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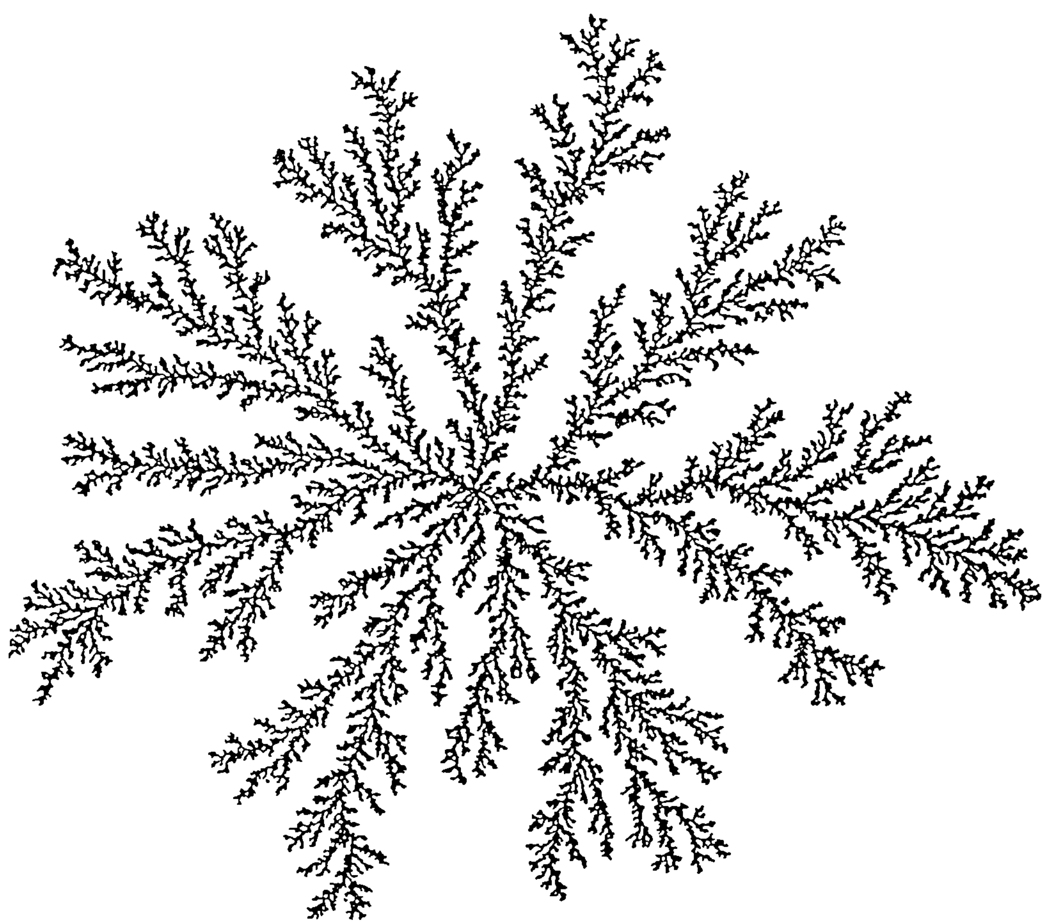

DLA fractal cluster of 10^6 particles

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CHIRALITY IN THE ELEMENTARY INTERACTIONS

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Abstract: *General survey of the chirality (parity violation) of the weak interaction is given, with some mathematical details and experimental consequences.*

