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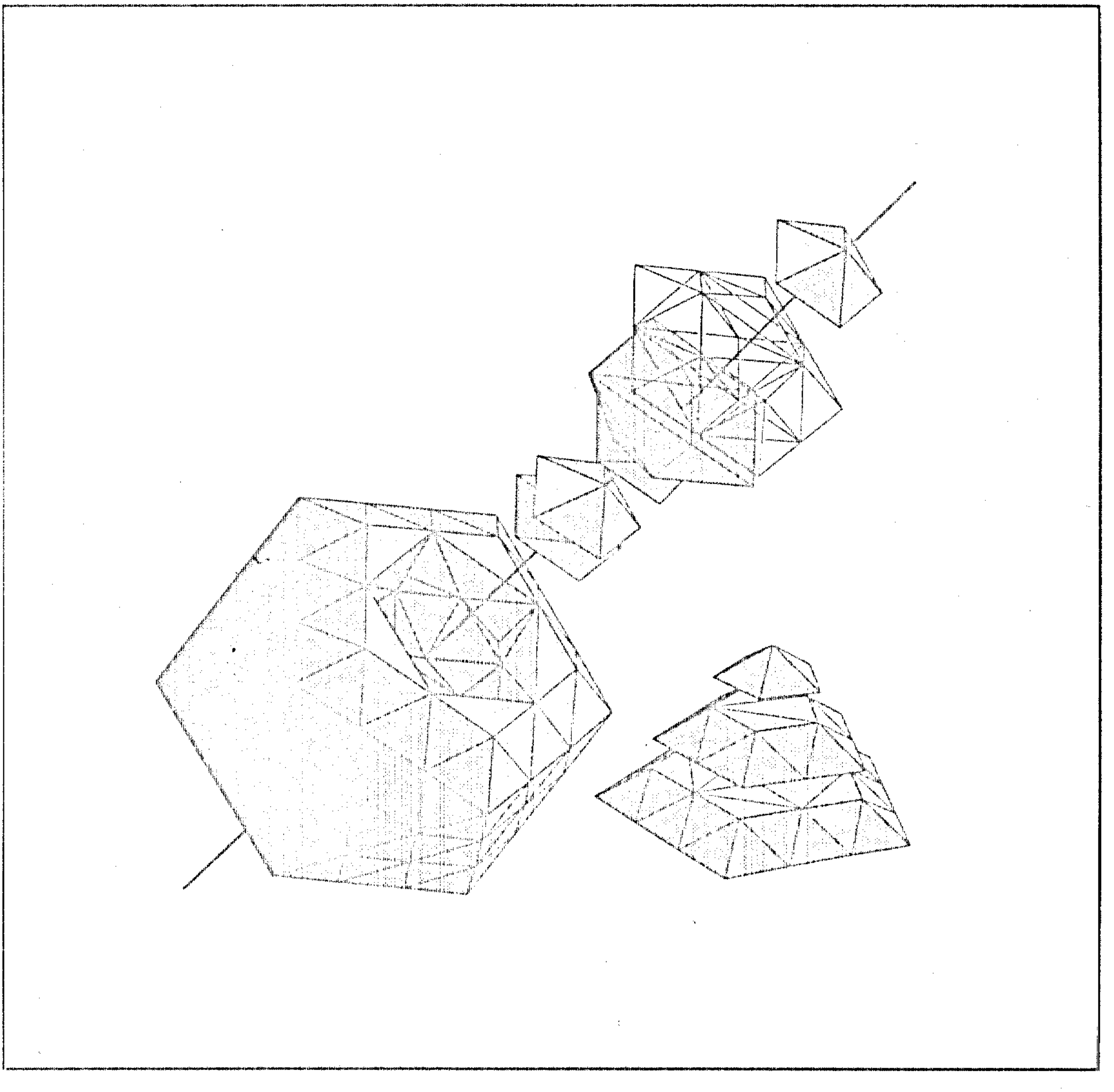

WALLPAPERS PRECISELY 17: AN EYE-OPENING CONFIRMATION

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Early geometry was constructive and visual; in time, geometry came to be expressed by abstract notation. Undoubtedly, the venerable Pythagoras beheld his extraordinary right triangle theorem rather than formulated it. Subsequent algebraic equations merely restate what was visually evident. As mainstream geometry increasingly became the province of the mathematician, graphic geometry, construction without calculation (in the "Platonic" tradition), became the province of the designer. These parallel paths can but benefit from interdisciplinary bridges such as the *Scientific American* column (1956-81) of Gardner and *The World of Mathematics* (1956) of Newman, whose aim was to "present mathematics as a tool, a language and a map; as a work of art and an end in itself."

A classic topic, attracting attention of both mathematician and designer, is that of the 17 *infinitely repeating patterns with two independent translations*, commonly called the 17 *wallpapers*. Various derivations (generally deficient as proofs) routinely utilize geometric and algebraic formulations, including group theory. In the proliferation of mathematical and other scientific texts, with redundant iterations of the problem, certain deficiencies have been perpetuated. The devising of a more visual approach to this highly visual problem can not only make the wallpaper lore more accessible to designers, but perhaps provide new insight into its rudiments and insinuate a proof.

