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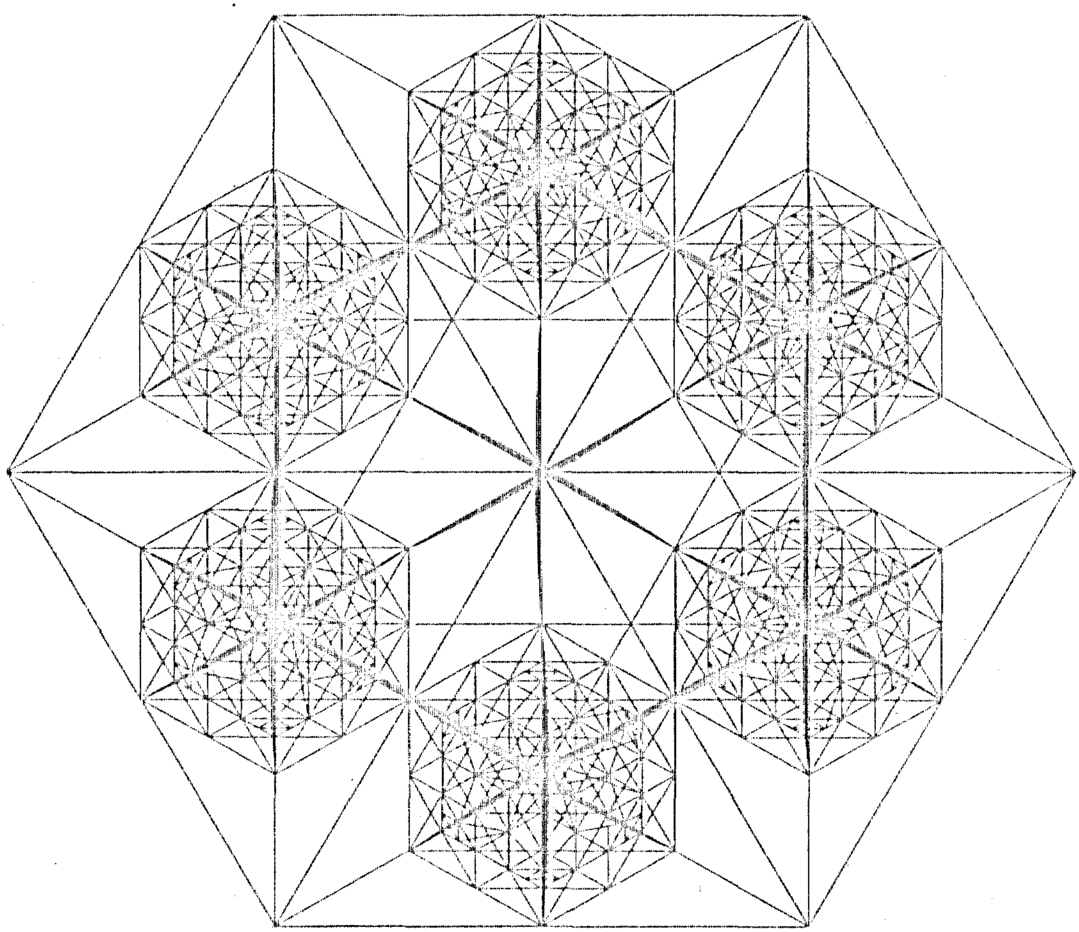

Large synergetical structure on hexagonal figure
Aerial view

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PERIODIC ANTISYMMETRY TILINGS

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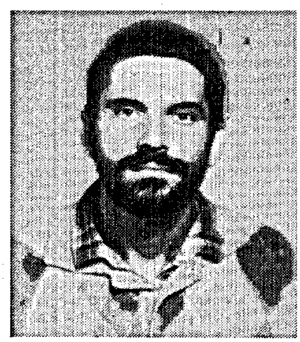
Shows: International Competition of Industrial Design and New Technology CEVISAMA-'87, Valencia, Spain, (1987); *Symmetry/Asymmetry*, National Gallery, Budapest, Hungary, (1989); 12th European Crystallographic Conference, Moscow, USSR, (1989); 2° Quadrienale Internazionale *La Ceramica Nell' Arredo Urbano*, Faenza, Italy, (1989).