1. ELEMENTARY PARTICLES AND THEIR INTERACTIONS

The elementary particles are the 'smallest', most fundamental, structureless and universal building blocks of our world. According to the generally accepted theory of the elementary particles (the so called *Standard Model*), we classify them into 4 groups: leptons, quarks, intermediate bosons and the Higgs scalar. Both the leptons and the quarks (see Tables 1 and 2) are classed into 3 families. Each family contains 2 types of elementary particles. In the lepton families one of the particles has zero charge and approximately zero mass. They are called neutrinos. The second particle species have an electric charge of -1 (in the unit of the electron's charge), and their masses are shown in Table 1, (1 GeV is approximately the mass of the Hydrogen atom; 1 GeV = 1000 MeV). The electron is stable, but the muon and the tau decay into other particles after a mean lifetime of 2×10^{-6} s (2 microseconds), and 3×10^{-13} s, respectively. All these leptons carry precisely the same amount of spin (intrinsic angular momentum): 1/2 . For each lepton there is a corresponding antilepton. The antiparticles have the same mass and spin as their respective particles but carry opposite values for other properties, such as electric charge. The antiparticle of the electron is called positron.

The lepton families are distinguished mathematically by lepton numbers; for example, the electron and the electron neutrino are assigned electron number 1, muon number 0 and tau number 0. Antileptons are assigned lepton numbers of the opposite sign. Although some of the leptons decay into other leptons, the total lepton number of the decay products is equal to that of the original particle. For example, the muon decays into an electron, an electron antineutrino and a muon neutrino: $\mu^- \rightarrow e^- \ \bar{\nu}_e \ \nu_\mu$.Total lepton number is unaltered in the transformation.

Electric charge must be conserved in all interactions, and the electron is the lightest charged particle. Therefore it is absolutely stable.

The quarks are also classified into 3 families (see Table 2). Their fractional charges (1/3 and 2/3 of the electron's charge) are never observed, because they form combinations in which the sum of their charges is integer. Barions consist of 3 quarks, the mesons consist of a quark-antiquark pair. For example, the most well-known barions, the proton and the neutron contain the light u and d quarks: $p - uud$; $n - udd$.

The top quark has not been observed when writing this article in the high energy experiments. If it exists, it's mass should be in the 100 GeV - 180 GeV interval, derived from theoretical and experimental investigations.

The six leptons and six quarks (with their antiparticles) are now thought to be the fundamental constituents of matter. Four forces (interactions) govern their relations: electromagnetism, gravity, strong and weak interactions. These interactions of the leptons and quarks are mediated by the intermediate bosons (see Table 3). The strong interaction between two quarks is mediated by the gluons, the electromagnetic force between two electrically charged particles is mediated by the photon. The heavy W^+, W^- and Z bosons are responsible for the weak interaction. The existence of the graviton is uncertain.

Name	Letter	Mass	Charge
electron neutrino	ν_e	≈ 0	0
electron	e^-	0.5 MeV	-1
muon neutrino	ν_μ	≈ 0	0
muon	μ^-	106 MeV	-1
tau neutrino	ν_τ	≈ 0	0
tau	τ^-	1.78 GeV	-1

Table 1: Leptons.

Name	Letter	Mass	Charge
up	u	$\approx$ 4 MeV	2/3
down	d	$\approx$ 6 MeV	-1/3
charm	c	1.5 GeV	2/3
strange	s	$\approx$ 150 MeV	-1/3
top	t	?	2/3
bottom	b	5 GeV	-1/3

Table 2: Quarks.

There is another hypothetical particle that has not been observed experimentally yet: the Higgs particle. It is neutral (0 electric charge), and its spin is also 0. According to the generally accepted theory of the elementary particles (the *Standard Model*), the interaction of the Higgs with the leptons, quarks, $W^\pm$, Z bosons is responsible for the masses of these particles. They get their masses through the so called spontaneous symmetry breaking (for further details see e.g., (Halzen and Martin, 1984; Quigg, 1985; Lukács, 1993a)).

Interaction	Intermediate boson	Mass	Spin
Strong	gluons	0	1
Electromagnetic	photon	0	1
Weak	$W^{\pm}$, Z	80 GeV, 91 GeV	1
Gravity	graviton (?)	0	2

Table 3: Fundamental interactions and their intermediate bosons

2. SYMMETRY PRINCIPLES IN PHYSICS

The notion of symmetry is central to the theories of the elementary particles. A transformation which does not alter the laws of nature is called symmetry of the nature (or symmetry transformation). The phenomena (events) of nature take place exactly in the same manner in the transformed world as in the original world. For example, the gravity or the Coulomb force between two particles has translation symmetry: the $F = x_1 - x_2$ force is unaltered after the $x \to x + a$ translation (here x_1 and x_2 are the coordinate vectors of the particles).

