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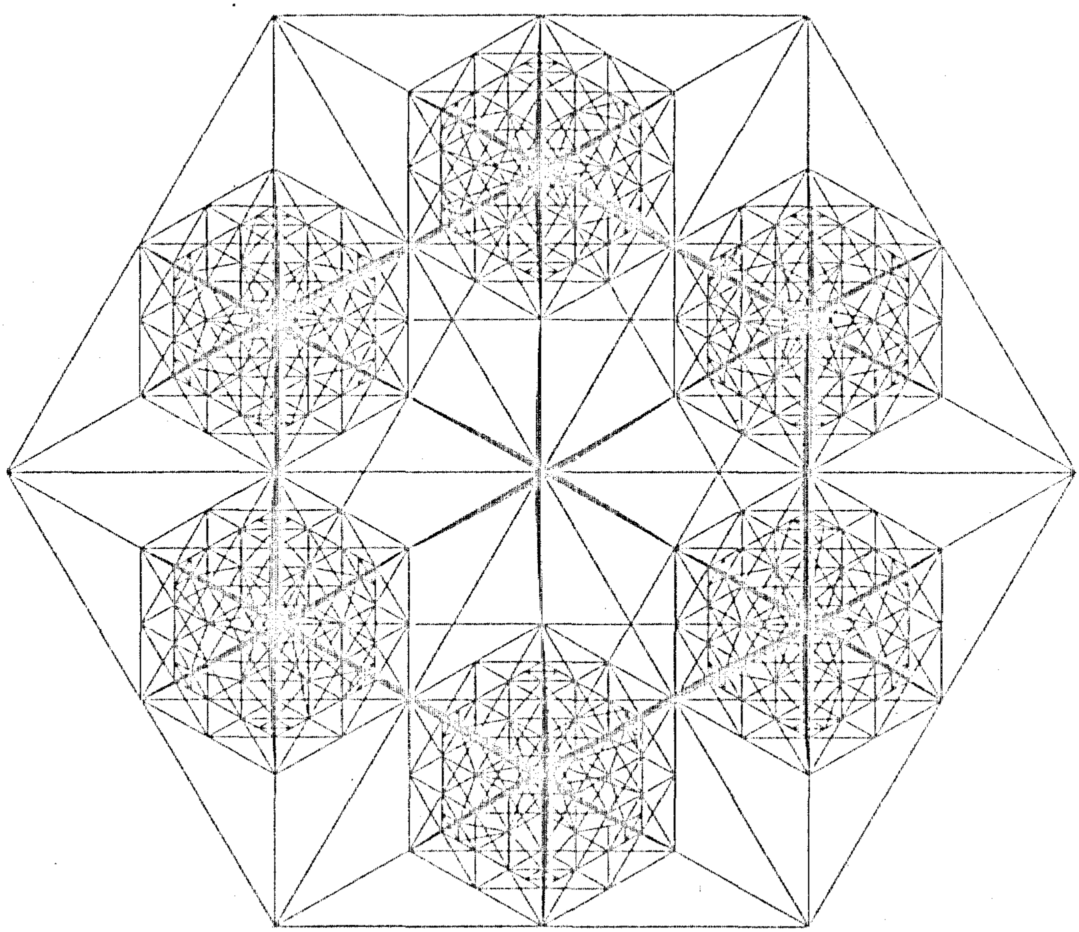

Large synergetical structure on hexagonal figure
Aerial view

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SYMMETRY IN THE ABORIGINAL ART OF AUSTRALIA

Donald W. Crowe

Mathematician, (b. Lincoln, Nebraska, U.S.A., 1927).
Address: Department of Mathematics, Madison, Wisconsin, University of Wisconsin, 53706, U.S.A, E-mail: crowe@math.wisc.edu .
Fields of interest: Geometry, combinatorial problems, finite geometries, color symmetry; real world analysis, with applications to anthropology.
Publications: The construction of finite regular hyperbolic planes of even order, *Colloquium Mathematicum*, 1965, 1, 247-250; *Excursions into Mathematics*, New York: Worth, 1969 (co-author); The geometry of African art I: Bakuba art, *Journal of Geometry*, 1 (1971), 169-182; II: A catalog of Benin patterns, *Historia Mathematica*, 2 (1975), 253-271; III: The smoking pipes of Begho, In: Davis, C., Grünbaum, B., and Sherk, F. A. eds., *The Geometric Vein, The Coxeter Festschrift*, New York: Springer, 1982, pp. 177-189; *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis*, Seattle: University of Washington Press, 1988 (co-author).