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Abstract: *Using 2-multiple antisymmetry mosaics corresponding to plane symmetry group* pmm, *from each set of six prototiles 210 different two-colored, two-dimensional periodic tilings, may be derived.*

As an intuitive concept, antisymmetry is present from the very beginnings of ornamental art, appearing with Neolithic 'black-white' ceramics (Fig. 1). In time, the analytical approach prevailed over the empirical one (Douat, 1722; Gombrich, 1979, pp. 70-72; Smith, 1987). As a scientific concept, the theory of antisymmetry belongs to the 20th century mathematics (Heesch, 1929).

Let a symmetry group G and the permutation group S_2 generated by the antiidentity transformation e_1, satisfying the relationship $e_1{}^2 = E$ and commuting with all elements of G, be given. If $S \in G$, $S^1 = e_1S = Se_1$ is the antisymmetry transformation derived from S. Every group G^1 isomorphic to G, derived from G by replacing some of its symmetries by the corresponding antisymmetries, is called antisymmetry (or black-white) group. Every antisymmetry group is uniquely defined by its generating symmetry group G and the symmetry subgroup H of G^1, that is by the group/subgroup symbol G/H ($G/H \simeq S_2$, $[G:H]=2$). In a general sense, the antiidentity transformation e_1 can be interpreted as the change of any bivalent geometric or non-geometric property commuting with the symmetries of G {e.g., (black white), (+ −), (over under), (S N), (convex concave) ...}. The antisymmetry groups can be interpreted by the corresponding black-white mosaics (Shubnikov, Belov, 1964, pp. 220).

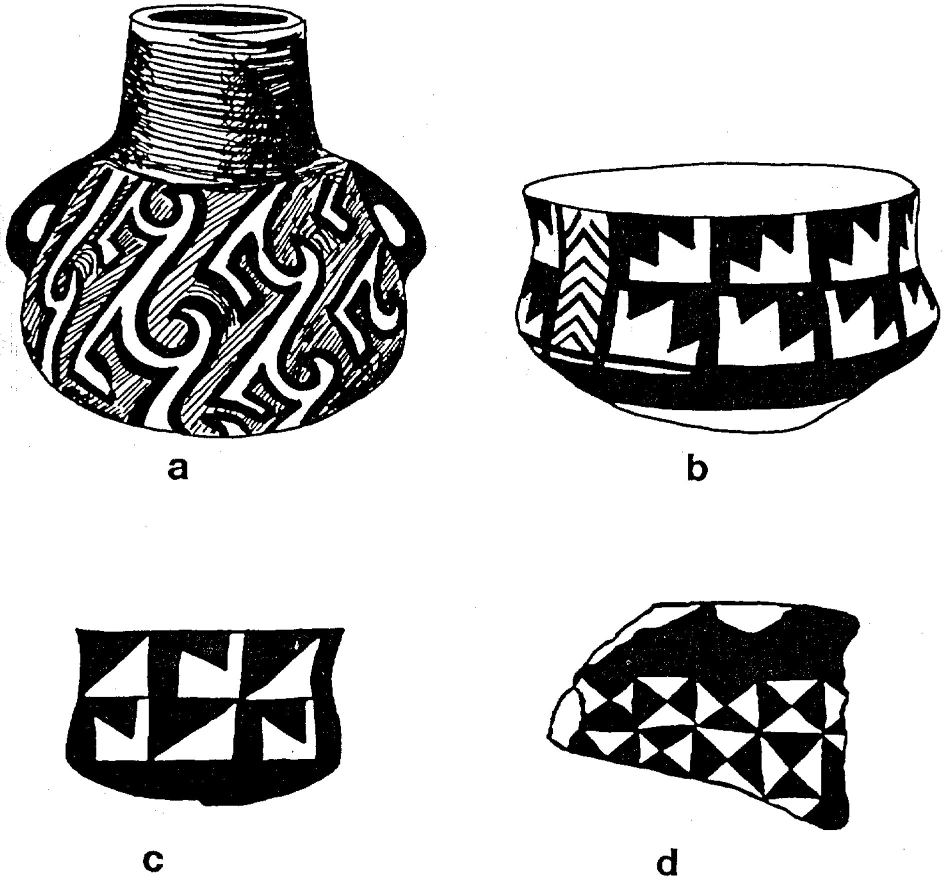


Figure 1: Neolithic antisymmetry ornaments: **a)** $p2/p1$, Rakhmani II, end of 5th mill. B.C. (Gimbutas, 1989, p. 281, Fig. 437); **b)** *pmg/pg*, Hacilar IIA, 5500-5300 B.C. (Mellaart, 1970, p. 98); **c)** $p2/p1$, Hacilar IIA, 5500-5300 B.C. (Mellaart, 1970, p. 42); **d)** *p4m/p4g*, Tell Brak, ~6000 B.C. (Frankel, 1979, back cover).