It is an experimental fact that certain physical quantities are not observable (unmeasurable). From these facts we can infer some symmetry principles, and from these principles we can deduce mathematically the conservation laws of nature (Lee, 1974). The table below contains some examples for these relations:

Not observable	Symmetry transformation	Conservation law
absolute space coordinate	space translation : $\underline{x} \to \underline{x} + \underline{a}$	momentum
absolute time	time translation : $t \to t + \tau$	energy
absolute direction in space	rotation: $\underline{x} \to R\underline{x}$	angular momentum
absolute right (left)	space reflection : $\underline{x} \to -\underline{x}$	parity
absolute phase of wave function	gauge transformation : $\psi \to e^{i\phi}\psi$	electric charge

Table 4: Unobservable quantities, symmetries and conservation laws

The symmetry principles in particle physics led to the discoveries of new laws of nature. For example, each of the four fundamental forces is now thought to arise from the invariance of a law of nature, such as the conservation of charge or energy, under a local symmetry operation, in which a certain parameter is altered independently at every point in space. The resulting theories are called gauge theories (Halzen and Martin, 1984; 't Hooft, 1980). The gauge group (local symmetry group) of the Standard Model is the $SU(3)_c \times SU(2)_L \times U(1)$ group.

3. SPACE REFLECTION SYMMETRY IN QUANTUM MECHANICS

Let ψ be an atomic wave function, $\hat{H}$ the Hamilton operator of the system, and $\hat{P}$ the space reflection operator:

$$(\hat{P}\psi)(\boldsymbol{x}) =: \psi'(\boldsymbol{x}) = \psi(-\boldsymbol{x})$$

If $\hat{H}(x) = \hat{H}(-\boldsymbol{x})$ (space reflection is symmetry of the system), and ψ is eigenfunction of the Hamiltonian ($\hat{H}\psi = E\psi$), then $\hat{P}\psi$ is also eigenfunction with the same eigenvalue (E), and $\hat{H}\hat{P} = \hat{P}\hat{H}$.

The eigenvalues of $\hat{P}$ are $P = \pm 1$ (because from $\hat{P}\psi = P\psi \rightarrow \hat{P}^2\psi = P^2\psi = \psi$). P is called parity of the system. If the physical system has symmetry under space reflection, then its parity is conserved.

According to the empirical Laporte-rule, the atomic wave functions change their parity while the atom emits a photon. In 1927 Wigner showed that the Laporte-rule is a consequence of the space reflection symmetry of the electromagnetic interaction.

Various investigations of atomic and nuclear transitions demonstrated unambigously that both the electromagnetic and the strong interactions have exact space reflection symmetry (they are parity conserving).

4. THE PARITY VIOLATION (SPACE REFLECTION ASYMMETRY) OF THE WEAK INTERACTION

Not only the atomic wave functions, but also the elementary particles have parity (*intrinsic parity*), similarly to their *intrinsic angular momentum* (spin). The parity of the leptons, barions, mesons, and photon can be measured with elementary processes proceeding through the electromagnetic and strong interactions. For example, the photon and the $\pi^{\pm}$, π^0 mesons have negative intrinsic parity.

The weak interaction was supposed to be also parity conserving until 1956. Then Lee and Yang pointed out that the conservation of parity as a universal principle was very inadequately supported by experimental evidence. They were first led to this finding by consideration of the various decays of the K-meson. Both the $K \rightarrow \pi\pi$ and $K \rightarrow \pi\pi\pi$ decay modes had been observed, while the $\pi\pi$ final state had $P = +1$ parity, the $\pi\pi\pi$ state had $P = -1$.