Abstract: *While aboriginal art first appears to have little to interest the student of symmetry, two aspects are discussed in this paper. The first is the presence of 'double bilateral' or sometimes just 'central' symmetry, in contrast to the simple bilateral symmetry of neighboring Papua New Guinea. The second is the rudimentary 3-color symmetry in relatively modern artifacts from the northern Queensland rain forest.*

INTRODUCTION

For the Pacific region little has been written dealing specifically with *symmetry*, in the general geometric sense which includes all the plane isometries of reflection, translation, rotation, and glide reflection, as well as two-color ('counterchange') symmetry. The mathematical tools for such a study have recently been made available to non-mathematical readers in the book *Symmetries of Culture* (Washburn and Crowe, 1988), and this is the first of a series of papers applying them to the description of Pacific symmetries. Subsequent papers will describe Tongan symmetries in several media, as well as the two-color symmetries of Fijian Cakaudrove-style *masi kesa*.

The present note contains some speculations about bilateral, double bilateral, and central symmetry in the traditions of the art of aboriginal Australia, as well as some recent 3-color patterns from northern Queensland.

The author particularly acknowledges the hospitality of Barrie Reynolds and James Cook, University of North Queensland during the time this work was germinating, and the special help of Mae Abernethy for much of the background material (Crowe and Nagy, 1992).

BILATERAL SYMMETRY

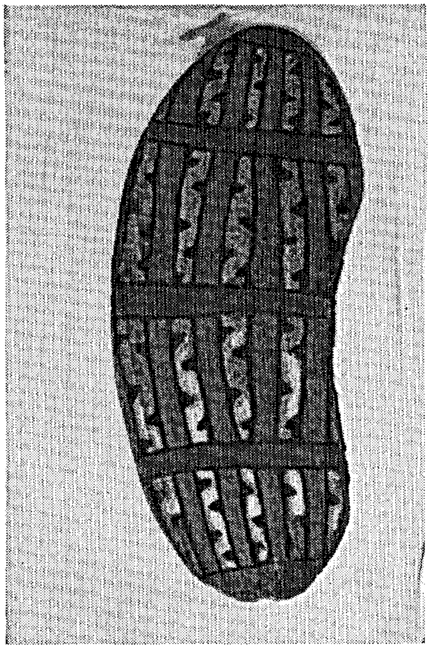
In the rain forests of northeast Australia, the aborigines traditionally produced several kinds of wooden artifacts from the abundantly available supply of forest wood. At first glance there appears to be little of interest to the student of symmetry in the symmetries of the painted decorations on these artifacts. Indeed, this might be felt to be the case for Australian aboriginal art in general, however appealing it is from a religious or artistic perspective. For that very reason it may be worthwhile to point out some geometric symmetries which *do* appear.

The most spectacular of the wooden rain forest artifacts are the shields, made to protect the bearer from the wooden spears and oversized wooden swords used in ceremonial combats called *bruns*. At the turn of this century in the Cardwell and Tully River areas of northern Queensland, these *bruns* were held with almost the frequency and regularity of markets elsewhere in the world. The *brun* "is essentially an entertainment – though the opportunity may be taken of wiping off old scores, and so settling disputes, either real or imaginary – and gives the men a chance of showing off their prowess and courage before the women" (Roth, 1902, p. 15).

"During the actual fighting spears are generally the first things to be thrown (generally at the legs, anywhere below the knees), then as they get nearer the boomerangs are let fly (at any portion of the body), and finally, when they get to close quarters the swords are used for striking at the head ..." (Roth, 1902, p. 16). Photographs, and museum specimens, show the immense size of the shields and the heavy wooden swords with which the close combat part of this battle is carried out (see Fig. 1). The shields cover most of the owner's body from knee to chin, and the entire width of a rather slender body. The sword is "about five inches wide up to the point, which is slightly rounded, and usually reaches from the foot to the shoulder. It is made of hard wood, with a short handle for only one hand, and is so heavy that any one not used to it can scarcely balance it perpendicularly with half-extended arm" (Lumholtz, 1889, p. 121). These shields, and a sword, can be seen in the old photograph reproduced in Figure 1.

The development of the swords reached an unpleasant modern extreme when the teeth were smoothed from giant metal crosscut saws used in logging, and the saw end was slightly rounded to produce a formidable weapon having the same general shape and size as the wooden sword. Such weapons, which can be seen in the local museum in Kuranda, Queensland, and in the Australian Museum in Sydney, would have had a devastating effect on the soft wooden shields. Indeed, to judge from museum specimens of shields, many were severely damaged along one long edge even by the wooden swords. One such is shown in Figure 2a.