A natural extension of antisymmetry, the l-multiple antisymmetry (Zamorzaev, 1976), is realized by introducing l qualitatively different antiidentity transformations e_1, ..., e_l, satisfying the relations $e_i^2 = E$, $e_ie_j = e_je_i$ $(i,j = 1, 2, ..., l)$ and commuting with the symmetries from G. The multiple antisymmetry groups can be visually interpreted by the corresponding multiple antisymmetry mosaics.

Among various applications of the l-multiple antisymmetry (e.g., in multidimensional geometry, mathematical crystallography, physics ...) (Zamorzaev and Palistrant, 1980), there is also its possible use in modular ornamental design. Let T be a prototile of some isohedral tiling with a symmetry group G (Grünbaum and Shephard, 1987, p. 31), divided in l disjoint subregions $T_1, T_2, ..., T_l$. Coloring each subregion by one from two different colors, we obtain 2^l colored prototiles. In the set of colored prototiles we also include the enantiomorphs of these 2^l prototiles. Their arrangements according to the laws of l-multiple antisymmetry result in a very large set of different two-colored periodic tilings.

In practice, we may use some suitable form of a prototile T (e.g., a rectangle, quadrangle, triangle with the angles 90°, 60°, 30°) and the corresponding tilings.

This will be illustrated by an example based on the rectangular lattice, the frieze symmetry group *mm* and the plane symmetry group *pmm*. From the rectangular prototile $T(0)$ divided into 2 subregions (triangular and trapezoidal), by different black-white coloring, the colored prototiles (1, 2, 3) and their enantiomorphs (1', 2') are obtained (Fig. 2). In this case antiidentity transformations $e_1 = (0\,1)\,(2\,3)$ and $e_2 = (0\,2)\,(1\,3)$ satisfy the conditions of 2-multiple antisymmetry.

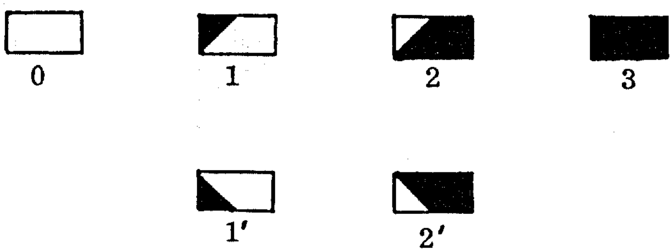


Figure 2

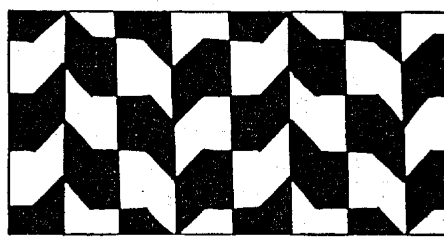
Because these prototiles are created for the sake of ornamental art, for the classification of the tilings obtained we may accept the very simple equality criterion: two tilings are equal if they are derived from the same set of prototiles and if there is a color-preserving isometry transforming one tiling into the other; otherwise, they are different. According to this equality criterion to the symmetry group of friezes *mm* and plane symmetry group *pmm* correspond antisymmetric characteristics 3.1 and 4.1, respectively (Jablan, 1990). Hence, the arrangement of such 6 colored prototiles, made according to the corresponding numerical schemes (i.e., 2-multiple antisymmetry mosaics), offers a great number of possibilities (e.g., 42 frieze types and 210 types of two-dimensional periodic black-white tilings derived from the symmetry group of friezes *mm* and ornaments *pmm* (Jablan, 1990)). Three of these possibilities, with their corresponding numerical schemes, are illustrated by Figure 3.