Space reflection symmetry of the world would require the 'mirror reflected' world (with the $x \to -x$ transformation) to be indistinguishable from the original world. Therefore, to check the mirror symmetry, we have to carry out two experiments that are mirror images of each other. If mirror symmetry holds, they should give the same results. On Figure 1 below we show the layout of the famous Co^{60} experiment (accomplished in 1956).

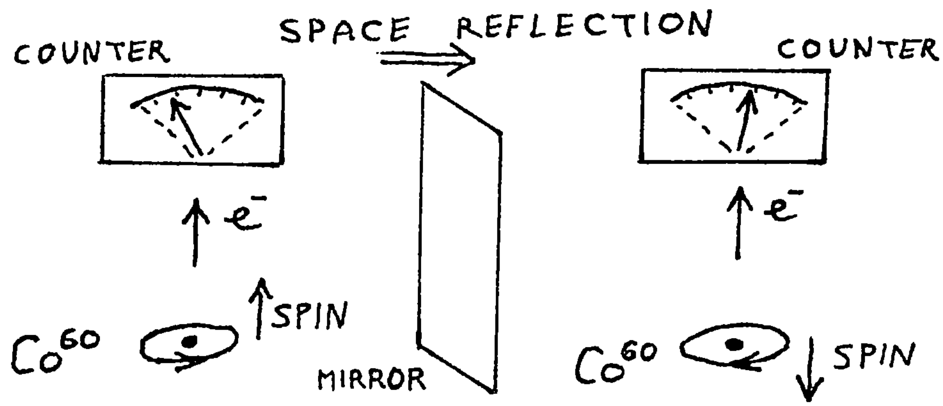

Figure 1: Co^{60} experiment.

The Co^{60} nucleus decays into Ni^{60} nucleus, electron (e^-) and electron antineutrino ($\bar{\nu}_e$). Mirror symmetry can be checked by measurement of the correlation between the outgoing electron's direction and the spin of the Co^{60}.

At the left hand side of Figure 1, the spin of the Co^{60} nucleus points upwards, at the right hand side downwards. This arrangement corresponds to mirror reflection. The different number of electrons going upwards in the two cases shows the violation of space reflection symmetry for the weak interaction.

Various experiments after 1956 showed that the spin of the antineutrino is always pointed towards its direction of motion (it is always right-handed), and the neutrino's spin is always opposite to the direction of motion (it is left-handed). This is another example of space reflection asymmetry (parity violation). After space reflection the left-handed neutrino would become right-handed, and right-handed neutrino does not exist in nature. World and mirror-world are distinguishable by physical experiments!

Let us introduce the notion of charge reflection (denoted by C). This transformation changes all particles to their antiparticles ($e^- \leftrightarrow e^+, \nu \leftrightarrow \bar{\nu}, ...$). The charge reflection is symmetry of the electromagnetic and strong interactions, and (similarly to space reflection) it is not symmetry of the weak interaction. The experiments show. however, that the combined CP transformation (charge + space reflections together) is very good symmetry of the weak interaction (for example, the right-handed $\bar{\nu}$ goes under this transformation into the existing left-handed ν).

In 1964 a small *CP* violation in *K*-meson decays was discovered. This *CP* violation could explain the matter-antimatter asymmetry of our world (according to the $SU(5)$ grand unified theory, the $X^{\pm}$ bosons with masses of $\approx 10^{15}$ GeV could have decayed asymmetrically into quarks-antiquarks and leptons-antileptons, about $t \approx 10^{-35}$ s after the big bang) (Wilczek, 1980; Lukács, 1993b).

5. MATHEMATICS OF THE WEAK INTERACTION AND OF PARITY VIOLATION

In the following we shall present some mathematical details of the weak interaction and its parity violation. First we introduce the notion of the Feynman-graph. These diagrams can illustrate the processes of the elementary particles, and also, one can use them to read the building blocks necessary for the calculations of the measurable quantities (according to some definite mathematical rules). One can deduce from each Feynman-graph a complex number: the amplitude of the corresponding elementary process that is illustrated by the graph. This complex amplitude can be used to calculate the measurable quantities of the process.

