning*. The Universe is one, indivisible, elementary unit, with Planck data. Maximal symmetry: no observable spatial structure; fields in symmetric states, conserved charges at 0 values. No parts. The whole Universe is *one* 'particle'.

2) *Just after beginning*. Some energy production must have happened, otherwise the Universe would have fluctuated back to its absence after t_{Pl}. Maybe a delayed phase transition happened, maybe Hawking radiation preserved the Universe; we do not know.

3) *Somewhere not far (?) below* T_{Pl}. Possibility for individual particles to be more or less separated and to drop out. (By uncorrelated fluctuations in the growing volume?)

4) *Somewhere between* T_{Pl} *and* 10^{15} *GeV*. Supersymmetry breaks down. Boson and fermion members of superpairs become distinguishable.

5) *Just above* 10^{15} *GeV*. Quantum fluctuations may start to create spatial inhomogeneities.

6) *In a range downwards from* 10^{15} *GeV*. Supercooling, inflation. Again a substantial part of the energy is created. This energy is new and do not necessarily follow the original pattern of inhomogeneity. Spatial symmetry is then restored (in some extent). Finally $SU(5)$ symmetry breaks down to $SU(3)\times SU(2)\times U(1)$. Afterwards baryons and leptons are practically separately conserved. The actual state is still symmetric for particle-antiparticle reflection.

7) *Just after the* SU*(5) breaking*. An effective (spontaneous?) *CP* breaking of the broken Grand Unification leads to faster decay of antiquarks and antileptons? (This point is rather obscure, for the details see (Barrow, 1983).)

8) *Down to* $\sim$*1000 GeV.* Probably no phase transition. Fluctuations may generate inhomogeneities but no macroscopic permanent structures exist. As far as we guess the present *point-like* particles already exist. The antiparticle/particle ratio continuously goes down.

9) *Somewhere in the range 1000 GeV.* The mixed $SU(2)\times U(1)$ Weinberg-Salam interaction breaks apart to the familiar electromagnetism and weak interaction. Again spontaneous symmetry breaking happens for some scalar bosons, and the weak coupling bosons W and Z (observed) get masses. The transition may be of first order, but already the time scale of the expansion and cooling is cca. 10^{-10} s, longer than the characteristic time of the electromagnetic interactions. So no substantial supercooling and inflation is expected.

10) *Between 1000 GeV and 200 MeV.* Standard expansion and cooling. $SU(3)$ symmetric state for *flavour abundances.*

11) *At cca 200 MeV.* Hadronisation of quarks in a probably first order transition (between 8 and 15 μs from beginning). Strong fluctuations, correlated in volumes containing $\sim$1 solar mass. At the end quarks and gluons are confined, protons, neutrons, some hyperons, and mesons are present. The state is no more $SU(3)$ symmetric for *flavours* (hyperons are less abundant), but there is still an $SU(2)$ symmetry of flavours (equal numbers of protons and neutrons).

12) *Between 200 and 1 MeV.* Standard expansion and cooling. Still particle-antiparticle symmetry *for leptons*.

13) *At 1 MeV.* ($\sim$ 1 s.) Neutrons start to decay but the lifetime is $\sim$1000 s.

14) *At 0.5 MeV.* Annihilation of e^-e^+ pairs. Only the slight (10^{-8}) e^- surplus survives. On the account of the actual CP symmetry the spatial homogeneity somewhat restores.

15) *Somewhere at 0.1 MeV.* ($\sim$1000 s.) The free neutrons vanish by decay; bound ones in d, t, γ and α survive and build up the primordial helium. Since it is detected, we have observational evidence about separate autonomous parts of the Universe from t = 1000 s.

16) *Afterwards, for a while.* Some nucleosynthesis. The matter is an $e+p$ plasma, opaque for photons.

17) *At* $\sim$*1 eV.* (300000 ys after beginning). Neutral atoms build up. The matter becomes transparent.

18) *Afterwards.* Macroscopic structures become possible. First proto-galaxy clusters are stable. Henceforth macroscopic homogeneity breaks down to random (homogeneous isotropic) distributions of spheres. This epoch ends with the start of fusion in protostars, and no great change happens until present in symmetry.

19) *Present.* No symmetry on human scales, spherical symmetry on terrestrial scales, more or less homogeneous isotropic distribution on large scales.