0	0	1	1'	0	0	1	1'
2'	2	3	3	2'	2	3	3

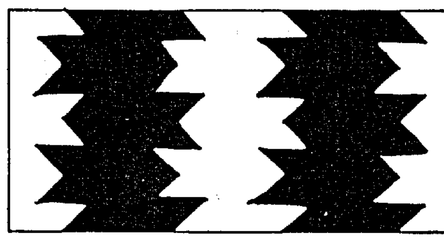
0	2'	1	3	0	2'	1	3
2'	0	3	1	2'	0	3	1

1	3	2	0	1	3	2	0
3	1	0	2	3	1	0	2

2	1'	3	0	2	1'	3	0
0	3	1'	2	0	3	1'	2
1	2'	0	3	1	2'	0	3
3	0	2'	1	3	0	2'	1
2	1'	3	0	2	1'	3	0
0	3	1'	2	0	3	1'	2
1	2'	0	3	1	2'	0	3
3	0	2'	1	3	0	2'	1



0	2'	3	1'	0	2'	3	1'
1'	3	2'	0	1'	3	2'	0
1	3	2	0	1	3	2	0
0	2	3	1	0	2	3	1
0	2'	3	1	0	2'	3	1
1'	3	2	0	1'	3	2	0
1	3	2	0	1	3	2	0
0	2	3	1	0	2	3	1



0	2'	3	1'	0	2'	3	1'
2'	0	1'	3	2'	0	1'	3
1	3	2	0	1	3	2	0
3	1	0	2	3	1	0	2
0	2'	3	1'	0	2'	3	1'
2'	0	1'	3	2'	0	1'	3
1	3	2	0	1	3	2	0
3	1	0	2	3	1	0	2

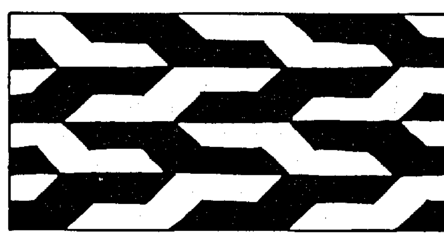


Figure 3

By a different choice of colored prototiles, following the same numerical schemes, it is possible to obtain the next series consisting of 42 frieze and 210 ornamental patterns, etc. (Fig. 4).

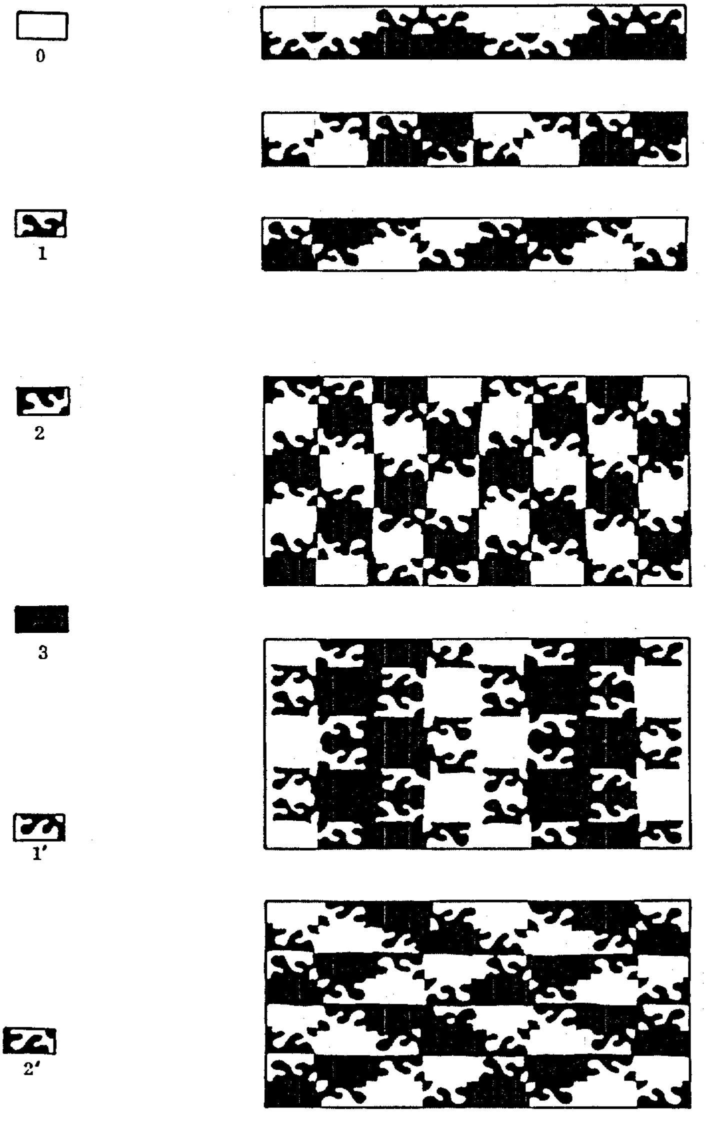


Figure 4

A survey of antisymmetry tilings derived from the prototiles (Fig. 2), made by a random choice, containing 80 of the obtainable 210 tilings, is illustrated by Figure 5.