Figure 1: Posed photograph showing shields and swords. From Cairns City Council, early 20th century? (James Cook University, Material Culture Unit, No. L 78.3.230a)



Figures 2a and 2b: Sword-damaged shield. es Cook University, Material Culture Unit, No. L 76.1.11); Shield with design exhibiting double bilateral symmetry. (James Cook University, Material Culture Unit, No. AAB 79.1.42)

Although the swords were not painted, the shields, whose oval shape had horizontal bilateral symmetry (i.e., top-bottom symmetry), and nearly vertical bilateral symmetry (i.e., left-right symmetry), were usually painted to match this 'double bilateral symmetry'. A relatively modern example, which shows typical symmetry and coloring, appears in Figure 2b.

The painting of the shields, like that of other rain forest artifacts, used a palette of four colors: red, yellow, white, and black. (In the black and white photographs the color of the darkest region is red, the next darkest yellow, and the lightest regions are white. The boundaries between regions, and the small triangles in Figure 2a, are black.) In the now more familiar aboriginal art of bark painting, as well as rock art, these are also the predominant colors. Perhaps for this reason, red, yellow, and black (without the white) are the colors appearing on the aboriginal flag in Australia.

The shields in Figure 2 are typical, in that the black is used to mark out regions which are then solidly filled with red, yellow or white, so that the resulting impression is that only those last three colors were used. In a modern field study (Abernethy, 1984) of the manufacture and decoration of a rain forest shield, the white and yellow pigments were clays, the red was obtained from the yellow by heating it over a wood-burning stove, and the black was made from charcoal mixed with blood and salt. By tradition, the blood was obtained by making a nose bleed, or by piercing a vein in the upper arm, but more recently cattle blood from a local slaughterhouse was used (Abernethy, 1984, p. 51). The painting was done with brushes made by crushing one end of a section of a common vine ('lawyer cane', so-called because of the entanglements it created for the rain forest traveler).

The symmetry of the painted decoration on these shields ordinarily conforms to the symmetry of the oval shape of the shield, not only in outline but in color. That is, the underlying pattern, outlined in black, has bilateral symmetry in two directions, and the solid regions are painted so that regions related by either of these symmetries (and hence also by half-turn symmetry) are the same color. This is readily apparent in the shields of Figure 2. Note that the symmetries of the shield do not interchange the colors; the symmetries *preserve* color. This is in contrast to the coloring of firesticks and boomerangs, to be discussed later, where the symmetries of the object (thought of as an infinite band) may interchange (i.e., permute) the three colors among themselves.

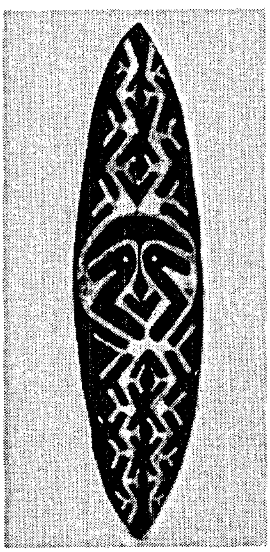


Figure 3: Ceremonial board (recent). Gulf of Papua. (James Cook University, Material Culture Unit)

An art student, slightly familiar with the shapes and designs found (generally carved) on shields and masks from New Guinea (the next closest large land mass to the north Queensland rain forest) might see these rain forest shields as being copies of, or otherwise closely related to, the well-known shields and masks from that abundant source of museum and private collections. A cursory glance at, for example, the photographs in (Newton, 1961) shows many shield-like objects from that region which is just across the Torres Strait from northern Queensland. A closer look reveals (i) that these are generally not shields, but are either masks or ceremonial boards, which were not used as shields, and (ii) that the symmetry of these objects is almost always left-right symmetry, but *not* top-bottom symmetry. This is a natural consequence of the fact that the carved surface commonly represents a face, or sometimes an animal, which, however abstracted and elaborated, does not have top-bottom symmetry. A typical modern example, from the Papuan Gulf, is shown in Figure 3.

From farther west in New Guinea the large Rockefeller collection of Asmat artifacts (Gerbrands, ed., 1967) does include many shields. However, in most cases the shape of the shield itself does not have top-bottom symmetry, but tapers somewhat towards the top. The carved designs on these shields again often have the symmetry of human or animal figures, not the double bilateral symmetry of the Queensland rain forest shields.

On other hand, elsewhere on the Australian continent itself there are suggestions that the double bilateral symmetry of the rain forest shield, or at least its half-turn symmetry, is an expression of an Australian aboriginal design theme. Two examples will be cited, both from the central, arid, regions of Australia.