20) *Future.* ???. (See also (Paál, 1993) for a *possible* future.)

11. CONCLUSIONS

The present paper had a double goal. First, we have emphasized that our knowledge about the *early* Universe is very limited. *This* statement of an expert is probably accepted by anybody else. However, hence other logical statements follow. For example, familiar models may not be applicable for the early Universe; our notions may be alien from those situations; very serious problems sleep almost undisturbed under many subsequent layers of familiar and quantitative problems. These statements are rather negative, and one can always question the validity of a model under strange circumstances, but now we wanted only to demonstrate that cosmology still needs continuous contemplation about its fundamental notions. However, in any definite time there is a *best* description, and our second goal was to show up the outlines of such a description, specially from the viewpoint of symmetries. We have shown that *(i)* the present Universe can be obtained from a maximally symmetric primordial one; *(ii)* the breakings of symmetries were spontaneous, therefore natural, and *(iii)* at some symmetry breakings internal and spatial symmetries changed oppositely (the breakdown of internal ones drove processes partially restoring the spatial ones).

In a very symmetric Universe complicated organisms (as ourselves) could not exist. In a very asymmetric one minds would have great difficulties in understanding. The present Universe is convenient for rational beings (according to observations). We do not know if it remains so forever. Present General Relativity does not predict further deterioration of the remainder of the spatial symmetries, and present particle physical theories do not predict the breakdown of the present effective $SU(3)\times SU(2)\times U(1)$ symmetry. However, they have not been constructed to predict it in the lack of any indication from measurements. One cannot exclude the existence of further scalar bosons with quartic potentials (Linde, 1984); if they exist, further symmetry breakings may happen. But we do not know to which vector boson they are coupled and with what strength.

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CHIRALITY IN THE ELEMENTARY INTERACTIONS

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Abstract: *General survey of the chirality (parity violation) of the weak interaction is given, with some mathematical details and experimental consequences.*

1. ELEMENTARY PARTICLES AND THEIR INTERACTIONS

The elementary particles are the 'smallest', most fundamental, structureless and universal building blocks of our world. According to the generally accepted theory of the elementary particles (the so called *Standard Model*), we classify them into 4 groups: leptons, quarks, intermediate bosons and the Higgs scalar. Both the leptons and the quarks (see Tables 1 and 2) are classed into 3 families. Each family contains 2 types of elementary particles. In the lepton families one of the particles has zero charge and approximately zero mass. They are called neutrinos. The second particle species have an electric charge of -1 (in the unit of the electron's charge), and their masses are shown in Table 1, (1 GeV is approximately the mass of the Hydrogen atom; 1 GeV = 1000 MeV). The electron is stable, but the muon and the tau decay into other particles after a mean lifetime of 2×10^{-6} s (2 microseconds), and 3×10^{-13} s, respectively. All these leptons carry precisely the same amount of spin (intrinsic angular momentum): 1/2 . For each lepton there is a corresponding antilepton. The antiparticles have the same mass and spin as their respective particles but carry opposite values for other properties, such as electric charge. The antiparticle of the electron is called positron.

The lepton families are distinguished mathematically by lepton numbers; for example, the electron and the electron neutrino are assigned electron number 1, muon number 0 and tau number 0. Antileptons are assigned lepton numbers of the opposite sign. Although some of the leptons decay into other leptons, the total lepton number of the decay products is equal to that of the original particle. For example, the muon decays into an electron, an electron antineutrino and a muon neutrino: $\mu^- \rightarrow e^- \ \bar{\nu}_e \ \nu_\mu$. Total lepton number is unaltered in the transformation.

Electric charge must be conserved in all interactions, and the electron is the lightest charged particle. Therefore it is absolutely stable.

The quarks are also classified into 3 families (see Table 2). Their fractional charges (1/3 and 2/3 of the electron's charge) are never observed, because they form combinations in which the sum of their charges is integer. Barions consist of 3 quarks, the mesons consist of a quark-antiquark pair. For example, the most well-known barions, the proton and the neutron contain the light u and d quarks: $p - uud$; $n - udd$.

The top quark has not been observed when writing this article in the high energy experiments. If it exists, it's mass should be in the 100 GeV - 180 GeV interval, derived from theoretical and experimental investigations.