Figure 2 below shows one of the simplest Feynman-graphs of the $e^-e^- \rightarrow e^-e^-$ collision process. The interaction of the electrons is mediated by the photon (γ), which is unobservable here (virtual).

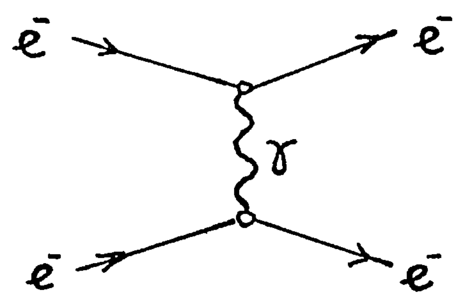

Figure 2: Simple Feynman-graph of the $e^-e^- \rightarrow e^-e^-$ process

We need 3 types of mathematical expressions for the calculation of the complex amplitudes :

- wave functions
- propagators
- vertices

5.1 Wave functions

They correspond to the outer lines of the graphs (on Figure 2 they are the e^- lines). The $\psi(x,t)$ wave function of the electron has 4 components:

$$\begin{pmatrix} \psi_1(x,t) \\ \psi_2(x,t) \\ \psi_3(x,t) \\ \psi_4(x,t) \end{pmatrix}$$

Here x is the space vector, t is for time. The space-time dependence of ψ for free electron is given by the Dirac-equation :

$$\left(i\gamma^0 \frac{\partial}{\partial t} + i \sum_{k=1}^{3} \gamma^k \frac{\partial}{\partial x^k} - m_e \right) \psi(\underline{x}, t) = 0$$

m_e is the electron mass, γ^0, γ γ , γ are the Dirac-matrices :

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad ; \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad , \ldots$$

(for further details see e.g., (Halzen and Martin, 1984)).

5.2 Propagators

They correspond to the inner lines (virtual particles, which are not observed at the given process). For particles with mass M and $p = (E, \boldsymbol{p})$ four-momentum (E is the energy, $\boldsymbol{p}$ the 3-momentum of the particle) the propagator is proportional to

$$1 / (p^2 - M^2)$$

The wave functions and the propagators characterize the particles, but not their interactions!

5.3 Vertices

The vertices are referred to the points where the outer and inner lines meet each other. They are complex matrices, and they determine the interactions of the particles in the process.

The vertex of the electromagnetic interaction is given by the

$$ie\gamma^\mu \ (\mu = 0, 1, 2, 3)$$

4×4 complex matrices ($e^2 \approx 4\pi/137$). This vertex corresponds to those points of the Feynman-graphs where 2 electron lines and a photon line meet each other:

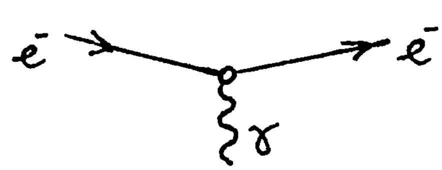

Figure 2a

It can be shown that this kind of interaction, given with the γ^μ ($\mu = 0, 1, 2, 3$) matrices, is parity conserving.

Let us now consider the $\nu_e e^- \to \nu_e e^-$ collision process. One of the Feynman-graphs can be seen below:

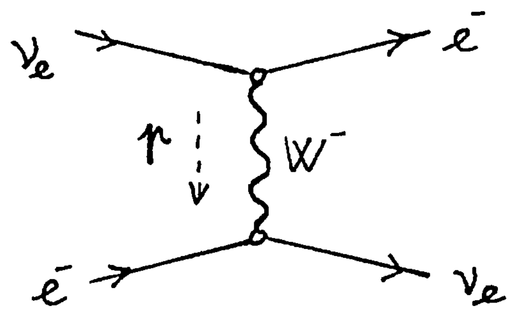

Figure 3: Feynman-graph for the $\nu_e e^- \to \nu_e e^-$ process.