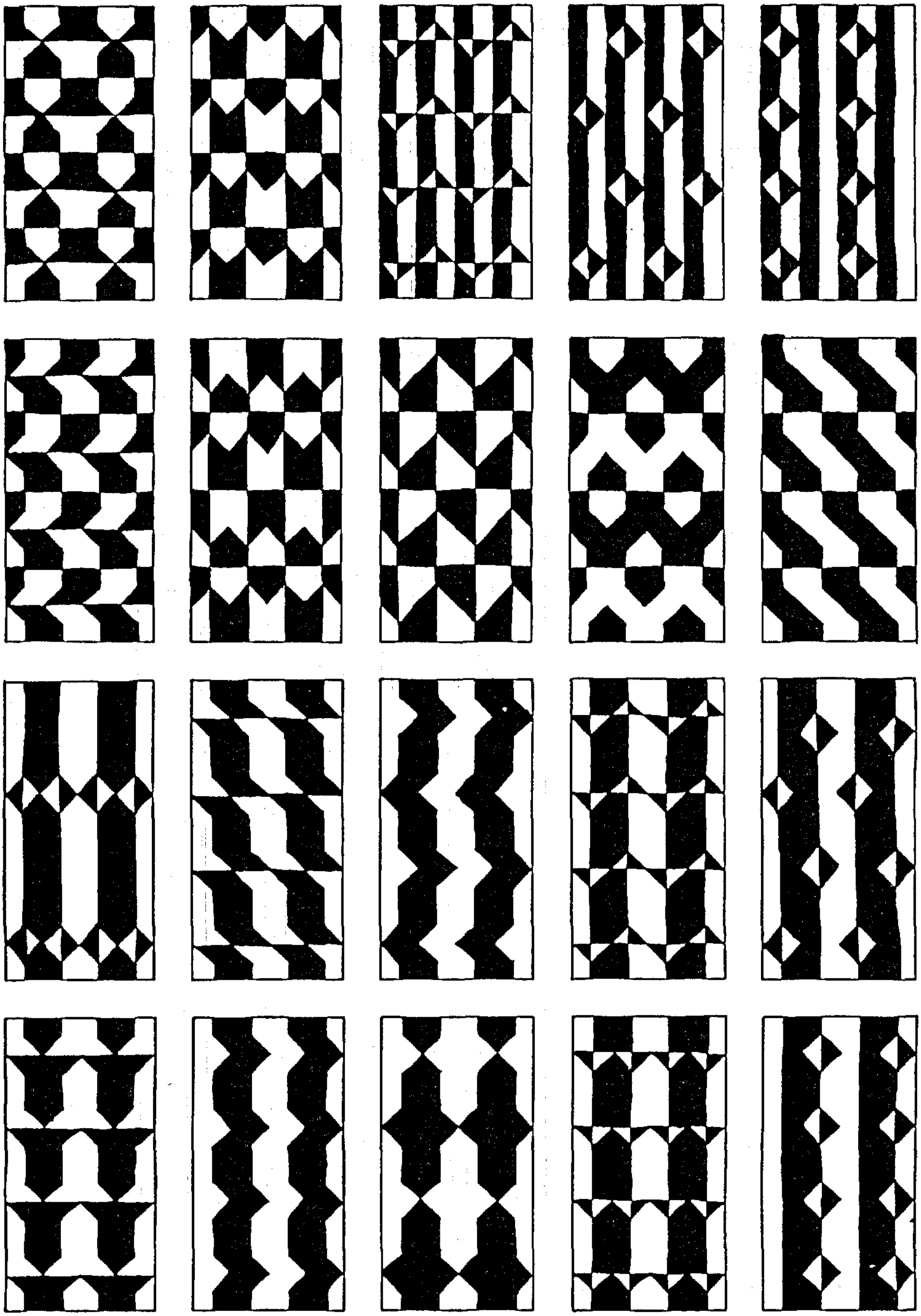