The six leptons and six quarks (with their antiparticles) are now thought to be the fundamental constituents of matter. Four forces (interactions) govern their relations: electromagnetism, gravity, strong and weak interactions. These interactions of the leptons and quarks are mediated by the intermediate bosons (see Table 3). The strong interaction between two quarks is mediated by the gluons, the electromagnetic force between two electrically charged particles is mediated by the photon. The heavy W^+, W^- and Z bosons are responsible for the weak interaction. The existence of the graviton is uncertain.

Name	Letter	Mass	Charge
electron neutrino	ν_e	≈ 0	0
electron	e^-	0.5 MeV	-1
muon neutrino	ν_μ	≈ 0	0
muon	μ^-	106 MeV	-1
tau neutrino	ν_τ	≈ 0	0
tau	τ^-	1.78 GeV	-1

Table 1: Leptons.

Name	Letter	Mass	Charge
up	u	≈ 4 MeV	2/3
down	d	≈ 6 MeV	-1/3
charm	c	1.5 GeV	2/3
strange	s	≈ 150 MeV	-1/3
top	t	?	2/3
bottom	b	5 GeV	-1/3

Table 2: Quarks.

There is another hypothetical particle that has not been observed experimentally yet: the Higgs particle. It is neutral (0 electric charge), and its spin is also 0. According to the generally accepted theory of the elementary particles (the *Standard Model*), the interaction of the Higgs with the leptons, quarks, $W^\pm$, Z bosons is responsible for the masses of these particles. They get their masses through the so called spontaneous symmetry breaking (for further details see e.g., (Halzen and Martin, 1984; Quigg, 1985; Lukács, 1993a)).

Interaction	Intermediate boson	Mass	Spin
Strong	gluons	0	1
Electromagnetic	photon	0	1
Weak	$W^{\pm}$, Z	80 GeV, 91 GeV	1
Gravity	graviton (?)	0	2

Table 3: Fundamental interactions and their intermediate bosons

2. SYMMETRY PRINCIPLES IN PHYSICS

The notion of symmetry is central to the theories of the elementary particles. A transformation which does not alter the laws of nature is called symmetry of the nature (or symmetry transformation). The phenomena (events) of nature take place exactly in the same manner in the transformed world as in the original world. For example, the gravity or the Coulomb force between two particles has translation symmetry: the $F = x_1 - x_2$ force is unaltered after the $x \to x + a$ translation (here x_1 and x_2 are the coordinate vectors of the particles).

It is an experimental fact that certain physical quantities are not observable (unmeasurable). From these facts we can infer some symmetry principles, and from these principles we can deduce mathematically the conservation laws of nature (Lee, 1974). The table below contains some examples for these relations:

Not observable	Symmetry transformation	Conservation law
absolute space coordinate	space translation : $\underline{x} \to \underline{x} + \underline{a}$	momentum
absolute time	time translation : $t \to t + \tau$	energy
absolute direction in space	rotation: $\underline{x} \to R\underline{x}$	angular momentum
absolute right (left)	space reflection : $\underline{x} \to -\underline{x}$	parity
absolute phase of wave function	gauge transformation : $\psi \to e^{i\phi}\psi$	electric charge

Table 4: Unobservable quantities, symmetries and conservation laws

The symmetry principles in particle physics led to the discoveries of new laws of nature. For example, each of the four fundamental forces is now thought to arise from the invariance of a law of nature, such as the conservation of charge or energy, under a local symmetry operation, in which a certain parameter is altered independently at every point in space. The resulting theories are called gauge theories (Halzen and Martin, 1984; 't Hooft, 1980). The gauge group (local symmetry group) of the Standard Model is the $SU(3)_c \times SU(2)_L \times U(1)$ group.

3. SPACE REFLECTION SYMMETRY IN QUANTUM MECHANICS

Let ψ be an atomic wave function, $\hat{H}$ the Hamilton operator of the system, and $\hat{P}$ the space reflection operator:

$$(\hat{P}\,\psi)\,(\boldsymbol{x}) =: \psi'(\boldsymbol{x}) = \psi(-\boldsymbol{x})$$

If $\hat{H}(x) = \hat{H}(-x)$ (space reflection is symmetry of the system), and ψ is eigenfunction of the Hamiltonian ($\hat{H}\psi = E\psi$), then $\hat{P}\psi$ is also eigenfunction with the same eigenvalue (E), and $\hat{H}\hat{P} = \hat{P}\hat{H}$.

