

# Symmetry: Culture and Science

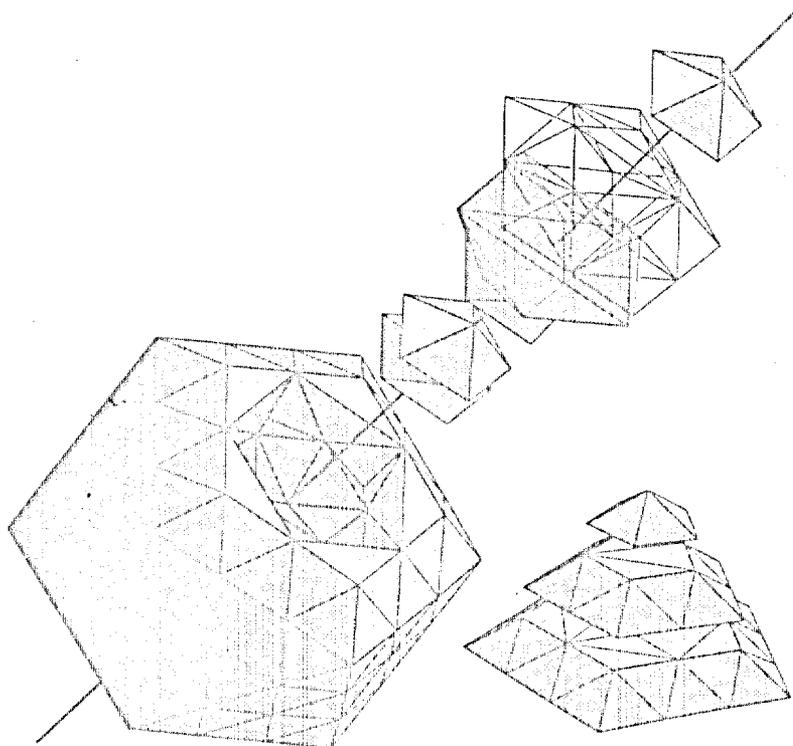
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EXPERIMENTAL STUDY OF WAVES IN FIBONACCI  
 AND PENROSE LATTICES

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Recently, wave propagation in modulated structure has been receiving considerable attention. In particular, the wave propagation in quasiperiodic systems is of great importance, since the phenomena have close relation to the physical properties of real quasicrystals. The essence of the phenomena is lying in the interference between multiply scattered waves in the modulated structure. In order to observe these effects experimentally, it is inevitable to preserve the phase coherency during the multiple scattering. In this sense the sound waves with little attenuation are favorable for the study of these phenomena. Third sound is a wave propagating in superfluid helium films, of which the attenuation is small. Since it propagates in the adsorbed helium film, the modulation of the solid surface can be used as the modulated structure in which third sound propagates. Recently, we have developed an experimental method to study the transmission spectra of third sound in one-dimensional lattices, which are discrete variations of the modulated structure. The modulation of the surface was done with aluminum strips (width  $d \sim 80\mu\text{m}$ ). Periodic [1], Fibonacci [2], Thue-Morse [3], and random [4] lattices were studied so far.

The Fibonacci sequence is generated by the following recurrence formula:

$$S_{n+1} = \{S_{n-1}, S_n\}, \text{ with } S_1 = \{B\} \text{ and } S_0 = \{A\}.$$

For example,  $S_2 = \{AB\}$ ,  $S_3 = \{BAB\}$ , and  $S_4 = \{ABBAB\}$ . We have fabricated the Fibonacci lattice by mapping A to an aluminum strip on a glass substrate, whereas B to the bare glass surface of the same width. The Fibonacci lattice is a prototype of the *one-dimensional* quasiperiodic lattice. The Penrose lattice can be produced by putting the aluminum pads on a glass surface according to the rule which generates Penrose tiling. The Penrose lattice is a prototype of the *two-dimensional* quasiperiodic lattice.

Figure 1 shows the transmission spectrum of third sound in the Fibonacci lattice, where  $k$  is the wave number of third sound. Two large transmission gaps, where the transmitting wave power is small, are observed. These transmission gaps locate at  $kd \sim 1/\tau^2$  and  $kd \sim 1/\tau$ , where  $\tau$  is golden ratio ( $\tau = (1 + \sqrt{5})/2$ ). The bands, where the wave is transmitted fairly well, are eroded away further by smaller gaps in a nested way. Figure 2 shows the spectrum of third sound in the Penrose lattice. The inset shows how the aluminum pads were distributed on the Penrose tiling. The length of the basis vector is expressed by  $a$ . The structure in the spectrum is weaker than the Fibonacci case. The similarity is noticed, however, between the spectrum in the Fibonacci lattice and that in the Penrose lattice. The spectra in the other directions have to be measured, since the lattice is two-dimensional.

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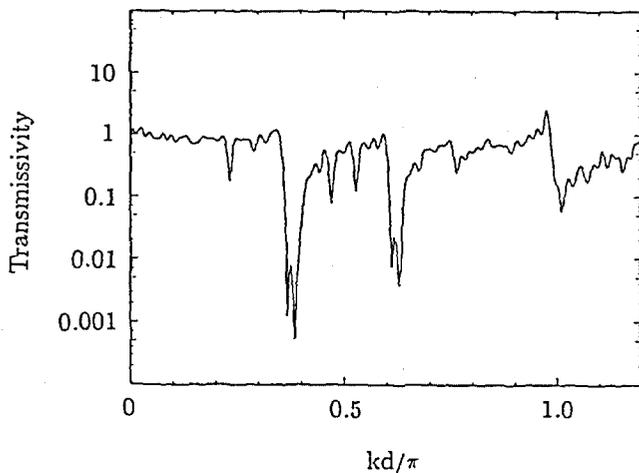


Figure 1: Transmission spectrum of third sound in the Fibonacci lattice.

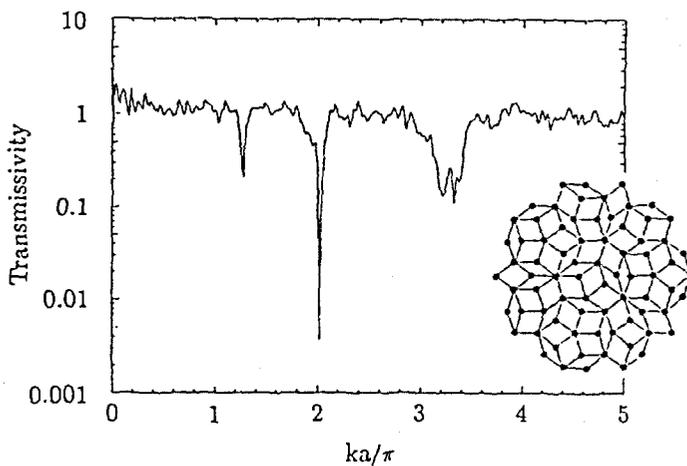


Figure 2: Transmission spectrum of third sound in the Penrose lattice.