In 1940 Charles Mountford handed sheets of brown paper, and red, yellow, white, and black crayons to a group of aboriginal men at Ernabella in the Musgrave Ranges of south-central Australia, and asked them to 'make marks' (Mountford, 1976). In his opinion, the resulting 300 drawings revealed a natural tendency to "... maintain an artistic balance between the design elements." (Mountford, 1976, p. 95). In support of this he cites the example redrawn as Figure 4, representing a myth concerning the Magellanic Clouds (prominent features of the southern sky).

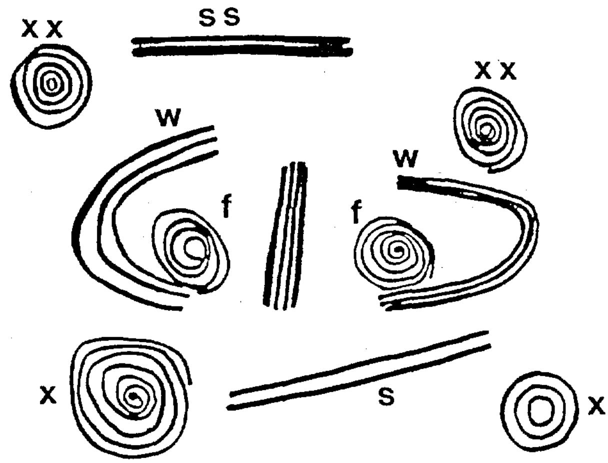


Figure 4: *Walanari brothers in the Magellan Clouds* (1940). Redrawn from (Mountford, 1976, Pl. 584).

In this drawing the symmetrically placed crescents, *W*, represent windbreaks of the men of the Magellanic Clouds, enclosing symmetrically placed fires, *f*. Between them is the body of a dead man from creation times. At *S* is a pair of lines representing the spear invented for the aborigines. The spiral and concentric circles, *X*, at lower left and right represent the two Magellanic Clouds. However, his informants stressed that these were the only two clouds in the sky. The upper spiral and concentric circles, *XX*, and the line pair *SS* were 'just nothing'. "They had been placed in the drawing to make it look 'more pretty', in other words to maintain an artistic balance." (Mountford, 1976, p. 95).

It is clear that the words 'artistic balance' here refer to a symmetry between the top and bottom of the picture, either bilateral symmetry or half-turn symmetry. The drawing, without the addition of the extra spiral and concentric circles and the line pair, already had (essentially) left-right symmetry (like the New Guinea masks and ceremonial boards referred to above). However, this central Australian artistic sensibility required the addition of these three extra 'ideal elements' to give top-bottom symmetry as well. (This may remind the mathematical reader of the introduction of the 'ideal elements' to the Euclidean plane in order to yield a symmetry between the axiom "two points determine a line", and its otherwise imperfect dual, "two lines determine a point".) The result resembles the double bilateral symmetry seen on rain forest shields.

In recent times, desert sand paintings have inspired a new art form, 'dot paintings'. The dot paintings often use symbols, much like the fire and windbreak symbols in Figure 4, which need to be explained by the artist to be intelligible to the uninitiated viewer. For such a viewer, trained to appreciate other art traditions, these dot paintings often appear to be *too* symmetric. For example, the *Bush-potato Dreaming* painting shown in Figure 5a might, in another art tradition, have used left-right symmetry to represent the approximate left right symmetry of a growing plant. But in this painting, as in many others of this type from central Australia, the artist has included a symmetry between the upper and lower masses of underground roots. Thus, some aboriginal urge toward the double bilateral symmetry of the rain forest shields is manifested once again.

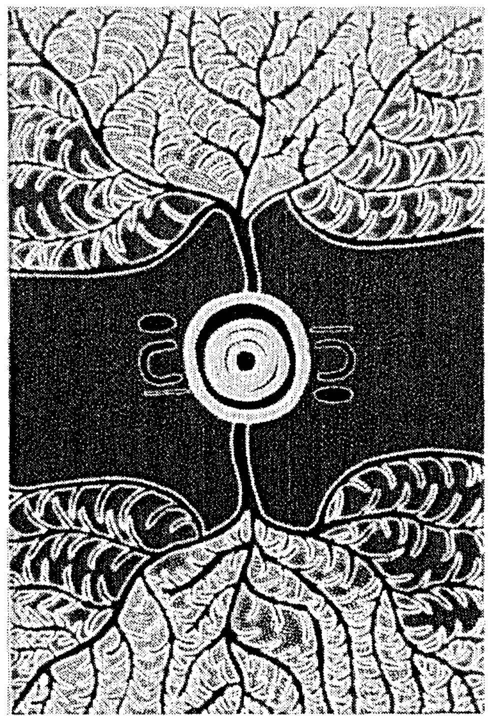


Figure 5a: *Bush-potato dreaming*, by Betty Egan (1989).