The eigenvalues of $\hat{P}$ are $P = \pm 1$ (because from $\hat{P}\psi = P\psi \Rightarrow \hat{P}^2\psi = P^2\psi = \psi$). P is called parity of the system. If the physical system has symmetry under space reflection, then its parity is conserved.

According to the empirical Laporte-rule, the atomic wave functions change their parity while the atom emits a photon. In 1927 Wigner showed that the Laporte-rule is a consequence of the space reflection symmetry of the electromagnetic interaction.

Various investigations of atomic and nuclear transitions demonstrated unambigously that both the electromagnetic and the strong interactions have exact space reflection symmetry (they are parity conserving).

4. THE PARITY VIOLATION (SPACE REFLECTION ASYMMETRY) OF THE WEAK INTERACTION

Not only the atomic wave functions, but also the elementary particles have parity (*intrinsic parity*), similarly to their *intrinsic angular momentum* (spin). The parity of the leptons, barions, mesons, and photon can be measured with elementary processes proceeding through the electromagnetic and strong interactions. For example, the photon and the $\pi^{\pm}$, π^0 mesons have negative intrinsic parity.

The weak interaction was supposed to be also parity conserving until 1956. Then Lee and Yang pointed out that the conservation of parity as a universal principle was very inadequately supported by experimental evidence. They were first led to this finding by consideration of the various decays of the K-meson. Both the $K \to \pi\pi$ and $K \to \pi\pi\pi$ decay modes had been observed, while the $\pi\pi$ final state had $P = +1$ parity, the $\pi\pi\pi$ state had $P = -1$.

Space reflection symmetry of the world would require the 'mirror reflected' world (with the $x \to -x$ transformation) to be indistinguishable from the original world. Therefore, to check the mirror symmetry, we have to carry out two experiments that are mirror images of each other. If mirror symmetry holds, they should give the same results. On Figure 1 below we show the layout of the famous Co^{60} experiment (accomplished in 1956).

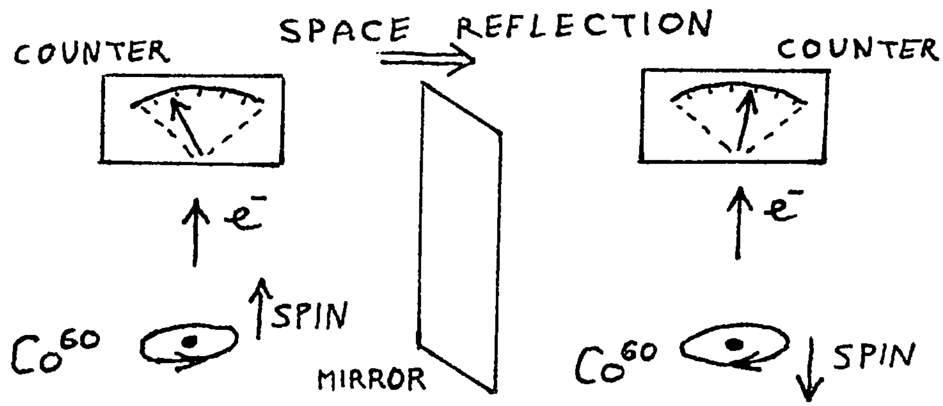

Figure 1: Co^{60} experiment.

The Co^{60} nucleus decays into Ni^{60} nucleus, electron (e^-) and electron antineutrino ($\bar{\nu}_e$). Mirror symmetry can be checked by measurement of the correlation between the outgoing electron's direction and the spin of the Co^{60}.

At the left hand side of Figure 1, the spin of the Co^{60} nucleus points upwards, at the right hand side downwards. This arrangement corresponds to mirror reflection. The different number of electrons going upwards in the two cases shows the violation of space reflection symmetry for the weak interaction.

Various experiments after 1956 showed that the spin of the antineutrino is always pointed towards its direction of motion (it is always right-handed), and the neutrino's spin is always opposite to the direction of motion (it is left-handed). This is another example of space reflection asymmetry (parity violation). After space reflection the left-handed neutrino would become right-handed, and right-handed neutrino does not exist in nature. World and mirror-world are distinguishable by physical experiments!