Figure 5

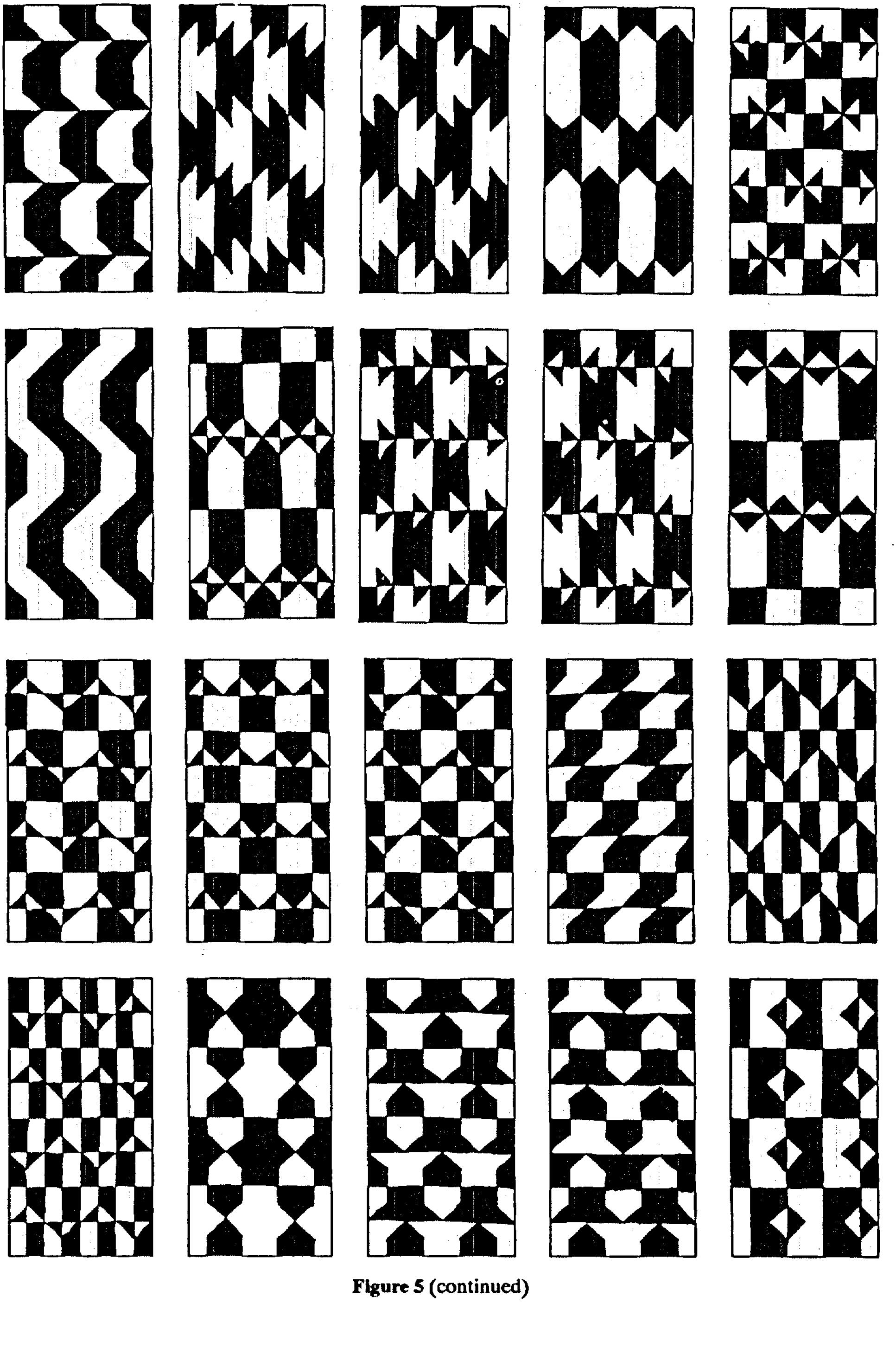