The 17 wallpapers were included in a basic design curriculum, instituted in 1960 by William Huff; a visual approach to their derivation (and proof)—to facilitate the instruction of design students—was explored in 1961 with mathematician Richard Durstine, who proposed a strategy that was not fully developed at the time. In 1990, Jack Holnbeck and I elected to work on different aspects of the 17 wallpapers as theses topics. A principal task of mine has been to pick up the threads of the Durstine proposal—while Holnbeck has scoured innumerable texts for their treatments of this persisting theme. As I worked out the Durstine strategy, a cumbersomely large chart resulted. In consultation with Dénes Nagy (1992), a somewhat different strategy was devised, based on preliminary work of Nagy's student, S. Prakash (1990). See TABLE.

A basic condition of the 17 wallpapers is that pattern coverage must be achievable through translation alone, along two different vectors. A second (symmetry defining) condition is that there is pattern coverage (pattern invariance), when *any* operation, inherent in any patterns, is effected. Prevailing literature implies a third condition: that upon the operation of any element of symmetry, inherent in the pattern, the lattice, as well as pattern, is left invariant. Weyl's *Symmetry* (1952) gives this impression: "Having found the 10 possible groups Γ of rotations and the Lattices L *left invariant* by each of them, one has to paste together a Γ with a corresponding L so as to obtain the full group of congruent mappings. . . While there are 10 possibilities for Γ, *there are exactly 17 essentially different possibilities for the full group of congruences* Δ [*italics* mine]."

Lockwood and Macmillan's *Geometric Symmetry* (1978) is the rare text, in apprising that *it cannot be said that the* [*lattice*] *is unchanged by a glide*. These authors, in their cautious, negatory statement, stop short of nailing it down for the outsider. Shubnikov and Koptsik, writing in *Symmetry in Science and Art* (1974) about 3-D space groups, bring to light Fedorov's recognition of *symmorphic* and *nonsymmorphic* groups, the latter, characterized by the presence of *glide* or *screw* symmetry. Wallpapers (2-D) cannot accommodate screw operations (3-D); they can and do incorporate glide.

Failing to pick up on Fedorov, the majority of texts either gloss over the thorny problem of the special nature of the glide operations by leaving the impression that all 17 patterns are achieved without event or they err in stating outright that all operations of a given pattern effect lattice coverage along with pattern coverage. To fail to instruct students about how the nonsymmorphic groups function in the 17 wallpapers is to fail to instruct them thoroughly about the analog that crystallographers have used to explain the 3-D space groups, not easily envisioned in the mind's eye.

A designer's simple device was concocted for the investigation of the concurrence of lattice coverage with pattern coverage. With duplicate copies of each pattern, one opaque, one transparent, any sort of move of the transparency over the opaque copy, which brings the pattern into coverage, effects a proper operation (translations, rotations), inherent in the pattern. It is discerned that both pattern coverage and lattice coverage occur in all instances. Improper operations (mirror reflections, glides) are simulated by flipping the acetate. It is revealed that in four cases involving glide (but not all) lattice coverage does not concur with pattern coverage. This investigation also challenges the crystallographer's dubious *centered rectangular lattice* (Buerger, et al.).

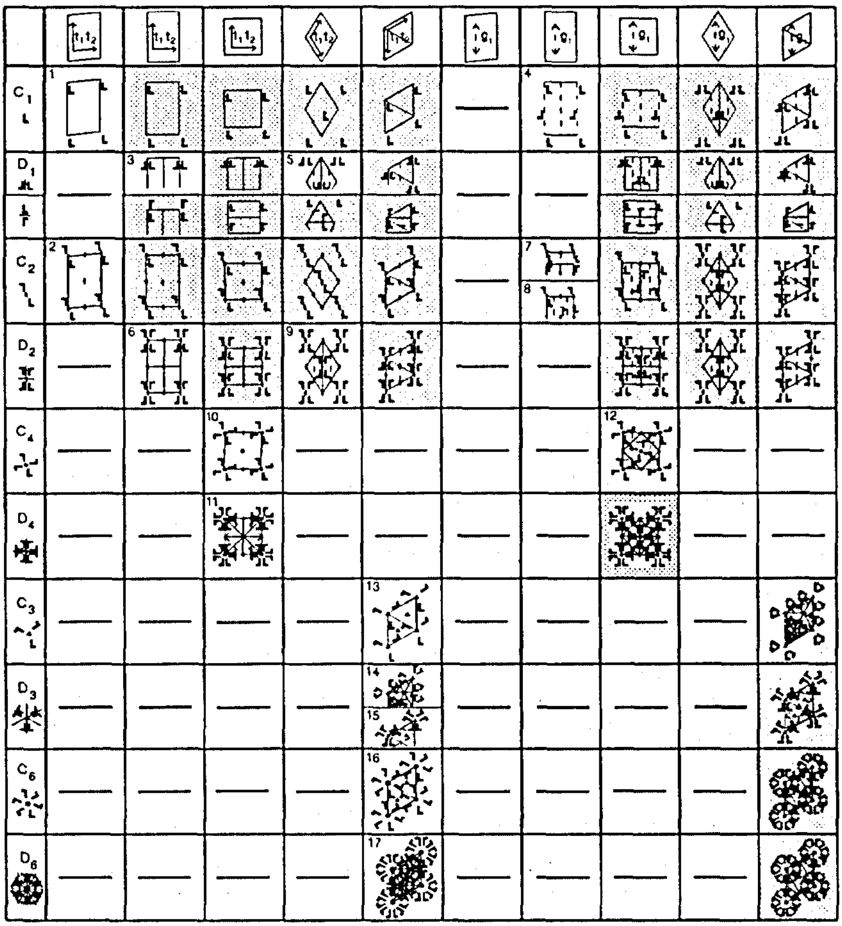

TABLE: The five lattices undergoing symmorphic and nonsymmorphic operations.