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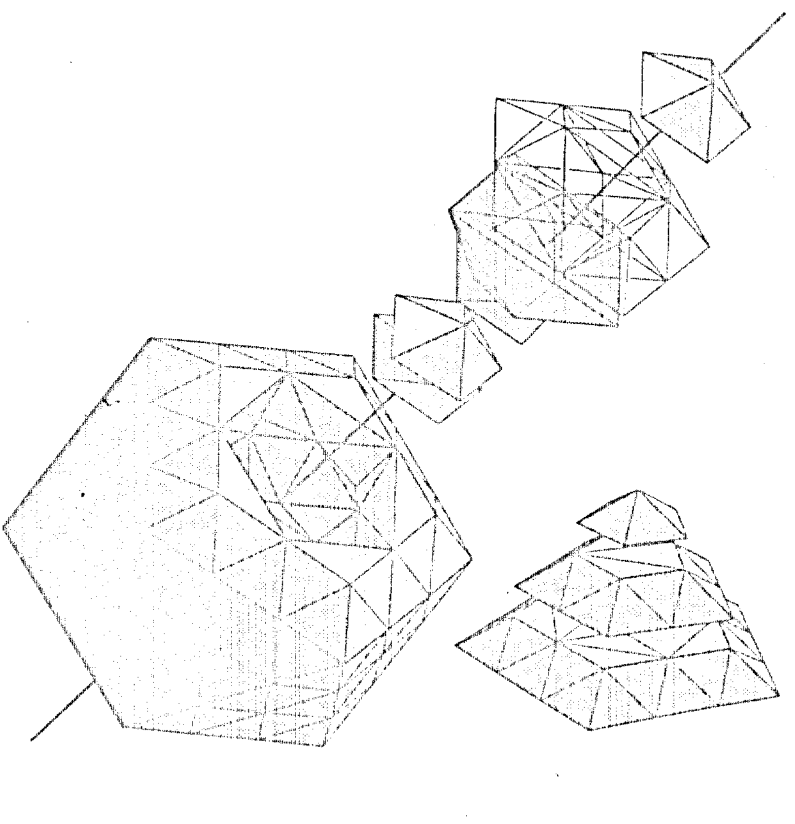


FLEXING POLYHEDRA

Caspar Schwabe
Ars Geometrica, AHA Gallery
Spiegelgasse 14, 8001 Zürich, Switzerland

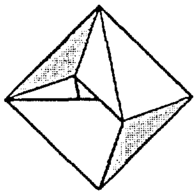
Flexing polyhedra are closed surfaces which are bordered by fixed, even polygons that can articulate along the edges. Such polyhedra permit of a deformation. Visualize a closed surface which is composed of flat pieces of cardboard and held together with adhesive tape along the edges. If the form of the polyhedron can change without tearing the tape or bending the cardboard, then we have a flexing polyhedron. The bellows of a camera, for example, function only with soft material; hence they are not genuine flexing polyhedra, being mathematically very impure.

In 1812 the well-known French mathematician Cauchy proved that convex polyhedra, i.e. those curved outwards, are immobile. In a generalisation of this principle, it was then postulated that concave polyhedra, i.e. those curved inwards, are also rigid. In 1897, however, a Belgian engineer named R. Bricard refuted this assumption. He discovered mobile octahedra strips, although they could not be completed as polyhedra because they showed some overlapping. Nevertheless it was regarded as impossible to construct a genuine flexing polyhedron.

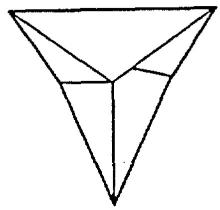
Only in recent years did R. Connelly with his revolutionary 36-sided polyhedron, succeed in modifying Bricard's model in such a way as to produce the world's first genuine flexing polyhedron. This polyhedron was later modified by N.H. Kuiper and P. Deligne to only 18 faces. To top this, in 1977, K. Steffen found his famous flexing polyhedron with only 14 faces and 9 vertices. All those flexing polyhedra are based on the model of Bricard and their mobility is severely limited by parts which impede one another. The mathematically pure flexing polyhedra discovered so far have constant capacity. It is therefore generally assumed that the volume of every possible flexing polyhedron remains constant during flexure.

Primary examples of flexing polyhedra with a variable capacity -although these are not mathematically pure examples- are W. Blaschke's flexing octahedra and M. Goldberg's double pyramid, resembling Siamese twins. More recent models, such as the diverse infinitesimal flexing polyhedra of W. Wunderlich and the 16-sided so called Quadricorn designed in 1981 by myself, allow rather more precise, effortless movements. The Quadricorn is the first practically perfect flexing polyhedron with a mathematically pure middle position and two flat boundary forms, i.e. its volume can be reduced to zero. As stated, the movements of all these flexing polyhedra with a variable capacity are not mathematically pure, for when they are in motion, tiny deformations hardly measurable and invisible to the naked eye will occur on the edges and surfaces. But one day , someone may discover a mathematically pure flexing polyhedron with a variable volume and thereby disprove the assumed constancy of capacity -who can tell?

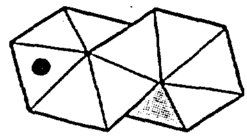
Bricard's
OCTAHEDRON



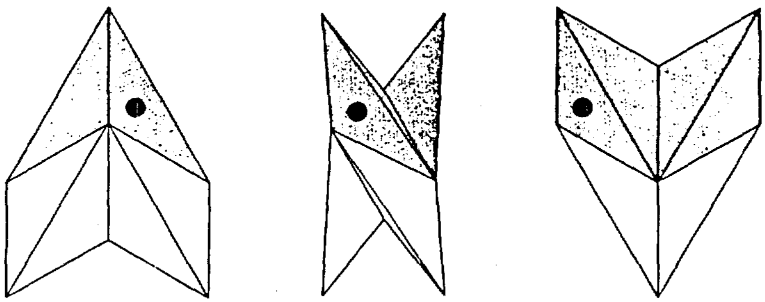
Steffen's
TETRADECAHEDRON



Goldberg's
SIAMESE TWINS



The QUADRICORN



flat position x — middle position — flat position y

One question remains: What sort of shape would a mathematically pure flexing polyhedron with a variable volume have?
Would it need to be symmetrical, or could it be of asymmetrical shape, a ring shaped torus or something resembling a so-called UFO? Might there be a space-filling flexing polyhedra? If so, this could be a dynamic structure where a single flexing polyhedron controls and determines the motion of all the adjacent flexing polyhedra. I firmly believe that any such discovery would have an enormous impact on physics.

You will find on the pages 213-221 the nets of the four flexing polyhedra illustrated above.

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- R. Connelly: A flexible sphere. Mathematical Intelligencer 3, 130-131 (1978).
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- W. Wunderlich, C. Schwabe: Eine Familie von geschlossenen gleichflächigen Polyedern die fast beweglich sind. Elemente der Mathematik. Vol. 41-4 (1986).
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