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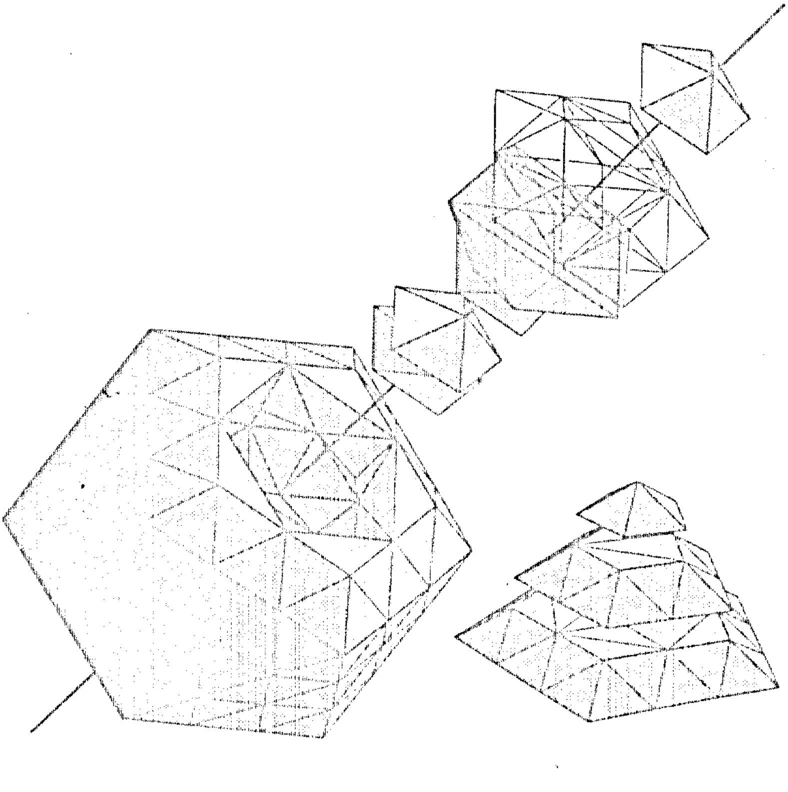

SOME GEOMETRICAL ATTEMPTS
Quasicrystals, Fractal Tesselation, Ideal Critical Pattern

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Some of geometrical attempts are proposed. The author is originally a theoretician mainly physics of condensed matters and focuses his interest on geometrical problems since he organized an interdisciplinary meeting "Morphysics" (morph + physics) in Kyoto in 1980.

1. Quasicrystals. [1], [2], [3]

[*A*] The author discovered three-dimensional Penrose tiling in 1985. There are two kinds of rhombohedral unit cells, A_6 and O_6. An A_6, beeing an expanded A_6, is consists of 55 A_6's and 34 O_6's and an O_6*, being an expanded O_6, is consists of 34 A_6's and 21 O_6's, so to express as

$$A_6^* = 55\,A_6 + 34O_6,$$

$$O_6^* = 34\,A_6 + 21O_6.$$

It is equivalent to

$$(\tau^{-1}A_6^* + \tau^{-2}O_6^*) = \tau^9(\tau^{-1}A_6 + \tau^{-2}O_6)$$

corresponding to the fact that their relative composition is golden mean $\tau = (1+\sqrt{5})/2 = 1.618$. The arrangement of A_6 and O_6 in A_6* and O_6* was discovered and the same procedure can be repeated as many times as desired. It is noted that the structure and then the procedure has some freedom. So-called projection model is contained in the present model. The author solved it as a puzzle and then he knows the feature of the structure very well. While the projection method is so useful that some users have only little knowledge about the structure. The ball and stick model of this structure is exhibited. The procedure can be expressed in another way. Every vertex is transformed into the centre of a flower dodecahedron, exactly speaking, which is a 60-hedron with icosahedral symmetry and consists of 20 A_6's.

[*B*] The concept of graphic geodesic line was introduced by the author and R. Collins to characterize any two-dimensional network with triangular meshes. All the geodesics were completely traced in the special case of triangulated Penrose tiling which was obtained by drawing all the minor diagonals in Penrose tiling of rhombic version. There are five sets of parallel lines, some circular roops and two hierarchical sets of rather complicated closed geodesics. This analysis figures out that Penrose tiling has rather strong fluctuations which cancel out in a small area.

There seems to appear a chaotic behaviour in the corresponding problem of octagonal Penrose tiling.

[*C*] Recently, the author found that the allocation scheme of the seats in an election in the proportional representation[4]. Though the time will not be enough to explain it, some copies of the concerned paper will be brought. It was written for physicists investigating quasicrystals or generallized crystallography The next publication in more geometrical description is now in preparation.

2. Fractal tessellation of a plane and a spherical surface. [5],[6]

Koch curve is a typical artificial fractal curve. When Koch curves are arranged in some symmetrical way, a fractal tessellation of a plane is obtained. There are five-fold case and six-fold case. By a similar way, some fractal tessellation of a spherical surface can be obtained. There are two cubic cases and an icosahedral case. The concept of similarity was extended in these cases since there is no similarity on a spherical surface that is not a linear space.

Peano curve, *covering* a cubic region, was extended to cover a spherical surface by a single curve as uniform as possible.

3. Ideal critical pattern [7].

You may regard a critical state as the state at the conceptual border of mixing and segrigating, though the concept is well defined in Physics. The motivation of this attempt is to combine intuitive sensibility and logic. The first aim is to realize some finite *ideal critical pattern* which can be the representative of the ensemble of all the critical configurations. The pattern should have both properties of mixing and segregating. One, finding some defects, tends to feel unsatisfactory in the pattern that he felt acceptable before.

[1] T. Ogawa; J. Phys. Soc. Jpn.54, 3205 (1985).
[2] T. Ogawa; Material Science Forum 22-24, 187.
[3] T. Ogawa and R. Collins; in Quasicrystals, Eds. T. Fujiwara and T. Ogawa,p.14, (1990, Springer).
[4] T. Ogawa and T. Ogawa; in Quasicrystals, Eds. K. H. Kuo and T. Ninomiya, p.394. (1991, World Scientific).
[5] T. Ogawa; in What are fractals? (in Japanese), (1989, Iwanami)
[6] T. Ogawa; Tokei Suuri, (Proceedings of the Institute of Statistical Mathematics, in Japanese) 37, 107 (1989),
[7] T. Ogawa, M. Himeno and T. Hirata; Forma, 6, 129 (1991).