The collision here is mediated by the W boson. The complex amplitude corresponding to this graph contains the

$$1 / (p^2 - M_W^2)$$

factor, coming from W propagator. The W mass is very large: $M_W \approx 80\ GeV = 80 \times 10^9\ eV$, therefore the above propagator factor is very small (p^2 is negative here). This explains the fact that the weak interaction, mediated by the heavy W boson, is very weak at small energies.

The $\nu_e e^- W$ vertex (corresponding to the points of the above graph where the lines of these particles meet each other) has the following form:

$$ic_w \gamma^\mu (1 - \gamma_5) \quad .$$

where :

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The presence of the γ_5 matrix is responsible for the parity violation of the weak processes!

The weak interaction vertices of the quarks have the above form, but with different c_W numbers :

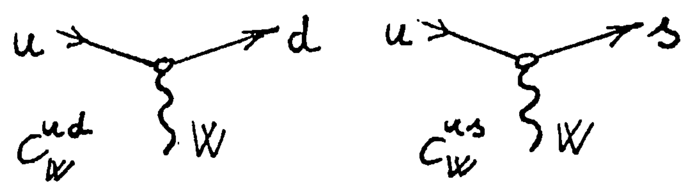

Figure 3a

The neutron decay ($n \to pe^-\bar{\nu}_e$), for example, can be reduced to $d \to ue^- \bar{\nu}_e$ quark decay (with some complications coming from the strong interaction of the quarks and gluons).

The $e^-e^- \to e^-e^-$ collision process can be mediated not only by photon, but also by the Z boson. The corresponding two Feynman-graphs :

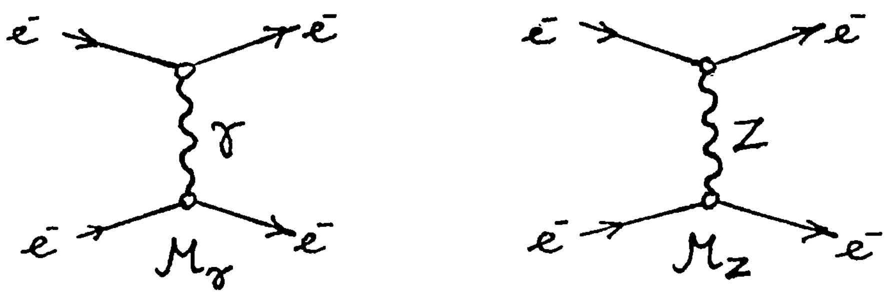

Figure 4: Photon and Z exchange graphs of the $e^-e^- \to e^-e^-$ process

For the calculation of the observable quantities of the $e^-e^- \to e^-e^-$ process we have to add the two complex numbers coming from the two graphs :

$$M = M_\gamma + M_Z$$

The eeZ vertex has the following form :

$$i\,(c_V\gamma^\mu - c_A\gamma^\mu\gamma_5)$$

The e, c_W, c_V, c_A coupling constants are real numbers, and have the same order of magnitude. The uuZ and ddZ vertices have similar forms, with different c_V and c_A constants. The propagator factors in the two amplitudes are the following :

$$\textit{photon}\ (\gamma) \to 1/p^2\ ; \quad Z\ \textit{boson} \to 1/(p^2 - M_Z^2)$$

The mass of the Z boson is very large ($M_Z \approx 91\ GeV = 91 \times 10^9\ eV$), therefore at small energies (where $|p^2| << M_Z^2$) the M_Z amplitude is very small compared to M_c:

$$|M_Z| << |M\gamma|$$

In the atoms, the atomic electrons interact with the quarks in the nucleus via both photon and Z exchange:

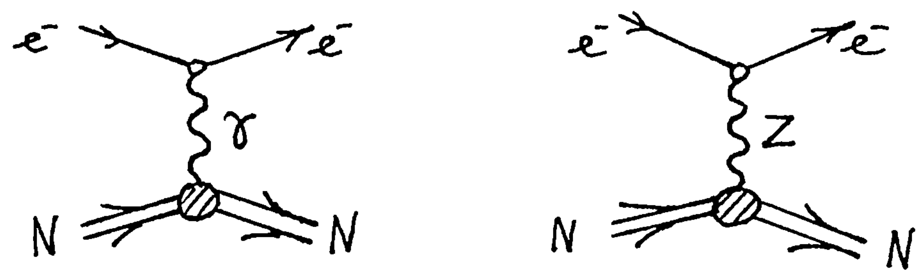

Figure 5: Photon and Z exchange graphs for the electron-nucleus interaction

The N nucleus contains neutrons (with *udd* quarks) and protons (with *uud* quarks). The *eeZ*, *uuZ* and *ddZ* interactions are parity violating (due to the γ_5 matrix in the vertices). Therefore, there are parity violating effects in atomic processes (Zeldovich, 1959). These effects are very small (due to the large mass of the Z boson), but can be observed. We mention that the c_W, c_V, c_A constants in the above vertex formulas are completely predicted by the $SU(2)_L \times U(1)$ unified theory of electromagnetic and weak interactions (Weinberg-Salam model) (Halzen and Martin, 1984).

6. EXPERIMENTAL DEMONSTRATION OF THE ATOMIC PARITY VIOLATION

The effect of the small parity violating contribution of the Z boson exchange in atomic processes can be shown by measurement of the optical activity of atoms (for other methods, see (Bouchiat and Pottier, 1984)).

Light is transverse wave motion – the electric field vector vibrates perpendicularly to its direction of propagation. The vibration can be arranged to take place in only one direction. This is the linearly polarized light. The direction of propagation and the electric field line determine the polarization plane. When the electric field vector rotates along a circle, the polarization of the light beam is called circular. The circularly polarized light contains photons with definite angular momentum (right-or left-handed photons). The linearly polarized beam is superposition of right- and left-handed circularly polarized beams with equal electric field amplitudes.

If a medium interacts differently with the right-handed and the left-handed photons, we call it optically active. When a linearly polarized light beam passes through such a medium, the polarization plane of the beam is rotated through some angle, and the beam emerges from the medium linearly polarized in a different direction. This rotation of the polarization plane is the consequence of the phase delay between the right-handed and left-handed circularly polarized beams.

Many crystals and molecular compounds exhibit rather large optical activity. This is due to the asymmetric arrangement of their atoms. The mirror images of these crystals and compounds rotate the polarization plane in the opposite direction and with the same angle. Optical activity here has nothing to do with parity violation.

The mirror image of a gas of atoms, however, is identical with the original gas. With mirror symmetric elementary interactions the atoms look the same in a mirror as they do in reality. Therefore, any optical rotation observed in an atomic gas is not caused by handedness (left-right asymmetry) in the geometry of the atoms, as it is in molecular gas, but by the handedness embodied in the laws of nature that govern the weak force.

The angle of optical rotation predicted by the electroweak theory (Weinberg-Salam model) is extremely small, about 10^{-5} degree under the most favourable experimental circumstances. The rotation degree enhances roughly with the cube of the atomic number (Bouchiat and Pottier, 1984). Therefore, the heavy atoms are preferred from the experimental point of view. Unfortunately, the theoretical calculations are rather difficult for the heavy atoms.

The first experiments performed in 1976-78 in Washington and Oxford with atomic Bismuth showed serious discrepancy between the predictions of the Weinberg-Salam model and the experimental results (Baird *et al.*, 1976; Sandars, 1977). The results of subsequent experiments carried out in 1980-82 were, however, in good agreement with this model (Bouchiart and Pottier, 1984; Barkov, 1981), which is nowadays the generally accepted unified theory of the electromagnetic and weak interactions (for historical details see Pickering, 1984, p. 294).

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