Of course this general bilateral symmetry of the main structure of this painting ignores the six small black symbols (left and right of the central concentric circles) representing two women with their digging sticks and wooden bowls. They are arranged in half-turn ('central') symmetry around the central circles. This feature, the reduction of structural bilateral double symmetry by the addition of other features

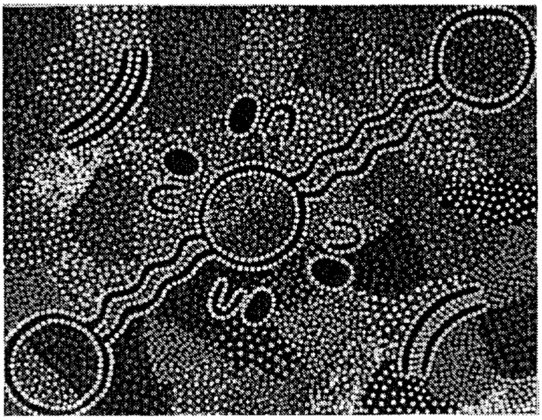


Figure 5b: *Water dreaming*, by Linda Umbajano (*ca*.1989).

(often more prominent than those in this example), is not uncommon. Figure 4 already illustrates this, since the lower right circles are matched by circles in the upper left while the lower left spiral is matched by a spiral in the upper right. It may even be conjectured that the leftward displacement of *SS* deliberately matches the rightward displacement of *S*. A variety of recent examples illustrating this same phenomenon can be seen in Plates 2, 4, 6, 20, and 21 of (Amadio and Kimber, 1988). An extreme version is seen in Figure 5b where the whole linear structure is carefully arranged in half-turn symmetry against an amorphous background.

COLOR SYMMETRY

We turn now to a different kind of symmetry which is found on some decorated firesticks and boomerangs from the rain forest. This is 'color symmetry', which takes into account the possibility of color permutations combined with geometric symmetry. The decorated objects to be discussed here were primarily made for sale to tourists during the 1960's and 1970's, but the traditional colors red, yellow, white, and black were used, as on the shields discussed above. They are now in the collection of the Material Culture Unit of James Cook University of North Queensland.

Firesticks were originally used by rain forest aborigines as soft 'hearths' against which a fire was made by twirling another harder stick until enough heat was generated by friction to ignite some suitable tinder. In their original form they were probably not decorated, but in the tourist versions shown in Figures 9b and 10 they are painted to represent fish, or human figures. In either case the two eye sockets could serve as the holes in which vertical hardwood sticks were twirled to produce fire. In the past, any convenient flat wood could be used, and museum specimens of shields sometimes have charred holes where they were used as fire hearths in this way.

The curved shape of a boomerang, a typical Australian artifact whose modern version has apparently survived from at least a 10,000 year ancestry (Hudson, 1989) is well-known. In discussing the patterns painted on boomerangs we ignore the curvature and treat them as if they were narrow straight strips.

Since both firesticks and boomerangs are generally longer in comparison to their width than shields, the symmetries involved can be thought of as those of (part of) an infinite strip. It is a known geometric fact (Washburn and Crowe, 1988, pp. 278-279) that, when classified by symmetry, there are just seven distinct such one-color strip patterns. Because black is, as in the shields, used merely to mark the boundaries between the colored regions, in effect the boomerangs and firesticks are colored with three colors. Moreover, in contrast to the shields, the three colors are often used more or less equivalently. In such examples the regions outlined in black fall into three roughly congruent parts, each of which is colored one of the three colors. Moreover the congruence between these parts is one of the symmetries of the one-color strip outlined in black.

The next three paragraphs are devoted to a brief mathematical discussion of the seven infinite strip patterns and their 'n-colorings'. The '3-colorings' of some of the firesticks and boomerangs are a special case of these general n-colorings. A convenient two-symbol notation for the seven infinite repeated patterns on a strip, and their symmetry groups, was introduced by Senechal (Washburn and Crowe, 1988, p. 58). In describing it the strip is imagined as stretching horizontally from left to right. Then 'horizontal reflection' means reflection in the (unique) horizontal midline of the pattern; 'vertical reflection' refers to reflection in any line (in the plane of the pattern) perpendicular to this. The two-symbol notation for such a strip is determined as follows:

> The first symbol is m if there is a vertical reflection; otherwise it is 1.
>
> The second symbol is m if there is a horizontal reflection; otherwise it is g if there is a glide reflection (but no horizontal reflection), 2 if there is half-turn (but no reflection or glide reflection), and 1 otherwise.