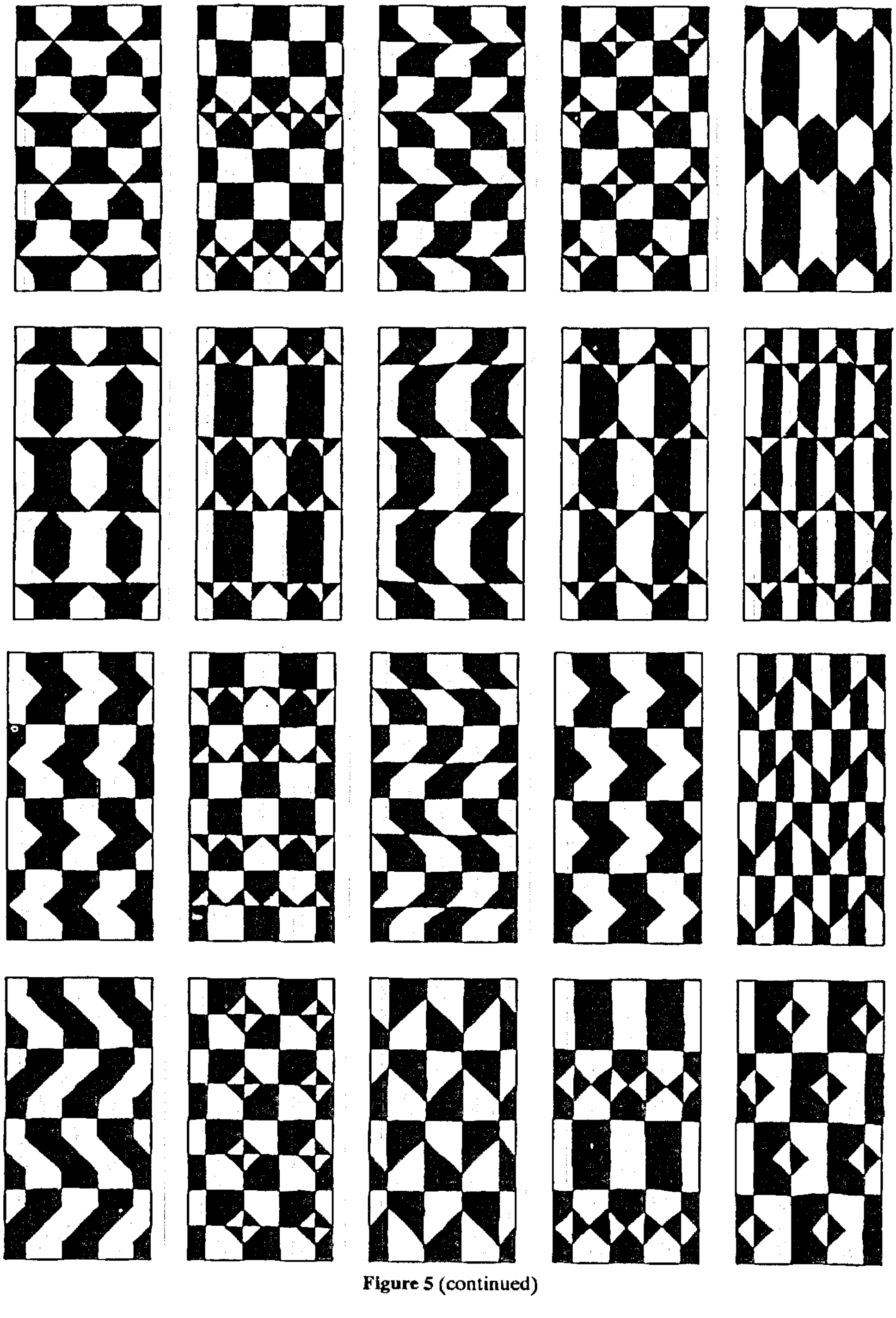