Let us introduce the notion of charge reflection (denoted by C). This transformation changes all particles to their antiparticles ($e^- \leftrightarrow e^+, \nu \leftrightarrow \bar{\nu}, ...$). The charge reflection is symmetry of the electromagnetic and strong interactions, and (similarly to space reflection) it is not symmetry of the weak interaction. The experiments show. however, that the combined CP transformation (charge + space reflections together) is very good symmetry of the weak interaction (for example, the right-handed $\bar{\nu}$ goes under this transformation into the existing left-handed ν).

In 1964 a small *CP* violation in *K*-meson decays was discovered. This *CP* violation could explain the matter-antimatter asymmetry of our world (according to the $SU(5)$ grand unified theory, the $X^{\pm}$ bosons with masses of $\approx 10^{15}$ GeV could have decayed asymmetrically into quarks-antiquarks and leptons-antileptons, about $t \approx 10^{-35}$ s after the big bang) (Wilczek, 1980; Lukács, 1993b).

5. MATHEMATICS OF THE WEAK INTERACTION AND OF PARITY VIOLATION

In the following we shall present some mathematical details of the weak interaction and its parity violation. First we introduce the notion of the Feynman-graph. These diagrams can illustrate the processes of the elementary particles, and also, one can use them to read the building blocks necessary for the calculations of the measurable quantities (according to some definite mathematical rules). One can deduce from each Feynman-graph a complex number: the amplitude of the corresponding elementary process that is illustrated by the graph. This complex amplitude can be used to calculate the measurable quantities of the process.

Figure 2 below shows one of the simplest Feynman-graphs of the $e^-e^- \rightarrow e^-e^-$ collision process. The interaction of the electrons is mediated by the photon (γ), which is unobservable here (virtual).

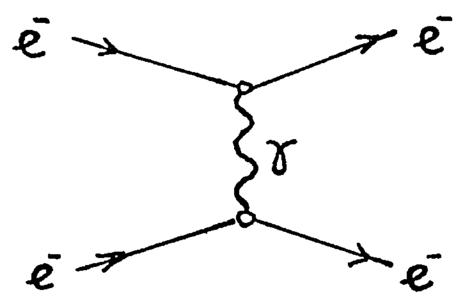

Figure 2: Simple Feynman-graph of the $e^-e^- \rightarrow e^-e^-$ process

We need 3 types of mathematical expressions for the calculation of the complex amplitudes :

- wave functions
- propagators
- vertices

5.1 Wave functions

They correspond to the outer lines of the graphs (on Figure 2 they are the e^- lines). The $\psi(x,t)$ wave function of the electron has 4 components:

$$\begin{pmatrix} \psi_1(x,t) \\ \psi_2(x,t) \\ \psi_3(x,t) \\ \psi_4(x,t) \end{pmatrix}$$

Here x is the space vector, t is for time. The space-time dependence of ψ for free electron is given by the Dirac-equation :

$$\left(i\gamma^0 \frac{\partial}{\partial t} + i \sum_{k=1}^{3} \gamma^k \frac{\partial}{\partial x^k} - m_e \right) \psi(\underline{x}, t) = 0$$

m_e is the electron mass, γ^0, γ γ, γ are the Dirac-matrices :

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad ; \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad , \ldots$$

(for further details see e.g., (Halzen and Martin, 1984)).

5.2 Propagators

They correspond to the inner lines (virtual particles, which are not observed at the given process). For particles with mass M and $p = (E, \boldsymbol{p})$ four-momentum (E is the energy, $\boldsymbol{p}$ the 3-momentum of the particle) the propagator is proportional to

$$1 / (p^2 - M^2)$$

The wave functions and the propagators characterize the particles, but not their interactions!

5.3 Vertices

The vertices are referred to the points where the outer and inner lines meet each other. They are complex matrices, and they determine the interactions of the particles in the process.

The vertex of the electromagnetic interaction is given by the

$$ie\gamma^\mu \; (\mu = 0, 1, 2, 3)$$

4×4 complex matrices ($e^2 \approx 4\pi/137$). This vertex corresponds to those points of the Feynman-graphs where 2 electron lines and a photon line meet each other:

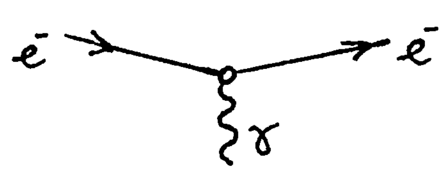

Figure 2a

It can be shown that this kind of interaction, given with the γ^μ ($\mu = 0, 1, 2, 3$) matrices, is parity conserving.