The resulting notations for the seven possible infinite strips (and their groups) are 11, 1*m*, *m*1, 12, *mm*, *mg*, and 1*g*. For convenient reference examples of each of them are shown in Figure 6.

Coxeter (1987) has given a particularly lucid description of all possible symmetric colorings of such a strip with n colors, called *n-colorings*. Such a coloring involves the symmetry type T (one of the above seven) of the uncolored pattern, and the type T' (also one of the seven) of the pattern consisting of one of the colors alone. We will also use T and T' to denote the symmetry groups of patterns of those types. In the case of strips (but, unfortunately not in the case of two-dimensional patterns) the group relation $[T : T'] = n$ ("the index of T' in T is n", that is, "the subgroup T' has exactly n cosets in T") completely determines the type of color distribution, each coset of T' in T corresponding to one of the n colors. Ambiguity arises only when the subgroup T' can be embedded in T in more than one way, which (fortuitously) cannot happen when T and T' are among the seven strip groups.

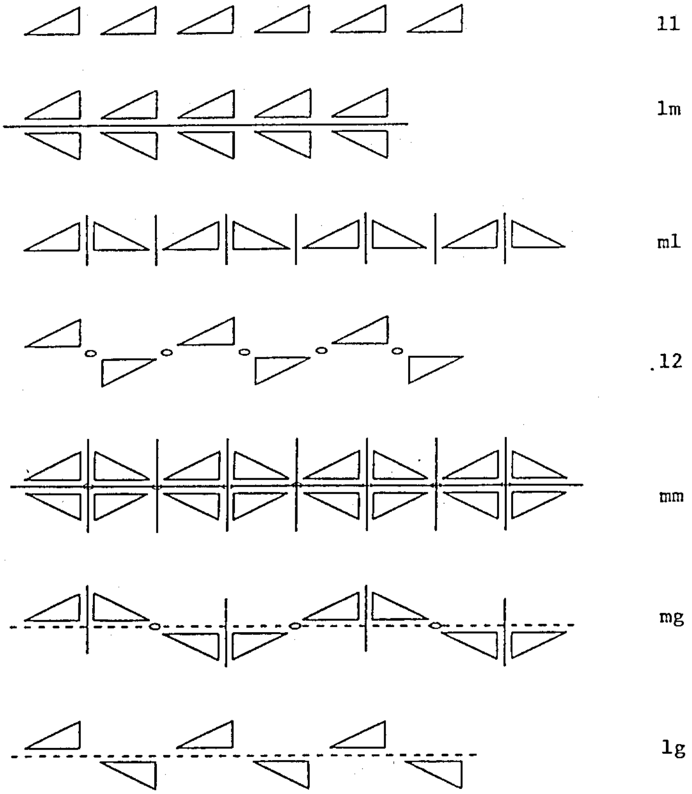


Figure 6: The seven uncolored strip patterns.

In case n is an odd number, the only possible group-subgroup relations turn out to be of the form $[T : T] = n$. That is, the symmetry type of the portion colored one of the n colors must be the same as the symmetry type of the strip as a whole. Hence, in those cases (which include our case, $n = 3$) there are only seven n-colorings of the seven strips, one for each of the seven one-color types. (For even numbers n there are 19 and 17 n-colorings, depending on whether n is divisible by 4 or not, see Coxeter, 1987, pp. 456-457.)

To return to firesticks and boomerangs, the maker of one of these objects first outlines the regions in black. Then he (or she, since in later times the painting of artifacts for sale was often a cooperative process involving both men and women (Abernethy, 1984, p. 64)) may try to fill in the regions with the three available solid colors according to various criteria. If the black outlined design itself has no symmetries (and consequently does not represent any of the seven strip types) it may be colored

(i) randomly, or

(ii) according to map-coloring principles, i.e., so that no two adjacent regions have the same color.

If the black outlined design itself has the symmetries of one of the seven strip patterns, then some additional possibilities are

(iii) the colored strip is a 3-coloring in the sense described above. That is, each of the three colors is used equivalently, and the coloring is represented by an equation $[T:T] = 3$, or

(iv) the colored strip is a $2:1:1$ coloring, in the sense described by (Grünbaum, Grünbaum, and Shephard, 1986, p. 652). (This is discussed in connection with Figure 10 below); or

(v) not all of the three colors appear in an essential way, so that the resulting painted pattern has the same symmetry as one of the seven uncolored strips.