Figure 5 (continued)



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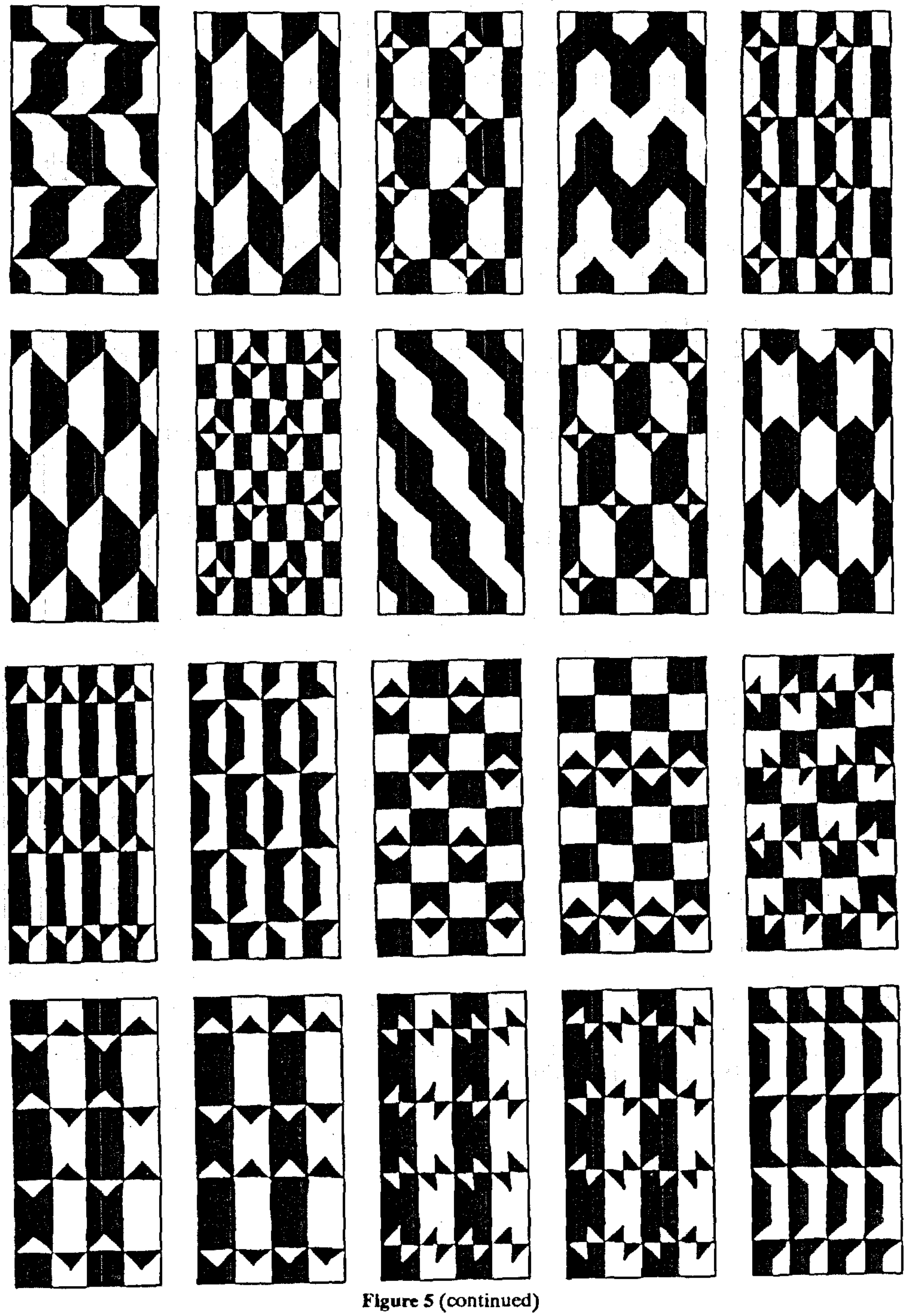

Figure 5 (continued)

For $l = 3$, from 12 colored prototiles result 168 frieze and 2520 two-colored two-dimensional periodic tilings, for $l = 4$ from 28 colored prototiles result 20160 two-colored two-dimensional periodic tilings derived from the symmetry groups *mm* and *pmm* (Jablan, 1990), etc.

Compared with existing decorative (e.g., ceramic) tiles, where from each prototile only one pattern results, the proposed modular tiles have the following practical advantages:

a) using a few colored prototiles (e.g., 6 prototiles, Fig. 2), a large number and variety of patterns are obtained;

b) each colored prototile is used equally often, since for the construction of each of the 42 frieze and 210 ornamental patterns, we need the same number of each of the colored prototiles;

c) the user may make individual choices of ornamental patterns using the same prototiles;

d) previously constructed ornamental patterns may be rearranged, in order to obtain new ones (in the case of removable tiles);

e) the assortment of ornamental patterns may be increased with a least possible investment, because each new series of 6 colored prototiles (e.g., Fig. 4) supplies 42 new frieze and 210 new ornamental patterns;

f) the production of colored prototiles is simple, since only 2 colors are needed;

g) the construction of ornamental patterns from a given set of colored prototiles is particularly easy, just by following the corresponding numerical scheme.

The modular design *Two-Colored Ornamental Tilings* got the award – honourable mention at the International Competition of Industrial Design and New Technology CEVISAMA-'87 (Valencia, Spain, 1987) and has been presented at the exhibition *Symmetry/Asymmetry* accompanying the symposium *Symmetry of Structure* (Budapest, Hungary, 1989), as well as at the 12th European Crystallographic Conference (Moscow, USSR, 1989). Its extension, *Colored Ornamental Tilings* has been presented at 2° Quadrienale Internazionale *La Ceramica Nell' Arredo Urbano* (Faenza, Italy, 1989) (Jablan, 1989) and in Moscow.

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