Let us now consider the $\nu_e e^- \to \nu_e e^-$ collision process. One of the Feynman-graphs can be seen below:

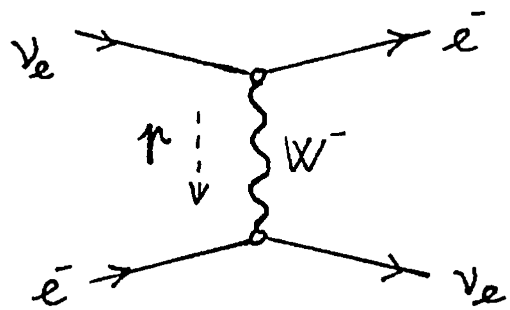

Figure 3: Feynman-graph for the $\nu_e e^- \to \nu_e e^-$ process.

The collision here is mediated by the W boson. The complex amplitude corresponding to this graph contains the

$$1 / (p^2 - M_W^2)$$

factor, coming from W propagator. The W mass is very large: $M_W \approx 80\ GeV = 80 \times 10^9\ eV$, therefore the above propagator factor is very small (p^2 is negative here). This explains the fact that the weak interaction, mediated by the heavy W boson, is very weak at small energies.

The $\nu_e e^- W$ vertex (corresponding to the points of the above graph where the lines of these particles meet each other) has the following form:

$$ic_W \gamma^\mu (1 - \gamma_5) \quad .$$

where :

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The presence of the γ_5 matrix is responsible for the parity violation of the weak processes!

The weak interaction vertices of the quarks have the above form, but with different c_W numbers :

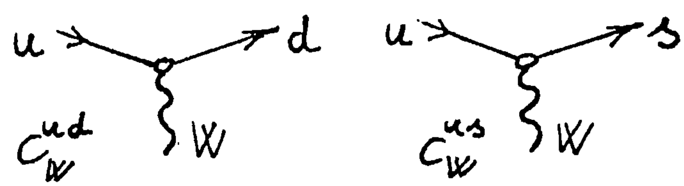

Figure 3a

The neutron decay ($n \to pe^- \bar{\nu}_e$), for example, can be reduced to $d \to ue^- \bar{\nu}_e$ quark decay (with some complications coming from the strong interaction of the quarks and gluons).

The $e^-e^- \to e^-e^-$ collision process can be mediated not only by photon, but also by the Z boson. The corresponding two Feynman-graphs :

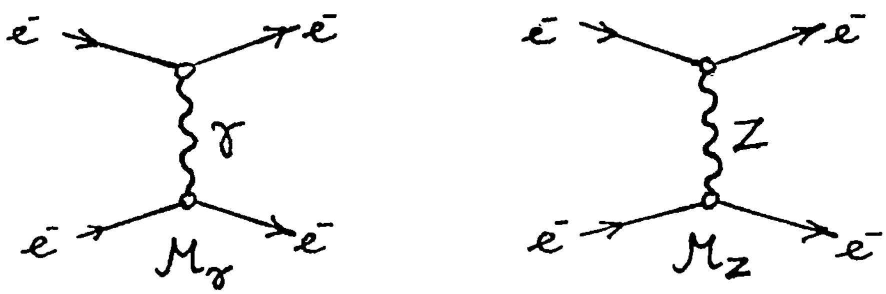

Figure 4: Photon and Z exchange graphs of the $e^-e^- \to e^-e^-$ process

For the calculation of the observable quantities of the $e^-e^- \to e^-e^-$ process we have to add the two complex numbers coming from the two graphs :

$$M = M_\gamma + M_Z$$

The eeZ vertex has the following form :

$$i\,(c_V\gamma^\mu - c_A\gamma^\mu\gamma_5)$$

The e, c_W, c_V, c_A coupling constants are real numbers, and have the same order of magnitude. The uuZ and ddZ vertices have similar forms, with different c_V and c_A constants. The propagator factors in the two amplitudes are the following :

$$photon\ (\gamma) \to 1/p^2\ ;\quad Z\ boson \to 1/(p^2 - M_Z^2)$$

The mass of the Z boson is very large ($M_Z \approx 91\ GeV = 91 \times 10^9\ eV$), therefore at small energies (where $|p^2| << M_Z{}^2$) the M_Z amplitude is very small compared to M_c:

$$|M_Z| << |M\gamma|$$

In the atoms, the atomic electrons interact with the quarks in the nucleus via both photon and Z exchange:

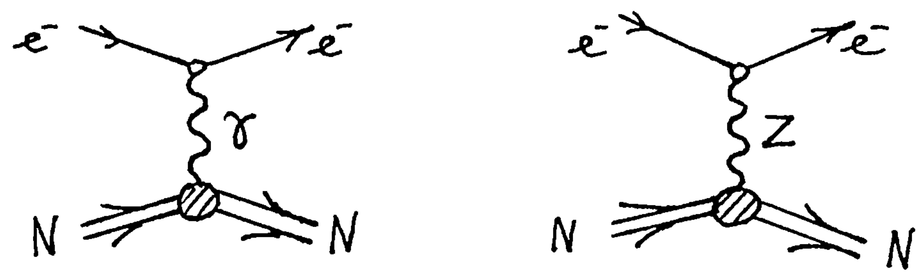

Figure 5: Photon and Z exchange graphs for the electron-nucleus interaction

The N nucleus contains neutrons (with *udd* quarks) and protons (with *uud* quarks). The *eeZ*, *uuZ* and *ddZ* interactions are parity violating (due to the γ_5 matrix in the vertices). Therefore, there are parity violating effects in atomic processes (Zeldovich, 1959). These effects are very small (due to the large mass of the Z boson), but can be observed. We mention that the c_W, c_V, c_A constants in the above vertex formulas are completely predicted by the $SU(2)_L \times U(1)$ unified theory of electromagnetic and weak interactions (Weinberg-Salam model) (Halzen and Martin, 1984).

6. EXPERIMENTAL DEMONSTRATION OF THE ATOMIC PARITY VIOLATION

The effect of the small parity violating contribution of the Z boson exchange in atomic processes can be shown by measurement of the optical activity of atoms (for other methods, see (Bouchiat and Pottier, 1984)).

Light is transverse wave motion – the electric field vector vibrates perpendicularly to its direction of propagation. The vibration can be arranged to take place in only one direction. This is the linearly polarized light. The direction of propagation and the electric field line determine the polarization plane. When the electric field vector rotates along a circle, the polarization of the light beam is called circular. The circularly polarized light contains photons with definite angular momentum (right-or left-handed photons). The linearly polarized beam is superposition of right- and left-handed circularly polarized beams with equal electric field amplitudes.

If a medium interacts differently with the right-handed and the left-handed photons, we call it optically active. When a linearly polarized light beam passes through such a medium, the polarization plane of the beam is rotated through some angle, and the beam emerges from the medium linearly polarized in a different direction. This rotation of the polarization plane is the consequence of the phase delay between the right-handed and left-handed circularly polarized beams.

Many crystals and molecular compounds exhibit rather large optical activity. This is due to the asymmetric arrangement of their atoms. The mirror images of these crystals and compounds rotate the polarization plane in the opposite direction and with the same angle. Optical activity here has nothing to do with parity violation.

The mirror image of a gas of atoms, however, is identical with the original gas. With mirror symmetric elementary interactions the atoms look the same in a mirror as they do in reality. Therefore, any optical rotation observed in an atomic gas is not caused by handedness (left-right asymmetry) in the geometry of the atoms, as it is in molecular gas, but by the handedness embodied in the laws of nature that govern the weak force.

The angle of optical rotation predicted by the electroweak theory (Weinberg-Salam model) is extremely small, about 10^{-5} degree under the most favourable experimental circumstances. The rotation degree enhances roughly with the cube of the atomic number (Bouchiat and Pottier, 1984). Therefore, the heavy atoms are preferred from the experimental point of view. Unfortunately, the theoretical calculations are rather difficult for the heavy atoms.

The first experiments performed in 1976-78 in Washington and Oxford with atomic Bismuth showed serious discrepancy between the predictions of the Weinberg-Salam model and the experimental results (Baird *et al.*, 1976; Sandars, 1977). The results of subsequent experiments carried out in 1980-82 were, however, in good agreement with this model (Bouchiart and Pottier, 1984; Barkov, 1981), which is nowadays the generally accepted unified theory of the electromagnetic and weak interactions (for historical details see Pickering, 1984, p. 294).

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