Each of these possibilities occurs in the Material Culture Unit collection. A number of objects appear to be designed and colored completely at random. Figure 7 illustrates one such. There are two pairs of adjacent regions (one at the center, and one near the left end) which are colored yellow, so that this coloring is not a 'map-coloring'. Ditchburn (1988) had the interesting idea that such apparently random designs implicitly recorded natural rain forest impressions, such as light spots filtering through the foliage of the canopy.

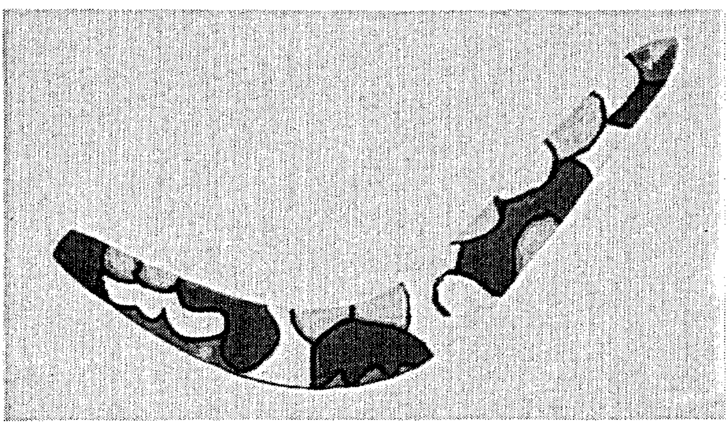


Figure 7: Boomerang with random design. (James Cook University, Material Culture Unit, No. AAB 79.1.437).

Figure 8 shows a design which is less irregular than the preceding. In fact it is colored strictly according to map-coloring principles, and the coloring emphasizes two kinds of partial symmetry. First, there is a strong suggestion of bilateral symmetry between the two ends of the boomerang. Second, the alternation of colors reveals a resemblance, along either end, to the more regular *3*-coloring shown in Figure 9c. This is no accident: the topological structure of the uncolored regions is such that whatever color is placed at one end the entire coloring must alternate towards the center in order to avoid having two adjacent regions the same color.

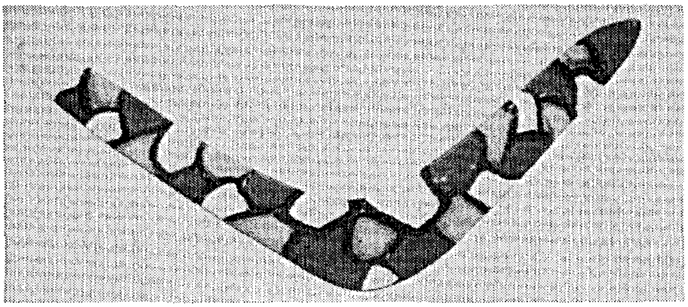
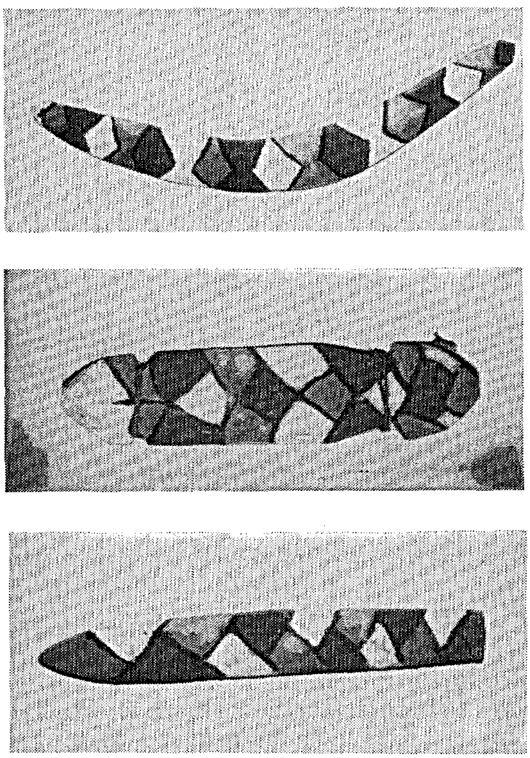


Figure 8: Boomerang with design having bilateral symmetry. (James Cook University, Material Culture Unit, No. AAB 79.1.176)

Figures 9a-c illustrate three of the seven possible 3-colorings of infinite strips. Using the Coxeter notation described above, 9a is a very convincing version of $[mm : mm] = 3$; 9b is, if not too strictly interpreted, a version of $[1m : 1m] = 3$; and 9c is $[1g : 1g] = 3$, albeit slightly compressed at one end.



Figures 9a, 9b, 9c: Boomerang and firesticks illustrating three possible 3-colorings of a strip. (James Cook University, Material Culture Unit. 9a: No. AAB 79.1.371; 9b: No. AAB 79.1.132; 9c: No. AAB 79.1.345).

Another systematic way of using three colors involves one of them as a special kind of background, which occupies a space congruent (by one of the strip symmetries) to the combined space occupied by the other two colors. Although such patterns apparently have not been systematically studied, we can call them 2 : 1 : 1 patterns, as in (Grünbaum, Grünbaum and Shephard, 1986). The firestick shown in Figure 10 is an example of a coloring of a strip pattern of type 1*m* in this way. Here the white and yellow portions (of the strip extended to infinity) are congruent to each other, and their union is congruent to the red (darkest) portion. From head to base the sequence of colors on this part of the infinite strip is *YRWRYRWRYRW*.

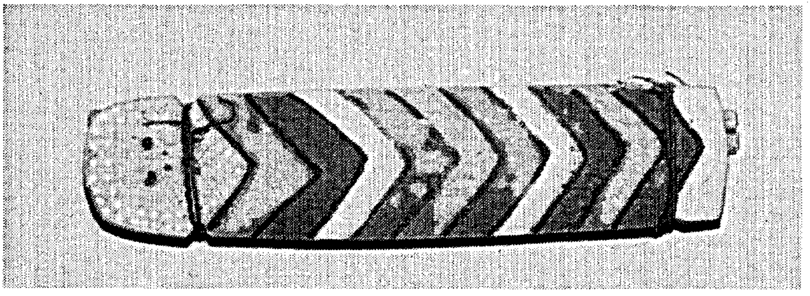


Figure 10: Firestick illustrating 2:1:1 symmetry. (James Cook University, Material Culture Unit, No. AAB 79.1.357).

Finally, Figure 11 shows two boomerangs whose colors are not interchanged at all by any symmetry of the underlying strip. (In this respect they are like the shields of Figure 2). In the lower example only two colors, white and yellow, are used (except for the usual black outline). They do not influence the symmetry type, which is *mm*. The upper boomerang illustrates the same symmetry, the only difference being that the border color is red instead of white.

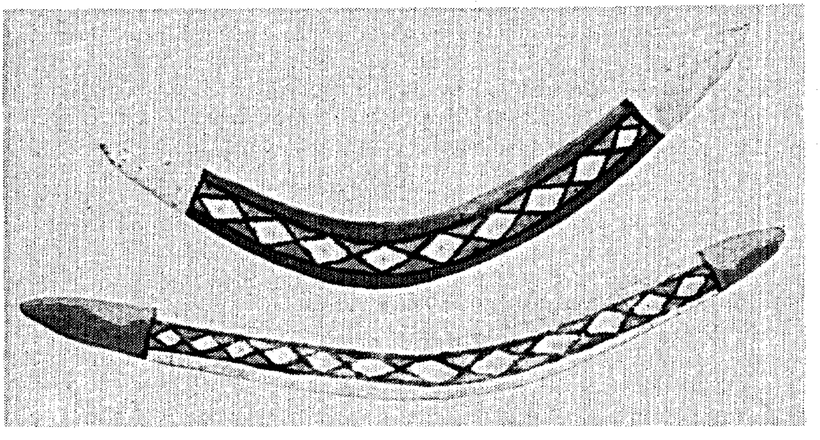


Figure 11: Two boomerangs illustrating uncolored strip type *mm*. (James Cook University, Material Culture Unit. Upper: No. AAB 79.1.330; Lower: No. AAB. 79.1.106).

CONCLUSION

Although symmetry is not the most striking feature of the aboriginal art of Australia, a closer examination of rain forest shields shows that they are commonly shaped and decorated in double-bilateral symmetry. A tendency toward double-bilateral symmetry can also be seen in aboriginal desert drawings and paintings. This type of symmetry contrasts with the superficially similar simple-bilateral symmetry of shields and masks from neighboring New Guinea. Likewise, the cruder 3-color painted symmetry of recent rain forest firesticks and boomerangs appears to have no analog in neighboring Pacific regions. Thus the symmetries discussed in this paper provide some support for the theory that individual cultural groups, particularly those in relative isolation, tend to develop their own characteristic artistic symmetries.

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